PART - A (1-20)

1. If X follows a binomial distribution with parameters n = 100 and $p = \frac{1}{3}$, then P(X = r) is maximum when r is equal to

(A) 16

(B) 32

(C) 33

(D) None of these

2. The system of equations

$$x + 2y + \lambda z = \mu$$
$$x + y + z = 6$$
$$x + 2y + 3z = 10$$

(A) has no solution for $\lambda = 3$ $\mu = 10$

(B) has infinite number of solution for any value of λ, μ

(C) has unique solution for $\lambda \neq 3$ $\mu = 2$

(D) has unique solution for $\lambda = 3$ $\mu = 3$

3. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

(A) $(x+y-2) = k(x-y)^3$

(B) $(x+y-2) = k(x-y)^{3/2}$

(C) $\sqrt{(x+y-2)} = k(x-y)^2$

(D) $(x+y-2) = k(x-y)^2$

4. Let X, Y, Z be i.i.d. U(0, 1) variate . If V = max {X, Y, Z} and W = min {X, Y, Z} then find the value of

$P\left\{1/3 < W \text{ and } V \le \frac{1}{2}\right\}$

(A) $\frac{1}{215}$

(B) $\frac{1}{261}$

(C) $\frac{1}{216}$

(D) $\frac{1}{217}$

5. The area of region $[(x,y): x^2 + y^2 \le 1 \le x + y]$ is

(A) $\frac{\pi^2}{5}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{3}$

(D) $\frac{\pi}{4} - \frac{1}{2}$

6. Let $\{X_n\}$ be a sequence of independent random variables such that $P\{X_K = \pm C_k\} = \frac{1}{2}, K \ge 1$, $C_K > 0$ where $C_K = K^{\alpha}$ then find the value of α for which the central limit theorem holds for sequence $\{X_n\}$.

(A) $\alpha \ge \frac{1}{2}$

(B) $\alpha \le \frac{1}{2}$

(C) $\alpha \ge \frac{-1}{2}$

(D) $\alpha \le \frac{-1}{2}$

7. Let X and Y be independent random variable with common distribution function F and density function f. Given that V = max {X, Y} has distribution function $F_V(v) = F(v^2)$ and density function $f_V(v) = 2f(v)F(v)$ then find E(V) and Var (V).

(A) $\frac{3}{2}, \frac{5}{2}$

(B) $\frac{5}{2}, \frac{3}{4}$

(C) $\frac{3}{4}, \frac{5}{4}$

(D) $\frac{3}{2}, \frac{5}{4}$

8. Let $\{X_n, n \ge 1\}$ be a sequence of independent Bernoulli random variables with mean $p \in (0,1)$. We construct an estimate $\hat{p}_n$ of p from $\{X_1, \ldots, X_n\}$. We know that $p \in (0.4, 0.6)$. Find the smallest value of n so that

$$\left(P\frac{|\hat{p}_n - p|}{p} \le 5\%\right) \ge 95\% .$$

(A) 2500

(B) 2300

(C) 2400

(D) 2000

9. Let X be uniformly distributed in [0, 2π] and Y = sin (X). Calculate the p.d.f. f_Y of Y.

(A) $\frac{1}{\sqrt{1-y^2}}$

(B) $\frac{1}{\pi\sqrt{1-y^2}}$

(C) $\frac{1}{\pi\sqrt{1-y}}$

(D) $\frac{1}{\pi\sqrt{1+y}}$

10. If X is chi-square variate with n.d.f then for large n, the distribution of is

(A) $N\left(\sqrt{2n},1\right)$

(B) $N(2n,1)$

(C) N(n,1)

(D) $N\left(\sqrt{\frac{2}{3}}1\right)$

11. Let X_1, X_2 ......,X_n be a random sample from the N(0, σ^2) distribution then Find a UMP test of size α for H_0 : $\sigma = \sigma_0$ versus $H\alpha$: $\alpha > \alpha_0$.

(A) $\sum x_i^2 \geq \sigma_0^2 \chi_\alpha^2(n)$

(B) $\sum x_i^2 \geq \chi_\alpha^2(n)$

(C) $\overline{X} \geq \sigma_0^2 \chi_\alpha^2(n)$

(D) $\sum X_i^2 \geq n^2 \sigma_0^2 \chi_\alpha^2(n)$

12. A sample of size n is drawn from each of the four normal populations which has the same variance σ^2 . The mean of the four populations are a + b + c, a + b - c, a - b + c and a - b - c then find the M.L.E for b is

(A) $\frac{\overline{x}_1+\overline{x}_2+\overline{x}_3+\overline{x}_4}{4}$

(B) $\frac{\overline{x}_1-\overline{x}_2+\overline{x}_3-\overline{x}_4}{4}$

(C) $\frac{\bar{x}_1 + \bar{x}_2 - \bar{x}_3 + \bar{x}_4}{4}$

(D) $\frac{\bar{x}_1 + \bar{x}_2 - \bar{x}_3 - \bar{x}_4}{4}$

13. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(A) $\alpha = \pm\frac{1}{\sqrt{2}}$

(B) $\beta = \pm\frac{1}{\sqrt{6}}$

(C) $\gamma = \pm\frac{1}{\sqrt{3}}$

(D) all of these

14. An arbitrary vector X is an eigen vector of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}, \text{ if } (a, b) = ...$$

(A) (0, 0)

(B) (1, 1)

(C) (0, 1)

(D) (1, 2)

15. If $a > 0$ $x_1 > 0$ and $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right)$

Then sequence $< x_n >$ is

(A) monotonic and divergent

(B) convergent, non monotonic

(C) divergent, non monotonic

(D) monotonic and convergent

16. If A and B are two independent events such that P(A) = $\frac{1}{2}$, P(B) = $\frac{1}{5}$, then

(A) $P\left(\frac{A}{B}\right) = \frac{1}{2}$

(B) $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$

(C) $P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$

(D) All of the above

17. A coin is tossed 2n times. The chance that the number of times one gets head is not equal to the number of times one gets tail is

(A) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$

(B) $1 - \frac{(2n!)}{(n!)^2}$

(C) $1 - \frac{(2n!)}{(n!)^2} \cdot \frac{1}{4^n}$

(D) None of these

18. Let $x_1, x_2, x_3, \ldots, x_n$ be the ranks of n individuals according to character A and $y_1, y_2, \ldots, y_n$ the ranks of the same individuals according to other character B such that $x_i + y_i = n + 1$ for i = 1, 2, 3, ..., n. Then the coefficient of rank correlation between the characters A and B is

(A) 1

(B) 0

(C) – 1

(D) None of these

19. The differential equation of the family of straight lines whose slope is equal to y-intercept is

(A) $(x + 1)\frac{dy}{dx} - y = 0$

(B) $(x + 1)\frac{dy}{dx} + y = 0$

(C) $\frac{dy}{dx} = \frac{x-1}{y-1}$

(D) $\frac{dy}{dx} = \frac{x+1}{y+1}$

20. The part of circle $x^2 + y^2 = 9$ in between y = 0 and y = 2 is revolved about y-axis. The volume of generating solid will be

(A) $\frac{46}{3}\pi$

(B) 12π

(C) 16π

(D) 28π

PART - B (21-40)

21. The area of the loop of the curve $y^2 = x^2(1 - x)$ is

(A) 15/8

(B) 4/15

(C) 8/15

(D) None of these

22. If $f(x) = \begin{cases} e^x; & x \le 0 \\ |1-x|; & x > 0 \end{cases}$, then

(A) f(x) is differentiable at x = 0

(B) f(x) is continuous at x = 0,1

(C) f(x) is differentiable at x = 1

(D) None of these

23. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that min f(x) > max g(x), then the relation between b and c is

(A) No real value of b and c

(B) $0 < c < b\sqrt{2}$

(C) $|c| < |b|\sqrt{2}$

(D) $|c| > |b|\sqrt{2}$

24. A special dice with (n + 1) faces is marked in its faces the numbers 0,

$$\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots\ldots, \frac{n-1}{n} \text{ and } \frac{n}{n}$$

The dice is unbiased. For the distribution of random variable corresponding to the number on the uppermost face, standard deviation is

(A) $\sqrt{\frac{n+2}{12n}}$

(B) $\sqrt{\frac{12n}{n+2}}$

(C) $\sqrt{\frac{n+3}{12n}}$

(D) $\sqrt{\frac{12n}{n+3}}$

25. If skulls are classified as A, B, C according as the length, breadth index is under 75, between 75 and 80 or over 80, find approximately (assuming that the distribution is normal) the mean of a series in which A are 58%, B are 38% and C are 4% being given that if

$$f(t) = \frac{1}{\sqrt{(2\pi)}}\int_0^t \exp\left(-t^2\right)dt \text{, then } f(.2) = 0.08 \text{ and } f(1.75) = 0.46.$$

