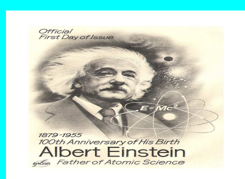

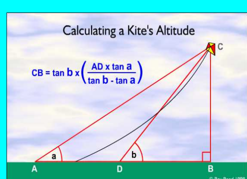

IIT-JAM - MATHEMATICS

MOCK TEST PAPER(According to new pattern)

- Attempt ALL the 60 questions.
- SECTION-A Consists of 30 questions. These questions are Multiple Choice Questions (MCQs),First 20 questions carries one marks for each,and remaining 10 questions carries two marks for each.
- Section-B Consists of 10 questions. These questions are Multiple Select Questions (MSQs), each question carries two marks.
- Section-C Consists of 20 Numerical Answer Type (NAT) questions each question carries two marks. For each NAT type question, the answer is a signed real number.
- In Section A, for all 1 mark questions,1/3 marks will be deducted for each wrong answer and for all 2 marks questions, 2/3 marks will be deducted for each wrong answer.There is no negative marking in Section B and Section C.

- ***Total marks : 100***
- ***Duration of test : 3 Hours***

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SECTION-A (1-30)

1. The solutions sin x and cos x of the differential equation $\frac{d^2y}{dx^2} + y = 0$ are

(A) Linearly independent

(B) Dependent

(C) These solutions does not exists

(D) None of these

2. The solution of $(4x + xy^2)\,dx + (y + x^2y)dy = 0$ is

(A) $(1 - x^2)(4 + y^2)$ = constant

(B) $(1 + x^2)(4 + y^2)$ = constant

(C) $(1 - x^2)(4 - y^2)$ = constant

(D) $(1 + x^2)(4 - y^2)$ = constant

3. A group of order 49 is always a

(A) Non-cyclic group

(B) Abelian group

(C) Non-abelian group

(D) Simple group

4. Let **A** be an n-by-n matrix with coefficients in F, having rows$\{a_1, . . . , a_n\}$. Then which one of the statement is true for the matrix A ?

(A) **A**' be a matrix obtained from **A** by an elementary row operation (interchanging two rows). Then D(**A**') = –D(**A**)

(B) **A**' be a matrix obtained from **A** by an elementary row operation (replacing the row a_i by $a_i + \lambda a_j$, with $\lambda \in F$, $i \neq j$). Then D(**A**') = D(**A**).

(C) **A**' be a matrix obtained from **A** by an elementary row operation (replacing a_i by μa_i, for $\mu \neq 0$ in F). Then D(**A**') = μD(**A**).

(D) All the three options are correct.

5. Let V be the vector space of real polynomials of degree atmost 2.which defines a linear operator T ; V $\to$ V by $T(x^i) = \sum_{j=0}^{i} x^j$, i = 0, 1, 2 then the matrix of T^{-1} with respect to the basis $(1, x, x^2)$ is

(A) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

6. Let f : (0, 2) $\to$ R be defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 2x-1 & \text{if } x \text{ is irrational} \end{cases}$$

then

(A) f is differentiable exactly at one point

(B) f is differentiable exactly at two point

(C) f is not differentiable at any point in (0, 2)

(D) f is differentiable at any point in (0, 2)

7. If $\sum_{n=0}^{\infty} a_n = L$

and if $\lim_{n\to\infty} na_n = 0$, then $\sum_{n=0}^{\infty} a_n$ is

(A) Converges to 0

(B) Converges to L

(C) Diverges to L

(D) Diverges to 0

8. Consider the differential equation $\frac{dy}{dx} = e^{x-y}\left(e^x - e^y\right)$ and y = 0 and $x \to -\infty$ then $y(\log_e 2)$ is

(A) $\log\left(1+2e^{-e^x}\right)$

(B) $-\left(1+2e^{-e^2}\right)$

(C) 0

(D) none of these

9. Consider the differential equation $y^3\left(\frac{dy}{dx}\right)+x+y^2=0$ which of the following statements is true ?