(A) m = 74.4, σ = 2.3

(B) m = 3.2 σ = 3.2

(C) m = 74.4, σ = 3.2

(D) m = 64.4, σ = 2.3

26. If given that $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ then

(A) $A^3 + (a^3 + b^3 + c^3)A = 0$

(B) $A^3 + (a^2 + b^2 + c^2)A = 0$

(C) $A^3 + (a^2 - b^2 - c^2)A = 0$

(D) $A^2 + (a^3 + b^3 + c^3)A = 0$

27. The characteristic equation $\begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ of matrix A is

(A) $(5 - \lambda)(3 - \lambda)(1 - \lambda) = 0$

(B) $(1 - \lambda)(3 - \lambda)^2 (5 - \lambda) = 0$

(C) $(1 - \lambda)^2 (3 - \lambda)(5 - \lambda)^2 = 0$

(D) $(1 - \lambda)(3 - \lambda)(5 - \lambda)^2 = 0$

28. Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then $(aI + bC)^4$ equals

(A) $a^4I + a^3bC$

(B) $a^4I + 2a^3bC$

(C) $a^4I + 3a^3bC$

(D) $a^4I + 4a^3bC$

29. Let A be a skew symmetric matrix then A^m is, where m is even.

(A) Symmetric matrix

(B) skew-symmetric matrix

(C) orthogonal matrix

(D) Hermitian matrix.

30. The number of value of K for which the system of equation

$(K + 1)x + 8y = 4k$

$Kx + (k + 3)y = 3k - 1$

has infinitely many solutions.

(A) 0

(B) 1

(C) 2

(D) ∞

31. If A is a 5 × 5 symmetric matrix over F then find the dimension of matrix A

(A) 10

(B) 15

(C) 20

(D) 25

32. If $A = (a_{ij})_{6 \times 6}$ be a matrix where $a_{ij} \in c$ and $a_{ij} = -a_{ji}$ over the field R then dimension of the matrix is

(A) 36

(B) 34

(C) 32

(D) 30

33. The five observation of the diameter of a sphere was recorded by a scientist as 6.33, 6.37, 6.36, 6.32 and 6.37 cm.

then estimate the mean and variance of the population respectively.

(A) 6.35, 5×10^{-4}

(B) 6.35, 5×10^{-2}

(C) 6.38, 5×10^{-2}

(D) 6.35, 5×10^{-4}

34. Consider a random sample of size 4 which is drawn from a normal population with unknown mean μ and variance σ^2. then among the estimators which are given below best estimator is

$$t_1 = \frac{x_1 + x_2 + x_3 + x_4}{4}, t_2 = x_1 + x_2 + \frac{x_3}{3}$$

(A) t_1

(B) t_2

(C) both t_1 and t_2

(D) neither t_1 nor t_2

35. If X and Y are standard normal variant with Coefficient of correlation ρ, then find the distribution of the statistic

$$Q = \frac{X^2 - 2\rho XY + Y^2}{(1-\rho^2)}.$$

(A) $\chi^2_{(1)}$

(B) $\chi^2_{(n)}$

(C) $\chi^2_{(2)}$

(D) N(0, 1)

36. Let X_1, X_2, X_3 inches be the excesses of the height of father, mother and son respectively above their mean values. A distribution of these variables gave the following approximate correlations and standard deviations.

$r_{12} = 0.28$, $r_{23} = 0.49$, $r_{31} = 0.51$

$\sigma_1 = 2.7$ $\sigma_2 = 2.4$, $\sigma_3 = 2.7$

Find out the regression of X_3 on X_1 and X_2.

(A) $X_3 = 0.42 X_1 + 0.40 X_2$

(B) $X_3 = 0.40 X_1 + 0.42 X_2$

(C) $X_3 = 0.42 X_1 + 0.41 X_2$

(D) $X_3 = 0.40 X_1 + 0.41 X_2$

37. A study has been made to compound the nicotine contents of two brands of cigarettes. Ten cigarettes of Brand A had an average nicotine content of 3·1 milligrams with a standard deviation of 0.5 milligram while eight cigarettes of Brand B had an average nicotine content of 2·7 milligrams with a standard deviation of 0·7 milligram.

Assuming that the two sets of data are independent random samples from normal populations with equal variance, construct a 95% confidence interval for the difference b/w the mean nicotine contents of the two brands of cigarettes.

(A) (–0·01, 1·00)

(B) (–0·20, 1·00)

(C) (–0·20, 0.20)

(D) (–1·00, 0·20)

38. The odd order moment about mean of the standard logistic distribution are

(A) greater than zero

(B) less than zero

(C) always zero

(D) does not exists

39. Let X and Y have bivariate normal distribution with parameters $\mu_X = 5$, $\mu_Y = 10$ $\sigma_x^2 = 1, \sigma_y^2 = 25$ and corr(X, Y) = ρ. If ρ > 0, then find ρ when p(4 < Y < 16/X 5) = 0.954

(A) ρ = 0.4

(B) ρ = 0.9

(C) ρ = 0.6

(D) ρ = 0.8 (∵ ρ > 0)

40. If T_1, T_2, T_3 are independent, unbiased estimates of θ and all have the same variance, which of the following unbiased estimates of θ would you prefer?

$(T_1 + 2T_2 + T_3)/4$, $(2T_1 + T_2 + 2T_3)/5$, $(T_1 + T_2 + T_3)/3$.

(A) $\frac{T_1 + 2T_2 + T_3}{4}$

(B) $\frac{2T_1 + T_2 + 2T_3}{5}$

(C) $\frac{T_1 + T_2 + T_3}{3}$

(D) $\frac{2T_1 + T_2 + 2T_3}{3}$

PART-C(41-50)

41. If random variable X, Y have joint pdf

$$f(x,y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

then find the conditional variance of X given $Y = \frac{1}{2}$

42. Variable X and Y have the joint pdf. given by :

$$f(x,y) = \frac{1}{3}(x+y); 0 \le x \le 1, 0 \le y \le 2.$$

Find (i) r(X, Y) (ii) The two lines of regression, and

(iii) The two regression curves for the means.

43. Obtain regression equation of Y on X for the following distribution :

$$f(x,y) = \frac{y}{(1+x)^4} \exp\left(-\frac{y}{1+x}\right); x, y \ge 0$$

44. If A is Hermitian matrix such that $A^2 = 0$, then show that A = 0 where 0 is the zero matrix.

45. Give an example of a matrix which is skew-symmetric but not skew-Hermitian.

46. Show that $\frac{X+1}{n+2}$ is a biased estimator of the binomial parameter θ. Is this estimator asymptotically unbiased?

47. If $X_1, X_2, ., X_n$ constitute a random sample from a population with the mean μ, what condition must be imposed on the constants $a_1, a_2, \ldots, a_n$ so that

$$a_1X_1 + a_2X_2 + ... + a_nX_n$$

is an unbiased estimator of μ ?

48. A random sample of size 10 was drawn from a normal population with an unknown mean and a variance of 44.1 inch^2.

Consider the observations 67, 71, 80, 76, 78, 82, 68, 72, 65 and 81 then obtain 95% confidence interval for the population mean.

49. Each of the following sets of observations is a random sample from a normal population.

Sets	Observation						
I	249,	242,	247,	250,	252		
II	251,	256,	255,	258			
III	266,	261,	265,	264			
IV	262,	260	263,	262,	261	264,	262

Test whether the population means are equal (Assume that the population standard deviations are same.)

50. Let $(X_1, X_2, \ldots, X_n)$ be jointly normal with $E X_i = 0$, $EX_i^2 = 1$ for all i, and cov $(X_i, X_j) = \rho$ if $|j - i| = 1$, and = 0 other wise. Then $S_n = \sum_{k=1}^{n} X_k$, is $N(0, \sigma^2)$, where

Show that $\sigma^2 = Var(S_n) = n + 2(n - 1)\rho$,

ANSWER KEY

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	C	C	A	C	D	C	D	C	B	A	A	D	D	B	D
Question	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	D	C	C	A	A	C	B	D	A	C	B	D	D	A	B
Question	31	32	33	34	35	36	37	38	39	40					
Answer	B	D	A	A	C	B	B	C	D	C					

HINTS AND SOLUTIONS

1.(C) Since , (n + 1)p = $\frac{101}{3}$ is not an integer,

therefore , P(X = r) is a maximum when r = $\left[\frac{101}{3}\right] = 33$

2.(C) The augmented matrix

$$[A \ : \ B] = \sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 6 \\ 1 & 2 & 3 & \vdots & 10 \\ 1 & 2 & \lambda & \vdots & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 6 \\ 0 & 1 & 2 & \vdots & 4 \\ 0 & 1 & \lambda - 1 & \vdots & \mu - 6 \end{bmatrix}$$

by $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & \vdots & 6 \\ 0 & 1 & 2 & \vdots & 4 \\ 0 & 0 & \lambda - 3 & \vdots & \mu - 10 \end{bmatrix} \text{by } R_3 \to R_3 - R_2.$$

Rank [A : B] = 3 = rank A if $\lambda \neq 3$

Rank A = Number of unknowns (system posses a unique solution.) for any value of μ.