(A) The differential equation is Bernoulli

(B) The differential equation is homogenous

(C) The differential equation can be converted to variable separable form by a suitable substitution

(D) By the substitution $y^2 + x = v$ differential equation can be resolved into homogenous form

10. Let $T : R^3 \to R^3$ be the linear transformation such that Y(1, 0, 1) = (0, 1 , –1) and T(2, 1, 1)= (3, 2, 1) Then T(–1, –2, 1)

(A) (6, –1, 5)

(B) (–1, 6, –5)

(C) (–6,–1,–5)

(D) (–1,–5, 6)

11. Let $T: R^3 \to R^3$ be the Linear transformation whose matrix with respect to the standard basis $\{e_1\ e_2\ e_3\}$ of R_3 is $\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ Then T

(A) has distinct eigenvalues

(B) has a non-zero null space

(C) has eigenvectors that span R^3

(D) maps the subspace spanned by e_1 and e_2 into it self

12. Let T: $R^2 \to R^3$ be the Linear transformation whose matrix with respect to standard basis of R^3 and R^2 is $\begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix}$ The T

(A) is one to one

(B) is one to one and onto both

(C) is onto

(D) has rank 1

13. Let $Y = \{y_n\}$ be a sequence such that

$$y_n = \frac{1}{1!} - \frac{1}{2!} + ... + \frac{(-1)^{n+1}}{n!}$$

Then– $n \to \infty$

(A) $\{y_n\}$ is monotonic sequence

(B) $\{y_n\}$ is cauchy sequence

(C) $|y_n - y_m| \to \infty$ as $n \to \infty$

(D) Y can not be calculated

14. Let V be the vector space of function $f : R \to R$ if W be its subsets then which of the following W is subspace of v

(A) W = {f(x) : f(1) = 0}

(B) W = {f(x) : f(3) = f(1)}

(C) W = {f(x) : f(–x) = –f(x)}

(D) W = {f(x) : f(4) = 3 + f(2)}

15. $\int_0^2 \int_{y^2}^4 \cos\left(x^{3/2}\right) dx\, dy =$

(A) $\frac{1}{3}\sin 8$

(B) $\frac{2}{3}\cos 8$

(C) $\frac{4}{3}\cos 8$

(D) $\frac{2}{3}\sin 8$

16. Find the surface of the solid formed by revolution about x-axis of the loop of the curve $x = t^2,\ y = t - \frac{1}{3}t^3$.

(A) π

(B) 2π

(C) 3π

(D) 4π

17. The general solution of the d.e $\frac{d^2y}{dx^2} + 4y = \sin^2 x$ is given by ,

(A) $y = C_1 e^{2x} + C_2 e^{-2x} + \sin x \cos x$

(B) $y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{8} - \frac{x}{8}\sin 2x$

(C) $y = (C_1 + C_2 \cos 2x)e^{-2x} - \frac{x}{8}\cos 2x$

(D) $y = C_1 \cos(2x + C_2) + \frac{1}{8}$

18. The mass of a solid right circular cylinder of height h and radius of base b, if density (mass per unit volume) is numerically equal to the square of the distance from the axis of the cylinder.is

(A) $\frac{1}{4}\pi hb^4$

(B) $\frac{1}{2}\pi hb^2$

(C) $\frac{1}{2}\pi hb$

(D) None of these

19. If f and g be continuous real valued functions on the metric space M. Let A be the set of all $x \in M$ s.t. f(x) < g(x)

(A) A is closed

(B) A is open

(C) Neither open nor closed

(D) None of these

20. The function sinx (1 + cosx) at $x = \frac{\pi}{3}$ is

(A) Maxima

(B) Minima

(C) Both (A) & (B)

(D) None of these

21. Evaluate . $\int_{\frac{1}{2}\pi-\alpha}^{\frac{1}{2}\pi} \sin\theta \cos^{-1}(\cos\alpha \operatorname{cosec}\theta)\, d\theta$

(A) $\frac{\pi}{2}(1+\cos\alpha)$

(B) $\frac{\pi}{2}(1-\cos\alpha)$

(C) $\frac{\pi}{2}(1-2\cos\alpha)$

(D) None of these

22. Using the method of Lagrange multipliers the greatest and smallest value that the function f (x,y) = xy takes on the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$ is

(A) 2,0

(B) -2,0

(C) 2,-2

(D) 1,2

23. If L(w) = w then w is a ———— of v.

(A) Subset

(B) set

(C) Superset

(D) Subspace

24. Let S be a closed surface and let denote the position vector of any point (x,y,z) measured from an origin O. then

$\int\int_s \frac{\vec{r}}{r^3} \cdot \hat{n}\, ds$ is equal to (if O lies inside S).

(A) 3π

(B) 2π

(C) 4π

(D) None of these

25. Let p(x) be a non-zero polynomial of degree N the radius of convergence of the power series $\sum_{n=0}^{\infty} p(n)x^n$

(A) depends on N

(B) is 1 for all N

(C) is 0 for all N

(D) is ∞ for all N

26. The function $f:\mathbb{R}\to\mathbb{R}$ satisfied $|f(x)-f(y)| \le c|x-y|$ for all x, y $\in$ and some constant c $\in$ Then,

(A) f must be bounded

(B) f must be continuous but may not be uniformly continuous

(C) f must be uniformly continuous but may be not differentiable

(D) f must be differentiable

27. Consider the system of linear equations

$$a_1x = b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

where ai bi ci di are real numbers for $1 \le i \le 3$ if $\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} \ne 0$ then the above system has

(A) At most one solution

(B) always exactly one solution

(C) more than one but finitely many solutions

(D) infinitely many solutions

28. Let $y_1(x) = 1 + x$ and $y_2(x) = e^x$ be two solutions of y"(x) + p(x)y'(x) + Q(x)y(x) = 0 then the set of initial conditions for which the above differential equation has No solution is .

(A) y(1)= 2, y'(1) = 1

(B) y(0) =1, y'(0) = 2

(C) y(0)= 2, y'(0) = 1

(D) None of these

29. Value of the $\oint_C y^3dx - x^3dy$ is (where C are the two circles of radius 2 and 1 centered at the origin with positive orientation.)

(A) $\frac{45\pi}{2}$

(B) $-\frac{50\pi}{2}$

(C) $\frac{50\pi}{2}$

(D) $-\frac{45\pi}{2}$

30. The series

$$1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+\ldots+\left(\frac{2}{3}\right)^{n-1}+\ldots \text{is}$$

(A) Divergent

(B) Convergent

(C) Both (A) & (B)

(D) None of these

SECTION - B (31-40)

31. Apply the method of variation of parameters to solve $x^2y_2 + xy_1 - y = x^2e^x$ then

(A) $y = c_1x + c_2\frac{1}{x} + e^x - x^{-1}e^x$

(B) $y = c_1 x + c_2 \frac{1}{x} - e^{x} - x^{-1} e^{x}$

(C) $xy = c_1 x^2 + c_2 + xe^x - e^x$

(D) $y = c_1 x + c_2 \frac{1}{x} - e^{x} + x^{-1} e^{x}$

32. Solution of the differential equation $(D^2 + 4)\, y = \sec 2x$.

(A) $y = c_1 \cos 2x + \frac{1}{4} c_2 \sin 2x + \cos 2x \log \cos 2x$

(B) $y = c_1 \cos x + c_2 \sin x + \frac{x}{2} \sin x + \cos 2x \log \cos 2x$

(C) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{x}{2} \sin 2x + \cos 2x \log \cos 2x$