Rank [A : B] = 3 and rank A = 2 if l = 3 and $\mu \neq 10$. System of equations is inconsistent (No solution).

Rank [A : B] = rank A system of equations is consistent. rank A < the number of unknowns. Then infinite number of sol.

3.(A) The given diff. equation is $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ ... (i)

Putting x + X + h and y = Y + k ... (ii)

the equation (i) reduces to $\frac{dY}{dX} = \frac{(X+h)+2(Y+k)-3}{(X+h)+(Y+k)-3}$

or $\frac{dY}{dX} = \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$ ... (iii)

Choose h and k such that

h + 2k – 3 = 0 and 2h + k – 3 = 0 ... (iv)

Solving the equations (iv) we have h = 1 = k,

$\therefore$ From (ii), x = X + 1 and y = Y + 1

or X = x – 1 and Y = y – 1 ... (v)

Also (iii) reduces to $\frac{dY}{dX} = \frac{X+2Y}{2X+Y}$

Putting Y = vX, $v + X\frac{dv}{dX} = \frac{X+2vX}{2X+cX}$ or $v + X\frac{dv}{dX} = \frac{1+2v}{2+v}$

or $X\frac{dv}{dX} = \frac{1+2v}{2+v} - v = \frac{1+2v-2v-v^2}{2+v} = \frac{1-v^2}{2+v}$

or $\frac{2+v}{1-v^2}dv = \frac{dX}{X}$ or $\left[\frac{1}{2}\cdot\frac{1}{1+v} + \frac{3}{2}\cdot\frac{1}{1-v}\right]dv = \frac{dX}{X}$,

resolving into partial fractions.

Integrating $\frac{1}{2}\log(1+v) - \frac{3}{2}\log(1-v) = \log X + \log C$, where C is an arbitrary constant.

Or $\log\left[\frac{(1+v)}{(1-v)^3}\right] = 2\log(CX) = \log(CX)^2$

or $(1+v)/(1-v)^3 = (CX)^2$

or $\frac{(1+Y/X)}{(1-Y/X)^3} = C^2X^2$ or $\frac{X+Y}{(X-Y)^3} = C^2$

or $\frac{(x-1)+(y-1)}{\{(x-1)-(y-1)\}^3} = C^2$ from (v) or $\frac{x+y-2}{(x-y)^3} = C^2$

or $(x + y – 2) = C^2(x – y)^3$.

4.(C) $\because \left\{\frac{1}{3} < W, V \le \frac{1}{2}\right\}$

$= \left\{\frac{1}{3} < X \le \frac{1}{2}, \frac{1}{3} < Y \le \frac{1}{2}, \frac{1}{3} < Z \le \frac{1}{2}\right\}$

then

$$P\left\{\frac{1}{3} < W < V \le \frac{1}{2}\right\} = P\left(\frac{1}{3} < X \le \frac{1}{2}\right)^3$$

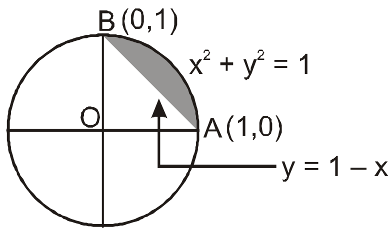
$$= \left[\int_{1/3}^{1/2} dx\right]^3$$

$$= \left(\frac{1}{6}\right)^3$$

$$= \frac{1}{216}$$

5.(D) $x^2 + y^2 = 1$, $x + y = 1$ meet when

$X^2 + (1 – x)^2 = 1 \Rightarrow x^2 + 1 + x^2 – 2x = 1$



$\Rightarrow 2x^2 - 2x = 0 \Rightarrow 2x(x-1) = 0$

$\Rightarrow x = 0, x = 1 \Rightarrow y = 1, y = 0$, i.e., A(1,0); B(0, 1)

Required area = $\int_0^1 [\sqrt{1-x^2} - (1-x)]\,dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - 1 + \frac{1}{2} = \frac{\pi}{4} - \frac{1}{2}$$

6.(C) Here E(X) = 0, $V(X_k) = c_k^2, \sigma_k = c_k, s_n = \sqrt{\sum c_k^2}$

$$E|X_k|^{2+\delta} = c_k^{2+\delta}$$

CLT holds for {x} if

$$\frac{\sum E|X_k|^{2+\delta}}{s_n^{2+\delta}} = \frac{\left(\sum c_k^{2+\delta}\right)}{\left(\sum c_k^2\right)^{\frac{2+\delta}{2}}} \to 0 \text{ as} \to \infty$$

Let $m_n = \max_{1\le k\le n} c_k \Rightarrow m_n \ge c_k$ for all $k \le n$

Hence $\sum c_k^{2+\delta} \le m_n^{\delta} \sum c_k^2$

so that

$$\frac{\sum E|X_k|^{2+\delta}}{s_n^{2+\delta}} \le (m_n / s_n)^{\delta}$$

Hence CLT holds for $\{X_n\}$ if $(m_n / s_n)^{\delta} \to 0$ as $n \to \infty$

Let us allow $c_k > k^{\alpha}$, $\alpha > 0$. There is a result in algebra that

$\sum_{k=1}^{n} k^{\alpha}$ behaves like $n^{\alpha+1}$ for large n

Hence , $m_n = \max_{1 \le k \le n} c_k = \max_{k \le n} k^{\alpha} = n^{\alpha}$

$$s_n^2 = \sum_{k=1}^{n} k^{2\alpha} \sim n^{2\alpha+1}$$

$$s_n \sim n^{\alpha+\frac{1}{2}}$$

$$\therefore \quad (m_n / s_n) \sim \frac{n^{\alpha}}{n^{\alpha+\frac{1}{2}}} \sim n^{\frac{1}{2}} \to 0 \text{ as } n \to 0$$

Hence CLT for such sequence whenever $\alpha > 0$. Now let us see what happens when $\alpha \le 0$.

In this case,

$$m_n = \max_{k \le n} k^{\alpha} = 1$$

$$\therefore \quad (m_n / s_n) \sim \frac{n^{\alpha}}{n^{\alpha+\frac{1}{2}}} \to 0 \text{ only if } \alpha > \frac{-1}{2}$$

When $\alpha = -\frac{1}{2}$ note that $\{X_n\}$ automatically become a sequence of uniformly bounded random variable with

$s_n = \sum_{k=1}^{n} k^{-1} \to \infty \text{ as } n \to \infty$. Hence CLT holds for $\{X_n\}$. Combining all these results, we have,

CLT holds for $\{X_n\}$ whenever $\alpha \ge -\frac{1}{2}, c_k = k^{\alpha}$.

7.(D) Since a, $f_V(v) = 2f(v)\,F(v) = 2e^{-v}(1 - e^{-v})$

$$E[V] = \int_0^{\infty} v f_V(v)\,dv = \int_0^{\infty} 2v\left(e^{-v} - e^{-2v}\right)dv$$

$$= 2\int_0^{\infty} v e^{-v}\,dv - \int_0^{\infty} v\left(2e^{-2v}\right)dv = 2 \times 1 - \frac{1}{2} = \frac{3}{2}$$

$$E\left[V^2\right] = \int_0^{\infty} v^2 f_V(v)\,dv = \int_0^{\infty} 2v^2\left(e^{-v} - e^{-2v}\right)dv$$

$$= \int_0^{\infty} 2v^2 e^{-v}\,dv - \int_0^{\infty} 2v^2 e^{-2v}\,dv$$

$$=\left\{\left[-2v^2e^{-v}\right]_0^{\infty}+\int_0^{\infty}4e^{-v}dv\right\}-\left(-v^2e^{-2v}+\int_0^{\infty}2ve^{-2v}dv\right)$$

$$=\left(0-4ve^{-v}\right]_0^{\infty}+\int_0^{\infty}4e^{-v}dv\Big)-\left(0-ve^{-2v}\right]_0^{\infty}+\int_0^{\infty}e^{-2v}dv\Big)$$

$$=\left(0-4e^{-v}\right]_0^{\infty}\Big)-\left(0-\frac{1}{2}e^{-2v}\right]_0^{\infty}\Big)=4-\frac{1}{2}=\frac{7}{2}$$