(D) $4y = 4c_1 \cos 2x + 4c_2 \sin 2x + 2x \sin 2x + \cos 2x \log \cos 2x$

33. If $f(x, y, z) = z^2 y^2 \log (x)$ then $f_{xx\,y\,zz}$ is not equal to :

(A) $\frac{12\, zy}{x^2}$

(B) $-\frac{12\, zy}{x^2}$

(C) $\frac{4\, zy}{x^2}$

(D) $\frac{5\, xyz}{-3}$

34. Let h be a continuous and differentiable function defined on $[0, 2\pi]$. Some function values of h and h' are given by the chart below:

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
h(x)	3	$\frac{\pi}{6}$	$\frac{\pi}{4}$	0	–2
h'(x)	$-\frac{\pi}{3}$	$\frac{3\pi}{2}$	–1	$\frac{\pi}{2}$	1

If $p(x) = \sin^2(h(2x))$,then $p'\left(\frac{\pi}{2}\right)$ is not equal to ____.

(A) 3

(B) – 2

(C) – 1

(D) 0

35. Consider the Region R is the first quadrant bounded by the curves $y = x^2$,

$y = x^2 / 5$, xy = 2 and xy = 4 then value of dxdy in terms of dudv if $u = x^2/y$,v = xy is

(A) $dxdy = \frac{1}{3v}$ dudv

(B) $dxdy = \frac{1}{3u}$ dudv

(C) $dxdy = \frac{1}{3uv}$ +dudv

(D) $6\,dxdy = \frac{2}{u}dudv$

36. If $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 & 5 \\ -1 & 0 & -2 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$ then find the value of x, y, z

(A) 2x = 2, 2y = 6, 2z = 8

(B) x = 1, y = –3, z = 4

(C) x = 1, y = 3, z = –4

(D) x = 1, y = 3, z = 4

37. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$

then which of the following statements are false ?

(A) (AB)' = A'B'

(B) B'A' = (AB)'

(C) AB = A'B'

(D) (AB) = B'A'

38. Which of the following are the wrong basis of the subspace spanned by the vectors α_1 = (1, 2, 3), α_2 = (2, 1, –1), α_3 = (1, –1, –4), α_4 = (4, 2, –2)?

(A) (α_2, α_3)

(B) (α_1, α_3)

(C) (α_1, α_2)

(D) (α_1, α_4)

39. If the matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$, where $i = \sqrt{-1}$, is unitary, then a is not equal to :

(A) -1

(B) 2

(C) 0

(D) 1

40. In the given set

$$A = \left\{ w_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i,\ w_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i,\ w_3 = 1 \right\}$$

the cube roots do not form.

(A) Group w.r.t. addition, the set of complex number c.

(B) Group w.r.t. multiplication, the set of complex number c.

(C) Group w.r.t. subtraction, the set of complex number c.

(D) Group w.r.t. division, the set of complex number c.

SECTION - C (41-60)

41. Evaluate the value of α, such that

$$\vec{V} = (x + 3y)\,\hat{i} + (y - 2z)\,\hat{j} + (x + \alpha z)\hat{k}$$

is solenoidal.

42. If u is harmonic on $\{(x,y)\ |x^2 + y^2 \le 1|\}$, and $\frac{\partial u}{\partial n}$ is the normal derivative of u on the boundary of the unit disc,then what is the value of $\int_0^{2\pi} \frac{\partial u}{\partial n} d\theta$?

43. Evaluate , $\int_C \mathbf{F}.d\mathbf{r}$ where $\mathbf{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ and curve C is $\mathbf{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ and $-1 \le t \le 1$.

44. In a group G, $a^5 = e$, $aba^{-1} = b^2$ for $a, b \in G$, $(b \neq e)$ What is the value of O(b) ?

45. What is the number of subgroups of S_4 of order 12 ?

46 Determine the radius of convergence for the following power series

. $\sum_{n=0}^{\infty} n!(2x+1)^n$

47. Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and S is the surface of the cube $0 \le x, y, z \le 1$

48. Evaluate limit . $\lim_{R\to\infty} \dfrac{\int_R^{\infty} r^n e^{-r^2/2} dr}{R^{n-1} e^{-R^2/2}}, n \in \mathbb{Z}_+$

49. Let $f\left(\dfrac{x+y}{2}\right) = \dfrac{f(x)+f(y)}{2}$ for all real x and y. If f'(0) exists and equals – 1 and f(0) = 1, then find f(2).