Var [V] = E[V]^2 - E[V]^2 = . $\frac{7}{2}-\left(\frac{3}{2}\right)^2=\frac{5}{4}$

8.(C) For Bernoulli Random variable we have E[X_n] = p and var [X_n] = p(1 - p). Let $\hat{p}=(X_1+......+X_n)/n$. We know that $\hat{p}_n\to asp$. Morever, from the CLT,

$$\sqrt{n}\frac{\hat{p}_n-p}{\sqrt{p(1-p)}}\to_D N(0,1) \qquad .$$

Now,

$$.\ =\frac{|\hat{p}_n-p|}{p}\le 0.05\Leftrightarrow\left|\sqrt{n}\frac{\hat{p}_n-p}{\sqrt{p(1-p)}}\right|\le 0.05\frac{\sqrt{np}}{\sqrt{1-p}}$$

Hence, for n large,

$$=P\left(\frac{|\hat{p}_n-p|}{p}\le 0.05\right)\approx P\left(N|0,1|\right)\le\frac{0.05\times\sqrt{np}}{\sqrt{1-p}}\Big)$$

we find $P\left(|N(0,1)|\le 2\right)\ge 0.95$. Hence $P\left(\frac{|\hat{p}_n-p|}{p}\le 0.05\right)$ if

$$\frac{0.05\times\sqrt{np}}{\sqrt{1-p}}\ge 2$$

i.e.,

$$n\ge 1600\frac{1-p}{p}=:n_0 .$$

Since we know that $p\in[0.4,0.6]$ the above condition implies $1067\le n_0\le 2400$. Hence the lowest value of n required to ensure a 95% accuracy is 2400.

9.(B) Since Y = g(X), we know that

$$f_y(y) = \sum \frac{1}{|g'(x_n)|} f_X(x_n)$$

where the sum is over all the x_n such that $g(x_n) = y$

For each $y \in (-1,1)$, there are two values of x_n in $[0, 2\pi]$ such that $g(x_n) = \sin(x_n) = y$.

For those values, we find t hat

$$|g'(x_n)| = |\cos(x_n)| = \sqrt{1-\sin^2(x_n)} = \sqrt{1-y^2}$$

and $f_X(x_n) = \frac{1}{2\pi}$

Hence,

$$. \; f_Y(y) = 2\frac{1}{\sqrt{1-y^2}}\frac{1}{2\pi} = \frac{1}{\pi\sqrt{1-y^2}}$$

10.(A) Since $X \sim \chi^2_{(n)}$, we have E(X) = n, Var (X) = 2n

$$\therefore \quad Z = \frac{X - E(X)}{\sigma_x} = \frac{X-n}{\sqrt{2n}} \sim N(0,1), \text{ for large n}$$

Consider, $P = \left(\frac{X-n}{\sqrt{2n}} \le z\right) = P\left(X \le n + z\sqrt{2n}\right) = P\left[\sqrt{2X} \le \left(2n + 2z\sqrt{2n}\right)^{1/2}\right]$

$$P = \left[\sqrt{2X} \le \sqrt{2n}\left(1 + z\sqrt{\frac{2}{n}}\right)^{1/2}\right] = \left[P\sqrt{2X} \le \sqrt{2n}\left(\frac{z}{\sqrt{2n}} - \frac{z^2}{4n} + ...\right)\right]$$

$$\approx P\left(\sqrt{2X} \le \sqrt{2n} + z\right), \text{ for large n.}$$

$$= P\left(\sqrt{2X} - \sqrt{2n} \le z\right), \text{ for large n.}$$

Since for large n, $(X-n)/\sqrt{2n} \sim N(0,1)$, from (*) we conclude that

$\sqrt{2X} - \sqrt{2n} \sim N(0,1)$, large n.

$\therefore \sqrt{2X}$ is asymptotically $N\left(\sqrt{2n}, 1\right)$

11.(A) $f(x;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2}$

$$\Rightarrow f(\vec{x};\sigma) = (2\pi)^{n/2}(\sigma^2)^{n/2} \cdot \exp\left[-\frac{1}{2\sigma^2}\sum x_i^2\right]$$

$$\Rightarrow c(\sigma) = -\frac{2}{2\sigma^2} \quad \text{and} \quad d(\vec{x}) = \sum x_i^2.$$

Note that c(σ) is an increasing function of σ.

So, a UMP test of size α is to reject H_0 if $\sum X_i^2 \geq k$ where k is chosen such that

$P\left(\sum X_i^2 \geq k; H_0\right) = \alpha.$

Now let's find k.

When H_0 is true, α = σ_0. Under this assumption, we need the distribution of $\sum x_i^2$.

A standard normal squared is a chi-squared, so let's standardize it :

$$\alpha = P\left(\sum X_i^2 \geq k : H_0\right)$$

$$= P\left(\sum X_i^2 \geq k : \sigma_0\right)$$

$$= P\left(\frac{\sum X_i^2}{\sigma_0^2} \geq \frac{k}{\sigma_0^2} : \sigma_0\right)$$

$$= P\left(\sum\left(\frac{x_i}{\sigma^0}\right)^2 \geq \frac{k}{\sigma_0^2} : \sigma_0\right)$$

$$. = P(W \geq k_1)$$

where $W \sim x^2(n)$ and $k_1 = k/\sigma_0^2$

We know that $k_1 = X_\alpha^2(n)$.

So, the UMP test of size α is to

$$\text{reject } H_0 \text{ if } \frac{\sum X_i^2}{\sigma_0^2} \geq X_\alpha^2(n)$$

$$\text{reject } H_0 \text{ if } \sum X_i^2 \geq \sigma_0^2 X_\alpha^2(n).$$

12.(D) Let the sample observations be denoted by x_{ij} , i = 1, 2, 3, 4; j = 1, 2, .., n. Since the four samples, from the four normal populations are independent, the likelihood function L of all the sample observations x_{ij}, (i = 1, 2, 3, 4, ; j = 1, 2, ..., n),

is given by : $$L=\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{4n}\cdot\exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{4}\sum_{j=1}^{n}\left(x_{ij}-\mu_i\right)^2\right\},$$

where μ_i, (i = 1, 2, 3, 4) is mean of the ith population. Therefore

$$L=\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{4n}\cdot\exp\left[-\frac{1}{2\sigma^2}\left\{\sum_j\left(x_{1j}-\mu_1\right)^2\right)+\sum_j\left(x_{2j}-\mu_2\right)^2\sum_j\left(x_{3j}-\mu_3\right)^2+\sum_j\left(x_{4j}-\mu_4\right)^2\right\}\right]$$

$\Rightarrow$

$$\log L=k-2n\log\sigma^2-\frac{1}{2\sigma^2}\left\{\sum_j\left(x_{1j}-a-b-c\right)^2+\sum_j\left(x_{2j}-a-b+c\right)^2+\sum_j\left(x_{3j}-a+b-c\right)^2+\sum_j\left(x_{4j}-a++c\right)^2\right\},$$

where k is a constant w.r.t. a, b, c and σ^2. The M.L.Es. for b is the solutions of the equations $\frac{\partial}{\partial b}\log L=0$ (maximum likelihood equations for estimating b) :

gives :

$$-\frac{1}{2\sigma^2}\left\{\sum_j\left(x_{1j}-a-b-c\right)(-2)+\sum_j\left(x_{2j}-a-b+c\right)(-2)+\sum_j\left(x_{3j}-a+b-c\right)(2)+\sum_j\left(x_{4j}-a+b+c\right)(2)\right\}=0$$

$$\Rightarrow\quad \sum_j x_{1j}+\sum_j x_{2j}-\sum_j x_{3j}-\sum_j x_{4j}+n\left[(-a-b-c)+(-a-b+c)-(-a+b-c)-(-a+b+c)\right]=0$$

$$\Rightarrow\quad \sum x_{1j}+\sum x_{2j}-\sum x_{3j}-\sum x_{4j}-4nb=0$$

$$\therefore\quad \hat{b}=\frac{1}{4}\left(\frac{1}{n}\sum x_{1j}+\frac{1}{n}\sum x_{2j}-\frac{1}{n}\sum x_{3j}-\frac{1}{n}\sum x_{4j}\right)\Rightarrow\hat{b}=\left(\bar{x}_1+\bar{x}_2-\bar{x}_3-\bar{x}_4\right)/4$$

where $\bar{x}_1$ is the mean of the ith sample.

13.(D) Let $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}, A' = \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}$

Since A is orthogonal, ∴ AA' = I

$$\Rightarrow \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equating the corresponding elements, we have

$$\left.\begin{matrix} 4\beta^2+\gamma^2 = 1 \\ 2\beta^2-\gamma^2 = 0 \end{matrix}\right\} \Rightarrow \beta = \pm\frac{1}{\sqrt{6}}, \gamma = \pm\frac{1}{\sqrt{3}}$$

$$\alpha^2+\beta^2+\gamma^2 = 1 \Rightarrow \alpha^2 + \frac{1}{6} + \frac{1}{3} = 1 \Rightarrow \alpha = \pm\frac{1}{\sqrt{2}}.$$

Hence (D) is correct answer.

14.(B) (1, 1) Since the matrix is triangular, the eigen values are I, a, b. If (X_1, X_2, X_3) is an arbitrary eigen vector, say corresponding to 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

We have, $X_1 = X_1$; $aX_2 = X_2$ which gives a = 1

and $bX_3 = X_3$ which gives b = 1; X_2, X_3 being not zero.

∴ (a, b) = (1, 1)

15.(D) $x_{n+1} - x_n = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right) - x_n = \frac{a - x_n^3}{3x_n^2}$

By AM GM inequality we know that if $\frac{x_1 + x_2 + x_3}{3} \geq (x_1\, x_2\, x_3)^{1/3}$ for $x_1, x_2, x_3 > 0$

Then this sequence is convergent

here $x_{n+1} = \frac{1}{3}\left(x_n + x_n + \frac{a}{x_n{}^2}\right) \geq a^{1/3}$

Since $< x_n >$ is decreasing and bounded below then it is monotonic and convergent both.

16.(D) P(A/B) = P(A) as independent event = $\frac{1}{2}$.

$P\{A/A \cup B\} = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$

{Since $A \cap (A \cup B) = A \cap [A + B - A \cap B] = A + A \cap B - A \cap B = a$}

$\Rightarrow \quad P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5} - \frac{1}{10}} = \frac{\frac{1}{2}}{\frac{6}{10}} = \frac{5}{6}$

Similarly $P\left(\frac{A \cap B}{A' \cup B'}\right) = 0$.

17.(C) The required probability

= 1 – probability of equal number of heads and tails

$= 1 - {}^{2n}C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{2n-n} = 1 - \frac{(2n)!}{n!n!}\left(\frac{1}{4}\right)^n = 1 - \frac{(2n)!}{(n!)^2}.\frac{1}{4^n}$.

18.(C) $x_i + y_i = n + 1$ for all i = 1, 2, 3, . . . n.

Let $x_i - y_i = d_i$, then $2x_i = n + 1 + d_i$

$\Rightarrow \quad d_i = 2x_i - (n + 1)$

$\therefore \quad \sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} \left[2x_i - (n+1)\right]^2$

$$= \sum_{i=1}^{n}\left[4x_i^2 + (n+1)^2 - 4(n+1)x_i\right]$$

$$= 4\sum_{i=1}^{n} x_i^2 + (n+1)^2(n) - 4(n-1)\Sigma x_i$$

$$= 4\,\frac{n(n+1)(2n+1)}{6} + (n+1)^2\,n - 4(n+1)\,\frac{n(n+1)}{2}$$

$$= \frac{n(n^2-1)}{3}$$

$\therefore$ $$r = 1 - \frac{6\sum d_i^2}{n(n^2-1)} = 1 - 2 = -1.$$

19.(A) Equation of line, whose slope = y – intercept, is

$$y = cx + c \qquad \Rightarrow y = c(x+1) \qquad ...(1)$$

Now, differentiating w.r.t x, we get $\frac{dy}{dx} = c$ ...(2)

$\therefore$ required dif. equation (By substituting (2) in (1)) is

$$y = \frac{dy}{dx}(x+1) \qquad \Rightarrow \frac{dy}{dx}(x+1) - y = 0$$

20.(A) The part of circle $x^2 + y^2 = 9$ in between y = 0 and y = 2 is revolved about y-axis. Then a frustum of sphere will be formed.

The volume of this frustum

$$= \pi \int_0^2 x^2 dy = \pi \int_0^2 (9 - y^2)\,dy$$

$$= p\left[9y - \frac{1}{3}y^3\right]_0^2 = \pi\left[9\times 2 - \frac{1}{3}(2)^3 - \left(9.0 - \frac{1}{3}.0\right)\right]$$

$$= \pi\left[18 - \frac{8}{3}\right] = \frac{46}{3}\pi \text{ cubic unit.}$$

21.(C) $y = \pm x\sqrt{1-x}\,dx$

Area of loop = $2\int_0^1 x\sqrt{1-x}\,dx$

Let 1 $x = t^2$

$\Rightarrow x = 1 - t^2 \Rightarrow dx = -2tdt$

$= 2\int_1^0 (1-t^2)t(-2t)dt = 4\int_0^1 (t^2 - t^4)dt = 4\left|\frac{t^3}{3} - \frac{t^5}{5}\right|_0^1$

$= 4\left[\frac{1}{3} - \frac{1}{5}\right] = 4\left(\frac{2}{15}\right) = \frac{8}{15}$.

22.(B) $f(x) = \begin{cases} e^x; & x \le 0 \\ (x-1); & 0 < x < 1 \\ (1-x); & x \ge 1 \end{cases}$

$Rf'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{1-h-1}{h} = -1$

$Lf'(0) = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\frac{e^{-h}-1}{-h} = 1$

So, it is not differentiable at x = 0.

Similarly, it is not differentiable at x = 1.

But it is continuous at x = 0, 1.

23.(D) $f(x) = (x + b)^2 + 2c^2 - b^2$ is minimum at $x = -b$ and $g(x)\ b^2 + c^2 - (x + c)^2$ is maximum at $x = -c$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2 \Rightarrow |c| > \sqrt{2}\,|b|$.

24.(A) Prob. of each number $= \frac{1}{n+1}$

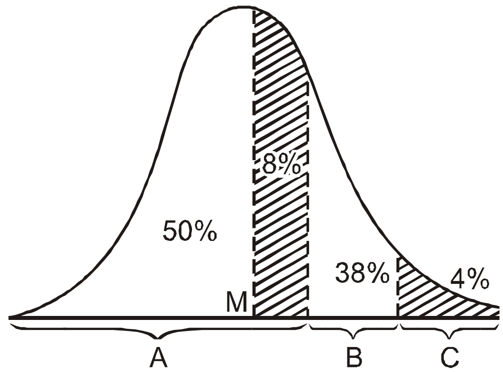
$E(X) = \frac{1}{n+1}\left\{\frac{1}{n}(0+1+2+...+n)\right\} = \frac{n(n+1)}{2n(n+1)} = \frac{1}{2}$

$E(X^2) = \frac{1}{n+1}\left\{\frac{1}{n^2}(1^2+2^2+3^2+....+n^2)\right\} = \frac{1}{(n+1)n^2}\times\frac{n(n+1)(2n+1)}{6} = \frac{2n+1}{6n}$

$\mu_2 = E(X^2) - \{E(X)\}^2 = \frac{2n+1}{6n} - \frac{1}{4} = \frac{n+2}{12n}$

$\sigma = \sqrt{\left(\frac{n+2}{12n}\right)}$

25.(C) Let the mean be m and S.D. be σ. As for A, the area to left of the ordinate at x = 75 is .58, the area between the mean and 75 is .08



Hence $\frac{75 - m}{\sigma} = .2$ ...(1)

Similarly the area above x = 80 is .04 or that between x = m and x = 80 is .5 – .04 = .46.

Hence $\frac{80 - m}{\sigma} = 1.75.$ ...(2)

Solving (1) and (2), we get

m = 74.4 and σ = 3.2.

26. (B) The characteristic equation of matrix A is

$$|A - \lambda I| = \begin{vmatrix} 0-\lambda & c & -b \\ -c & 0-\lambda & a \\ b & -a & 0-\lambda \end{vmatrix} = 0$$

$= -\lambda(\lambda^2 + a^2) - c(\lambda c - ab) - b(ac + \lambda b) = 0$

$= -\lambda^3 - \lambda(a^2 + b^2 + c^2) = 0$

i.e. $\lambda^3 + \lambda(a^2 + b^2 + c^2) = 0$

if we use Cayley Hamilton theorem then we can say that A satisfies it's characteristic equation

i.e. $A^3 + A(a^2 + b^2 + c^2) = 0$

[we can verify it easily also]

27. (D) The characteristic equation of matrix A is given as

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 5-\lambda & -2 & 6 & -1 \\ 0 & 3-\lambda & -8 & 0 \\ 0 & 0 & 5-\lambda & 4 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (5 - \lambda) [(3 - \lambda) [(5 - \lambda) (1 - \lambda)]] = 0$

$\Rightarrow \quad (3 - \lambda) (5 - \lambda) (1 - \lambda) (5 - \lambda) = 0$

or $\quad (1 - \lambda) (3 - \lambda) (5 - \lambda)^2 = 0$

which is required characteristic equation.