50. If $u = \log_e(x^2 + y^2) + \tan^{-1}\left(\dfrac{y}{x}\right)$, then find out the value of . $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$

51. Let I = $\int_C \dfrac{e^y}{x}dx + (e^y \ln x + x)dy$, where C is the positively oriented boundary of the region enclosed by $y = 1 + x^2$, $y = 2$, $x = \dfrac{1}{2}$,then what is the value of I?

52. If **A** = $(3x^2 + 6y)$**i** – 14yz**j** + $20xz^2$**k**, evaluate $\int_C \mathbf{A}.dr$ from (0, 0, 0) to (1, 1, 1) along the following paths C :

$x = t$, $y = t^2$, $z = t^3$.

53. If $\vec{V} = \hat{i}\dfrac{x}{x^2+y^2} + \hat{j}\dfrac{y}{x^2+y^2}$ what is the value of this integral $\int_C \vec{v}.dr$ the circular path $x^2 + y^2 = 1$?

54. For which value of the system of linear equation

$$3x - y + \lambda z = 1,$$

$$2x + y + z = 2,$$

$$x + 2y - \lambda z = -1 ?$$

have no sol.

55. If $\int_0^1 \frac{x^5 + x^4 + x^2}{x} \sqrt{4x^5 + 5x^4 + 10x^2}\, dx = \alpha\, (19)^{3/2}$

then what is the value of α ?

56. On applying the mean value theorem on integral, then find out the average value of f(x) = 3 $-\frac{3}{2}$ x or on [0, 2].

57. Evaluate $\int_{-2}^{2} \left(x^4 - 4x^2 + 6\right) dx.$

58. Evaluate $\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta \, d\theta$

59. If $\int_{-2}^{2} 3f(x)dx = 12, \int_{-2}^{5} f(x)dx = 6$ and $\int_{-2}^{5} g(x)dx = 2$

then determine the value of . $\int_{-2}^{5} \left[\frac{f(x) + g(x)}{5}\right] dx$

60. What is the value of $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\, dy\, dx$?

ANSWER KEY

[redacted]

HINTS AND SOLUTIONS

1.(A) We find that

$$W(\sin x, \cos x) = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -\sin^2 x - \cos^2 x = -1 \neq 0$$

for all real x. Thus, since W(sin x , cos x) $\neq 0$ for all real x, we conclude that sin x and cos x are indeed linearly independent solutions of the given differential equation on every real interval.

2.(B) $(4x + xy^2)\, dx + (y + x^2 y)dy = 0$

Dividing by $(1 + x^2)(y^2 + 4)$, we have

$$\left(\frac{x}{1+x^2}\right)dx + \frac{y}{(4+y^2)}\,dy = 0$$

On integrating it, we get ,

$$\frac{1}{2}\ln\left(1+x^2\right) + \frac{1}{2}\ln\left(4+y^2\right) = 0$$

or $\quad \ln (1 + x^2)^{1/2} + \ln (4 + y^2)^{1/2} = c$

or $\quad (1 + x^2)^{1/2} (4 + y^2)^{1/2}$ = new constant

or $\quad (1 + x^2)(4 + y^2)$ = new constant

3.(B) Since, we know that every group of order p^2 is cyclic, where p is a prime integer.

$\therefore$ Group of order p^2 is abelian also.

$\therefore$ Group of order 7^2 i.e, 49 is abelian and cyclic.

4.(D) By a well known result we know that if A be an $n \times n$ matrix with coefficients in F, having rows $\{a_1, a_2, \ldots, a_n\}$, then the following statements are true.