28. (D) Given that $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

then

$(aI + bC)^2 = a^2I^2 + b^2C^2 + 2bCI$

$= a^2I + 2abC \qquad [\because C^2 = 0]$

again

$(aI + bC)^2 (aI + bC)^2 = (a^2I + 2abC) (a^2I + 2abC)$

$= a^4I^2 + 2a^3bC\,I + 2a^3bCI + 4a^2b^2C^2$

$= a^4I + 4a^3bC \qquad$ [again $C^2 = 0$]

$$\left\{ \text{as } C^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

29. (A) Given that A be a skew-symmetric matrix

i.e. $A^T = -A$

first of all consider m is +ve integer then

$(Am)^T \quad = (AAA.........\text{ m times})^T$

$= (A^TA^TA^T ....\text{ m times})$

$= [(-A)(-A)(-A)......\text{ m times}]$ [∵ A is skew symmetric]

$= (-1)^m A^m$

but m is even $\Rightarrow (A^m)^T = A^m$

So, A^m is symmetric matrix

30. (B) If system of equation has infinitely many solution then

$$\frac{K+1}{k} = \frac{8}{K+3} = \frac{4k}{3k-1}$$

by first two factors we know that

$(k + 1)(k + 3) = 8k$

$\Rightarrow \quad k^2 + 4k + 3 - 8k = 0$

$\Rightarrow \quad k^2 - 4k + 3 = 0$

$\Rightarrow \quad k^2 - 3k - k + 3 = 0$

$\Rightarrow \quad (k - 3)(k - 1) = 0$

So, $K = 3, 1$ i.e.

31. (B) Given that $A = (a_{ij})$ be an $n \times n$ symmetric matrix. i.e. for symmetric matrix $a_{ij} = a_{ji}$, $(i \neq j)$ so the number of independent entries are $a_{ij} (i < j)$ and a_{ii},

i.e. $\frac{n(n-1)}{2} + n = \frac{n(n+1)}{2}$

Here n = 5 then dimension of 5 × 5 symmetric matrix is $\frac{5(5+1)}{2} = 15$

32. (D) Given that 6 × 6 matrix $A = (a_{ij})_{6\times6}$ such that $a_{ij} \in$ c and $a_{ij} = -a_{ji}$ over R.

which show that A is skew-Hermitian matrix if A is n × n skew hermitian matrix then the number of independent entries are

$$n(n - 1) = n^2 - n$$

but here $\underline{n = 6}$, so dimension of 6 × 6 skew symmetric matrix is $6^2 - 6 = 36 - 6 = 30$

33. (A) We know that unbiased of mean and variance of population are $\bar{X} = \frac{\Sigma x_i}{n}$ and

$S^2 = \frac{1}{n-1}\Sigma\left(X_i - \bar{X}\right)^2$ respectively.

Here $\bar{X} = \frac{6.33 + 6.37 + 6.36 + 6.32 + 6.37}{5}$

$= 6.35$ cm.

and $S^2 = \frac{1}{n-1}$ $[(6.33 - 6.35)^2 + (0.02)^2 + (0.01)^2 + (-0.03)^2 + (0.02)^2]$

$= \frac{1}{4}$ $[0.0022]$

$= 0.0005$ cm^2

i.e. $S^2 = 5 \times 10^{-4}$ cm^2

34. (A) $V(t_1) = \text{Var}\left[\frac{X_1 + X_2 + X_3 + X_4}{4}\right]$

$= \frac{1}{16}\left[\text{Var}(X_1) \ldots\ldots\ldots + \text{Var}(X_4)\right]$

$= \frac{1}{16}$ $[\sigma^2 + \sigma^2 + \sigma^2 + \sigma^2]$ $\quad [\because \text{Var}(X_i) = \sigma^2]$

$= \frac{4\sigma^2}{16}$

$= \frac{\sigma^2}{4}$

again $v(t_2) = \text{Var}\left[X_1 + X_2 + \frac{X_3}{3}\right]$

$= \text{Var}(X_1) + \text{Var}(X_2) + \frac{1}{9}\text{Var}(X_3)$

$= \sigma^2 + \sigma^2 + \frac{\sigma^2}{9}$

$$= \frac{19}{9}\sigma^2$$

Since $Var(t_1) < Var(t_2)$ $\left[\text{as } \frac{1}{4} < \frac{19}{9}\right]$

Thus. t_1 is best estimator as it's variance is less than variance of t_2.

35. (C) $M_Q(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{tQ} f(x,y)\,dx\,dy$

$$= \frac{1}{2\pi\sqrt{(1-\rho^2)}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(tQ)\times\exp\left[-\frac{1}{2(1-\rho^2)}\{x^2 - 2\rho xy + y^2\}\right]dx\,dy$$

$$= \frac{1}{2\pi\sqrt{(1-\rho^2)}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(tQ - \frac{Q}{2}\right)dx\,dy$$

$$= \frac{1}{2\pi\sqrt{(1-\rho^2)}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{Q}{2}(1-2t)\right]dx\,dy$$

Put $\sqrt{(1-2t)}\ x = u$ and $\sqrt{(1-2t)}\ y = v \Rightarrow dx = \frac{du}{\sqrt{(1-2t)}}$ and $dy = \frac{dv}{\sqrt{(1-2t)}}$

Also $Q = \frac{1}{(1-\rho^2)}[x^2 - 2\rho xy + y^2] = \frac{1}{(1-\rho^2)}\left[\frac{u^2 - 2\rho uv + v^2}{1-2t}\right]$

$$\therefore \quad M_Q(t) = \frac{1}{2\pi\sqrt{(1-\rho^2)}}(1-2t)\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2(1-\rho^2)}(u^2 - 2\rho uv + v^2)\right]du\,dv$$

$$= \frac{1}{(1-2t)}\cdot 1 = (1-2t)^{-1}$$

which is the m.g.f. of chi-square (χ^2) variant with n (= 2) degrees of freedom.

36. (B) Regression equation of X_3 on X_1 and X_2 is

$$X_3 = b_{31.2}X_1 + b_{32.1}X_2$$

and $b_{31.1} = -\frac{\sigma_3}{\sigma_1} \times \frac{\omega_{31}}{\omega_{33}}$

$$b_{32.1} = \frac{\sigma_3}{\sigma_2} \times \frac{\omega_{32}}{\omega_{33}}$$

where $\omega = \begin{vmatrix} 1 & r_{21} & r_{31} \\ r_{12} & 1 & r_{32} \\ r_{13} & r_{23} & 1 \end{vmatrix}$

Therefore $\omega_{33} = 1 - r^2_{12}$, $\omega_{31} = r_{21}\, r_{32} - r_{31}$

and $\omega_{32} = (r_{32} - r_{12}\, r_{13})$

$$\therefore \quad b_{31.2} = \frac{-2.7}{2.7} \times \frac{0.28 \times 0.49 - 0.51}{1-(0.28)^2} = \frac{.3728}{.9216} = 0.40$$

$$b_{32.1} = +\frac{2.7}{2.4} \times \frac{0.49 - 0.28 \times 0.51}{1-(0.28)^2}$$

$$= \frac{2.7}{2.4} \times \frac{.3474}{.9216} = 0.42$$

Therefore the plane of regression of X_3 on X_1 and X_2 is

$$X_3 = 0.40X_1 + 0.42X_2$$

37. (B) First we substitute $n_1 = 10$, $n_2 = 8$, $s_1 = 0.5$, and $s_2 = 0.7$ into the formula for s_p, and we get

$$s_p = \sqrt{\frac{9(0.5) + 7(0.49)}{16}} = 0.596$$

Then, substituting this value together with $n_1 = 10$, $n_2 = 8$, and $t_{0.025,16} = 2.120$ into the confidence-interval formula which is given as :

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2.n_1+n_2} - 2 \cdot s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2.n_1+n_2-2} \cdot s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

we find that the required 95% confidence interval is

$$(3.1 - 2.7) - 2.120(0.596)\sqrt{\frac{1}{10}+\frac{1}{8}} < \mu_1 - \mu_2$$

$$< (3.1 - 2.7) + 2.120(0.596)\sqrt{\frac{1}{10}+\frac{1}{8}}$$

which reduces to

$$0.20 < \mu_1 - \mu_2 < 1.00$$

Thus the 95% confidence limits are –0.20 and 1.00 milligrams.

38. (C) Since we know that the p.d.f. of logistic distribution is given as

$$F_x(x) = \left[1 + \exp\left\{-\frac{(x-\alpha)}{\beta}\right\}\right]^{-1} ; \beta > 0$$

$$= \frac{1}{2}\left[1 + \tanh\left\{\frac{(x-\alpha)}{2\beta}\right\}\right] ; \beta > 0$$

when we compute all the odd order moments about mean of the standard logistic distribution then we get all these are equal to zero.

39. (D) Since X, Y ~ BVN (μ_X, μ_Y, $\sigma_X{}^2, \sigma_Y{}^2, \rho$), the conditional distribution of Y given X = x is also normal.

$$(Y \mid X = x) \sim N\left[\mu = \mu_Y + \frac{\rho\sigma_Y}{\sigma_X}(x - \mu_X), \sigma^2 = \sigma_Y{}^2(1-\rho^2)\right].$$ Therefore

$$(Y \mid X = 5) \sim N\left[\mu = 10 + \rho \times \frac{5}{1}(5-5), \sigma^2 = 25(1-\rho^2)\right] = N\left[\mu = 10, \sigma^2 = 25(1-\rho^2)\right]$$

We want ρ so that P(4 < Y < 16 | X = 5) = 0.954

where $Z = \frac{Y-\mu}{\sigma} = \frac{Y-10}{5\sqrt{(1-\rho^2)}} \sim N(0,1)$

$$\Rightarrow \quad P\left(\frac{4-10}{\sigma} < Z < \frac{16-10}{\sigma}\right) = 0.954$$

$$\Rightarrow \quad P\left(\frac{-6}{\sigma} < Z < \frac{6}{\sigma}\right) = 0.954 \quad \text{... (1)}$$

But we know that if Z ~ N (0, 1), then $P(-2 < Z < 2) = 0.954$ ... (2)

Comparing (1) and (2) , we get

$$\frac{6}{\sigma} = 2 \Rightarrow \sigma = 3 \Rightarrow \sigma^2 = 9 = 25\,(1-\rho^2)$$

$$\therefore \quad 1-\rho^2 = \frac{9}{25} \Rightarrow \rho^2 = \frac{16}{25} \Rightarrow \rho = \frac{4}{5} = 0.8 \quad (\because \rho > 0)$$

40. (C) The last expression, viz. $(T_1, T_2, T_3)/3$ would be the most preferable; because it is the mean of T_1, T_2, T_3 and hence the Best Linear Unbiased Estimator, i.e. has the minimum variance among all linear function which may be proposed as unbiased estimators of θ.

41. $f_Y(y) = \int_0^1 (x,y)\,dx$

$$= \int_0^1 [2-x-y]\,dx$$

$$= \left[(2-y)x - \frac{x^2}{2}\right]_0^1$$

$$= 2 - y - \frac{1}{2}$$

$$= \frac{3}{2} - y \quad ,0 \le y \le 1$$

then

$$f_{X/Y}\left(\frac{x}{y}\right) = \frac{f(x,y)}{f_Y(y)}$$

$$= \frac{2-x-y}{\frac{3}{2}-y}, \; 0 \le x \le 1, \; 0 \le y \le 1$$

$$E\left(X/Y=\frac{1}{2}\right)=\int_0^1 x\cdot\left[\frac{2-x-1/2}{\frac{3}{2}-\frac{1}{2}}\right]dx$$

$$=\int_0^1 x\left(\frac{3}{2}-x\right)dx$$

$$=\left[\frac{3}{2}\left(\frac{x^2}{2}\right)-\frac{x^3}{3}\right]_0^1$$

$$=\frac{3}{4}-\frac{1}{3}$$

$$=\frac{5}{12}$$

$$E\left(X^2/Y=\frac{1}{2}\right)=\int_0^1 x^2\cdot\left[\frac{2-x-\frac{1}{2}}{\frac{3}{2}-\frac{1}{2}}\right]dx$$

$$=\int_0^1 x^2\left(\frac{3}{2}-x\right)dx$$

$$=\frac{3}{2}\left[\frac{x^3}{3}\right]_0^1-\left[\frac{x^4}{4}\right]_0^1$$

$$=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

then

$$\text{Var}\left(X/Y=\frac{1}{2}\right)=E\left(X^2/Y=\frac{1}{2}\right)-\left[E\left(X/Y=\frac{1}{2}\right)\right]^2$$

$$=\frac{1}{4}-\left(\frac{5}{12}\right)^2$$

$$=\frac{1}{4}-\frac{25}{144}$$

$$=\frac{36-25}{144}$$

$$=\frac{11}{144}$$

42. The marginal p.d.f.'s of X and Y are given by :

$$f_1(x)=\int_0^2 f(x,y)\,dy=\frac{1}{3}\int_0^2 (x+y)\,dy=\frac{1}{3}\left|xy+\frac{y^2}{2}\right|_{y=0}^{y=2}$$

$$\Rightarrow \quad f_1(x)=\frac{2}{3}(1+x);0\le x\le 1 \qquad \text{... (1)}$$

$$f_2(y)=\int_0^1 f(x,y)\,dx=\frac{1}{3}\int_0^1 (x+y)\,dx=\frac{1}{3}\left|\frac{x^2}{2}+xy\right|_{x=0}^{x=1}$$

$$\Rightarrow \quad f_2(y)=\frac{2}{3}(1+2y);\ 0\le y\le 2 \qquad \text{... (2)}$$

The conditional distributions are given by :

$$f_3(y\,|\,x)=\frac{f(x,y)}{f_1(x)}=\frac{1}{2}\left(\frac{x+y}{1+x}\right);\ f_4(x\,|\,y)=\frac{f(x,y)}{f_2(y)}=\frac{(x+y)}{2(1+2y)} \qquad \text{.. (3)}$$

$$E(Y\,|\,x)=\int_0^2 y\cdot f_3(y\,|\,x)\,dy=\frac{1}{2(1+x)}\int_0^2 y(x+y)\,dy$$

$$=\frac{1}{2(1+x)}\left|\frac{xy^2}{2}+\frac{y^3}{3}\right|_{y=0}^{y=2}=\frac{3x+4}{3(x+1)}$$

and $E(X\,|\,y)=\int_0^1 x f_4(x\,|\,y)\,dx=\frac{1}{2(1+2y)}\int_0^1 (x^2+xy)\,dx=\frac{2+3y}{12(1+2y)}$

(iii) Hence the regression curves for means are :

$$y=E(Y\,|\,x)=\frac{3x+4}{3(x+1)} \quad \text{and} \quad x=E(X\,|\,y)=\frac{2+3y}{12(1+2y)}.$$

From the marginal distributions, we shall get

$$E(X)=\int_0^1 x f_1(x)dx=\frac{5}{9},\ E(X^2)=\int_0^1 x^2 f_1(x)dx=\frac{7}{18}$$

$$\Rightarrow \quad Var(X)=\sigma_X{}^2=\frac{7}{18}-\left(\frac{5}{9}\right)^2=\frac{13}{162}$$

Similarly, we shall get

$$E(Y)=\frac{11}{9}.\ E(Y^2)=\frac{16}{9};\sigma_Y{}^2=\frac{16}{9}-\left(\frac{11}{9}\right)^2=\frac{23}{81}$$

Also $E(XY)=\int_0^1\int_0^2 xy\,f(x,y)dx\,dy=\frac{1}{3}\int_0^1\int_0^2(x^2y+xy^2)dx\,dy$

$$=\frac{1}{3}\left\{\left(\int_0^1 x^2dx\right)\left(\int_0^2 y\,dy\right)+\left(\int_0^1 x\,dx\right)\left(\int_0^2 y^2dy\right)\right\}$$

$$=\frac{1}{3}\left[\frac{1}{3}\times 2+\frac{1}{2}\times\frac{8}{3}\right]=\frac{2}{3}$$

$\therefore$ Cov(X, Y) = E(XY) – E(X) E(Y) $=\frac{2}{3}-\frac{5}{9}\times\frac{11}{9}=-\frac{1}{81}$

(i) $r(X,Y)=\frac{Cov(X,Y)}{\sigma_X\cdot\sigma_Y}=\frac{-\frac{1}{81}}{\sqrt{\frac{13}{162}\times\frac{23}{81}}}=-\left(\frac{2}{299}\right)^{1/2}$

(ii) The two lines of regression are :

$$Y-E(Y)=\frac{Cov(X,Y)}{\sigma_X{}^2}[X-E(X)] \quad\Rightarrow\quad Y-\frac{11}{9}=-\frac{2}{13}\left(X-\frac{5}{9}\right)$$

and $X-E(X)=\frac{Cov(X,Y)}{\sigma_Y{}^2}[Y-E(Y)] \quad\Rightarrow\quad X-\frac{5}{9}=-\frac{1}{23}\left(Y-\frac{11}{9}\right).$

43. Marginal p.d.f. of X is given by :

$$f_1(x)=\int_0^\infty f(x,y)dy=\frac{1}{(1+x)^4}\int_0^\infty ye^{-y/(1+x)}dy$$

$$=\frac{1}{(1+x)^4}\cdot\Gamma 2.(1+x)^2$$ (Using Gamma Integral)

$$= \frac{1}{(1+x)^2}; x \geq 0$$

The conditional p.d.f. of Y (for given X) is given by :

$$f(y|x) = \frac{f(x,y)}{f_1(x)} = \frac{y}{(1+x)^2}\exp\left(-\frac{y}{1+x}\right); y \geq 0$$

Regression equation of Y on X is given by :

$$y = E(Y|X) = \int_0^\infty y f(y|x)dy = \frac{1}{(1+x)}\int_0^\infty y^2 e^{-y/(1+x)}dx$$

$$= \frac{1}{(1+x)^2}\cdot \Gamma 3.(1+x)^3 \qquad \text{[Using Gamma Integral]}$$

$\therefore$ $y = 2(1 + x)$ $\qquad [\because \Gamma_3 = 2! = 2]$

Hence the regression of Y on X is linear.