(a) if A' be a matrix obtained from A by an elementary row operation (interchanging two rows). Then

$$D(A') = -D(A)$$

(b) if A' be a matrix obtained from a by an elementary row operation (replacing the row a_i by λa_j, with $\lambda \in F$, $i \neq j$). Then

$$D(A') = D(A)$$

(C) if A' be a matrix obtained from A by an elementary row operation (replacing a_i by μa_i, for $\mu \neq 0$ in F). Then

$$D(A') = \mu D(A)$$

i.e. all the three options are correct.

5.(B) $T(x_0, x_1, x_2) = (x_0, x_0 + x_1, x_0 + x_1 + x_2)$

Let basis are (1, 0, 0), (0, x, 0), and $(0, 0, x^2)$

Then $T(1, 0, 0) = (1, 1, 1)$

$T(0, x, 0) = (0, x, x)$

$T(0, 0, x^2) = (0, 0, x^2)$

$$\Rightarrow T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & x & x \\ 0 & 0 & x^2 \end{bmatrix}$$

$|T| = 1(x^3) = x^3$

At x = 1, $T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $|T| = 1$

Cofactors of T

$T_{11} = 1$ $T_{12} = 0$ $T_{13} = 0$

$T_{21} = -1$ $T_{22} = 1$ $T_{23} = 0$

$T_{31} = 0$ $T_{32} = -1$ $T_{33} = 1$

$\therefore$ adj. T = Transpose of co-factors matrix = $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Hence $T^{-1} = \frac{1}{|T|}$ adj. $T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

6.(A) Given that f(0, 2) $\to$ R then

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 2x-1 & \text{if } x \text{ is irrational} \end{cases}$$

$$f'(x) = \begin{cases} 2x & \text{if } x \text{ is rational} \\ 2 & \text{if } x \text{ is irrational} \end{cases}$$

$\Rightarrow$ f(x) is differentiable only when x = 1

i.e., f(x) is differentiable, exactly at one point

7.(B) Given that $\sum_{n=0}^{\infty} a_n = L$ ....(1)

and $\lim_{n\to\infty} na_n = 0,$ ....(2)

By a well known th. we know that

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} |ka_k| = 0. \qquad ....(3)$$

Given $\in > 0$, it follows from (2), (3), and (1) that there exists N $\in$ I such that

$$|na_n| < \frac{\in}{3} \qquad (n \geq N), \qquad ....(4)$$

$$\frac{1}{n}\sum_{k=1}^{n} |ka_k| < \frac{\in}{3} \quad (n \geq N), \qquad .....(5)$$

and such that

$$\left|L-\sum_{k=0}^{\infty}a_k x^k\right|<\frac{\in}{3} \qquad \left(1-\frac{1}{N}<x<1\right). \qquad .....(6)$$

For any $n \in I$ and $x \in (0, 1)$ we have

$$L-\sum_{k=0}^{n}a_k = L-\sum_{k=0}^{\infty}a_k x^k+\sum_{k=0}^{\infty}a_k x^k-\sum_{k=0}^{n}a_k = L-.\sum_{k=0}^{\infty}a_k x^k+\sum_{k=1}^{n}a_k(x^k-1)+\sum_{k=n+1}^{\infty}a_k x^k$$

Hence

$$\left|L-\sum_{k=0}^{n}a_k\right|\le\left|L-\sum_{k=0}^{\infty}a_k x^k\right|+\sum_{k=1}^{n}|a_k|.(1-x^k)+\sum_{k=n+1}^{\infty}|a_k|x^k = I_1 + I_2 + I_3, \text{ say.} \qquad .....(7)$$

For any $n \ge N$, choose x such that $1 - 1/n < x < 1 - 1/(n + 1)$. Then $1 - 1/N \le 1 - 1/n < x < 1$, and so, by (6),

$$I_1 = \left|L-\sum_{k=0}^{\infty}a_k x^k\right|<\frac{\in}{3}$$

Now $I - x^k = (1 - x)(1 + x + x^2 + ... + x^{k-1}) \le k(1 - x)$, for any $k \in I$.

Hence, since $1 - x < 1/n$, we have

$$1 - x^k \le k(1 - x) < \frac{k}{n}. \qquad .....(8)$$

By (8) and (5) we then have (since $n \ge N$)

$$I_2 = .\sum_{k=1}^{n}|a_k|(1-x^k)<\frac{1}{n}\sum_{k=1}^{n}|ka_k|<\frac{\in}{3}$$

To estimate I_3 we have, using (4),

$$I_3 = \sum_{k=n+1}^{\infty}|ka_k|\frac{x^k}{k}<\frac{\in}{3}\sum_{k=n+1}^{\infty}\frac{x^k}{k}\le\frac{\in}{3(n+1)}\sum_{k=n+1}^{\infty}x^k\le\frac{\in}{3(n+1)}\sum_{k=0}^{\infty}x^k=\frac{\in}{3(n+1)(1-x)}.$$

But x < 1 – 1/(n + 1) and so 1 – x > 1/(n + 1). Thus (n + 1) (1 – x) > 1 and so

$$I_3 \le \frac{\epsilon}{3(n+1)(1-x)} < \frac{\epsilon}{3}.$$

From (7) we then have

$$\left|\sum_{k=0}^{n} a_k - L\right| \le I_1 + I_2 + I_3 < \epsilon \quad (n \ge N)$$

which proves that $\sum_{k=0}^{\infty} a_k$ converges to L.

8.(D) Given differential equation $\frac{dy}{dx} = e^{x-y}\left(e^x - e^y\right)$

multiply both sides of the given equation by e^y we get

$$e^y \frac{dy}{dx} = e^{x-y} e^y \left(e^x - e^y\right)$$

$$e^y \frac{dy}{dx} = e^x \left(e^x - e^y\right)$$

$$\Rightarrow e^y \frac{dy}{dx} + e^x e^y = e^{2x}$$

Take $e^y = t$

$$e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + te^x = e^{2x}$$

Integrating factor $= e^{\int e^x dx} = e^{e^x}$

solution is

$te^{e^x} = c + \int e^{2x} e^{e^x} dx$

where c is an arbitrary constant

$t \cdot e^{e^x} = c + \int e^x e^{e^x} e^x dx$

where $u = e^x$

$du = e^x dx$

$\Rightarrow t \cdot e^x = c + \int u \cdot e^u du = c + (u - 1)e^4$

$t \cdot e^x = c - (e^x - 1)\ e^{e^x}$

$\Rightarrow e^y = ce^{-x} + (e^x - 1)$

$\Rightarrow e^y = c\,e^{-(e^x)} + e^x - 1$

as $x \to -\infty$

$y \to 0$

$\Rightarrow e^0 = ce^0 + 0 - 1$

$\Rightarrow -2 = c$

so $e^y = -2e^{-e^x} + e^x - 1$

now $y \log\left(e^x - 1 - 2e^{-e^x}\right)$

when $x = \log 2$

$y(\log 2) = \log\left(2 - 1 - 2e^{-e^x}\right) = \log\left(-1 - e^{-e^x}\right)$ = not defined.

9.(D) Given differential equation

$$y^3\left(\frac{dy}{dx}\right)+x+y^2=0$$

now on observing we can see that. The differential equation is neither Linear and homogeneous nor separable now putting $y^2 = v-x$

or $$y\frac{dy}{dx}' = \frac{1}{2}\left(\frac{dv}{dx}-1\right)$$

The given equation reduces to $(\upsilon-x)\frac{1}{2}\left[\frac{du}{dx}-1\right]+x+(v-x)=0$