44. Given that $A = [a_{ij}]_{n\times n}$ be Hermitian matrix of order n,

s.t. $A^\theta = A$

i.e., if $A = \begin{bmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{22} & a_{22} & \dots\dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$

then $A^\theta = \begin{bmatrix} \overline{a_{11}} & \overline{a_{12}} & \dots\dots & \overline{a_{1n}} \\ \overline{a_{21}} & \overline{a_{22}} & \dots\dots & \overline{a_{2n}} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{a_{n1}} & \overline{a_{n2}} & \dots\dots & \overline{a_{nn}} \end{bmatrix}$

Given that $A^2 = O$ [where O is the zero matrix]

Now, let $AA^\theta = [b_{ij}]_{n\times n}$ and if $AA^\theta = 0$ then

each element of AA^θ is zero i.e. the elements of all the principle diagonal are also zero.

i.e. $b_{ii} = 0 \ \forall \ i = 1, 2, \ldots\ldots, n$

Here $b_{ii} = a_{i1}\overline{a_{i1}} + a_{i2}\overline{a_{i2}} + \ldots.. + a_{in}\overline{a_{in}}$

$$= |a_{i1}|^2 + |a_{i2}|^2 + \ldots\ldots.. + |a_{in}|^2$$

but $b_{ii} = 0 \Rightarrow |a_{i1}|^2 + |a_{i2}|^2 + \ldots\ldots.. + |a_{in}|^2 = 0$

$$\Rightarrow |a_{i1}|^2 = 0,\ |a_{i2}|^2 = 0,\ \ldots\ldots\ldots\ldots |a_{in}|^2 = 0$$

$$\Rightarrow |a_{i1}| = 0,\ |a_{i2}| = 0,\ \ldots\ldots\ldots\ldots |a_{in}|^2 = 0$$

$$\Rightarrow a_{i1} = 0,\ a_{i2} = 0,\ \ldots\ldots\ldots\ a_{in} = 0$$

i.e. each element of i^{th} row of A is zero.

i.e. each element of each row of A is zero

Hence A = 0

45. Let us consider a matrix (complex)

$$A = \begin{bmatrix} 0 & 2+3i \\ -2-3i & 0 \end{bmatrix}$$

So, $A^T = \begin{bmatrix} 0 & -2-3i \\ 2+3i & 0 \end{bmatrix}$

$$= -\begin{bmatrix} 0 & 2+3i \\ -2-3i & 0 \end{bmatrix}$$

$$= -A$$

which shows that A = –AT

Thus, A is skew-symmetric matrix.

Again,

$$A^{\theta} = (\overline{A})$$

$$= \begin{bmatrix} 0 & -2+3i \\ 2-3i & 0 \end{bmatrix}$$

$\neq -A$

Hence, A is not skew-Hermitian matrix.

46. Given that X is an binomial statistics with parameter n, θ

then $E(X) = n\theta$

Now

$$E\left[\frac{X+1}{n+2}\right] = \frac{1}{n+1}E(X+1) = \frac{1}{n+2}\left[E(X)+1\right]$$

$$= \frac{1}{n+2}[n\theta+1]$$

Since $E\left[\frac{X+1}{n+2}\right] = \frac{n\theta+1}{n+2} \neq \theta$

which shows that given statistics $\frac{X+1}{n+2}$ is a biased estimator of θ.

Now consider $\theta = \frac{1}{2}$ then $E\left[\frac{x+1}{n+2}\right] = \frac{1}{2} = \theta$

Hence, for $\theta = \frac{1}{2}$ the estimator is asymptotically unbiased.

47. Since $X_1, X_2, \ldots\ldots X_n$ are in constitute random sample from a population with mean μ.

i.e. $E(X_i) = \mu$ $[i \in N]$

then $E[a_1X_1 + a_2X_2 + \ldots\ldots + a_nX_n] = \mu$ (given that)

[where $a_1, a_2, \ldots\ldots a_n$ are constants]

$\Rightarrow a_1E(X_1) + a_2E(X_2) + \ldots\ldots + a_nE(X_n) = \mu$

$\Rightarrow a_1\mu + a_2\mu + \ldots\ldots a_n\mu = \mu$

$\Rightarrow \boxed{a_1 + a_2 + \ldots\ldots\ldots + a_n = 1}$

i.e. for $a_1 + a_2 + ... + a_n = 1$ the estimator $a_1X_1 + a_2X_2 + ... a_nX_n$ is unbiased of μ.

48. Given that $n = 10$

$\sigma^2 = 44.1$ inch^2

Now the sample mean $\bar{x} = \frac{\Sigma x_i}{n}$

$$= \frac{67+71+80+76+78+82+68+72+65+81}{10}$$

$$= \frac{740}{10}$$

$= 74$

Here σ^2 is known then the 95% confident interval for population mean μ is given

$$\left(\bar{X} = 1\cdot 96\sqrt{\frac{\sigma^2}{n}},\ \bar{X} + 1.96\sqrt{\frac{\sigma^2}{n}}\right)$$

$$\Rightarrow \left(740 - 1.96\sqrt{\frac{44.1}{10}};740 + 1.96\sqrt{\frac{44.1}{10}}\right)$$

$\Rightarrow$ (740 – 1.96 × 2.1, 740 + 1.96 × 2.1)

$\Rightarrow$ $\boxed{69.88, 78.11}$ **Ans.**

which is required 95% interval.

49. The observation, reduced by 250, and also the preliminary results for calculating the various sum of squares, are shown in below

Calculations for analysis of variance

	Sets				
	I	II	III	IV	
	–1	1	16	12	
	–8	6	11	10	
	–3	5	15	13	
	0	8	14	12	
	2			11	
				14	
				12	
Total	$T_1 = -10$	$T_2 = 20$	$T_3 = 56$	$T_4 = 84$	$T = 150$
Total of Sqs.	78	126	798	1018	$\Sigma\Sigma x_{ij}^2 = 2020$
Sample size	$n_1 = 5$	$n_2 = 4$	$n_3 = 4$	$n_4 = 7$	$N = 20$

$$\text{C.F.} = \frac{150^2}{20} = 1125$$

Total SS = 2020 – 1125 = 895

$$\text{SSB} = \left[\frac{(-10)^2}{5} + \frac{20^2}{4} + \frac{56^2}{4} + \frac{84^2}{7}\right] - 1125$$

= (20 + 100 + 784 + 1008) – 1125 = 787

SSE = 895 – 787 = 108

The various Sums of Squares (S.S.) along with the degree of freedom (d.f.) are shown in the following table.

Analysis of variance table

Source of Variation	S.S	d.f.	M.S.	F value Observed	F value Tabulated
Between Sets	787	3	262.3	38.9	$F_{.05} = 3.24$
Within Sets (Error)	108	16	6.75		$F_{.01} = 5.29$
Total	895	19	—	—	—

Since the observed value F is larger than the 1% tabulated value corresponding to d.f., we reject the null hypothesis and conclude that the means of normal populations are not equal.

50. $E\left\{\frac{Y_n^2}{1+Y_n^2}\right\} = E\left\{\frac{S_n^2}{n^2+S_n^2}\right\}$

$$= \frac{2}{\sigma\sqrt{2\pi}}\int_0^\infty \frac{x^2}{n^2+x^2} e^{-x^2/2\sigma^2} dx$$

$$= \frac{2}{\sqrt{2\pi}}\int_0^\infty \frac{y^2\left[n+2(n-1)\rho\right]}{n^2+y^2\left[n+2(n-1)\rho\right]} e^{-y^2/2} dy$$

$$\le \frac{n+2(n-1)\rho}{n^2}\int_0^\infty \frac{2}{\sqrt{2\pi}} y^2 e^{-y^2/2} dy \to 0 \quad \text{as } n \to \infty.$$

which show that WLLN holds for the sequence $\{x_n\}$