writhing $y^3\frac{dy}{dx}=y^2\left(y\frac{dy}{dx}\right)$

or $$(u-x)\frac{du}{dx}-(v-x)+2v=0$$

or $$\frac{dv}{dx}=\frac{u+x}{x-v}$$

which is a homogeneous equation

10.(C) Since $T : R^3 \to R^3$

such that $T(2, 1, 1) = (3, 2, 1)$

$T(1, 0, 1) = (0, 1, -1)$

But let $(-1, -2, 1) = a(2, 1, 1) + b(1, 0, 1)$

on solving $-1 = 2a + b$

$-2 = a + 0$

$a + b = 1$

$\Rightarrow$ $b = 3$

so on applying transformation T on (1) bothsides

T(1, -2, 1) = T(–2, (2, 1,1) + 3(1, 0, 1) = –2 T(2, 1, 1) + 3T(1, 0, 1)

= –2 (3, 2, 1)+ 3(0, 1, –1) = (–6, –1, –5)

11.(C) Let T : $R^3 + R^3$

such that the matrix for T is given by

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

First we check out the eigenvalues of A is given by

$$|A - \lambda I| = \begin{bmatrix} 1-\lambda & 3 & 3 \\ -3 & -5-\lambda & -3 \\ 3 & 3 & 1-\lambda \end{bmatrix}$$

$= (1 - \lambda)\,[-(5 + \lambda)(1 - \lambda) + 9] - 3[-3\,(1-\lambda)+ 9] + 3\,[-9 + 3\,(5 + \lambda)]$

$= -\lambda^3 - 3\lambda 2 + 4 = -(\lambda - 1)\,(\lambda + 2)^2$

The Eigen vectors for l =1 is $u_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

as $\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

and Eigen vectors for $\lambda = -2$ are $v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

and if $P = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Then $P^{-1} AP = D$

where $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

so set $\{\upsilon_1\ \upsilon_2\ \upsilon_3\}$ spans R^3 as dim R^3 =3

12.(A) Let iof $T : R^2 \to R^3$ be a Linear transformation such that matrix a with respect to standard basis of T is $\begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix}$

here clearly the columns of A are linearly independent b $\Rightarrow$ T is one one mapping since matrix is 3 × 2 the column of A span R^3 if A has 3 pivot positions but it is contradiction as A has 2 columns only

$\Rightarrow$ Associated Linear transformation is not onto

Rank of matrix = Rank of Linear transformation =2

13.(B) Let $Y = (y_n)$ be the sequence of real numbers given by

$$y_1 : \frac{1}{1!},\ y_2 : \frac{1}{1!} - \frac{1}{2!}, ...,\quad y_n : \frac{1}{1!} - \frac{1}{2!} + ... + \frac{(-1)^{m+1}}{m!}.$$

Clearly, Y is not a monotone sequence. However, if m > n, then

$$y_m - y_n = \frac{(-1)^{n+2}}{(n+1)!} + \frac{(-1)^{n+3}}{(n+2)!} + \dots + \frac{(-1)^{m+1}}{m!}$$

Since , $2^{r-1} \le r!$ it follows that if m > n, then

$$|y_m - y_n| \le \frac{1}{(n+1)!} + \frac{1}{(n+2)!} + \dots\dots + \frac{1}{m!}$$

$$\le \frac{1}{2^n} + \frac{1}{2^{n+1}} + \dots + \frac{1}{2^{m-1}} < \frac{1}{2^{n-1}}.$$

Therefore, it follows that (y_n) is a Cauchy sequence. Hence it converges to a limit y. At the present moment we cannot evaluate y directly; however, passing to the limit (with respect to m) in the above inequality, we obtain

$$|y_n - y| \le 1/2^{n-1}.$$

Hence we can calculate y to any desired accuracy by calculating the terms yn for sufficiently large n. The reader should to this and show that y is approximately equal to 0.632 120559.

14.(D) (a) Let $\hat{0}$ denote the zero polynomial, so $\hat{0}(x) = 0$ for every value of x.

$\hat{0} \in W$, since $\hat{0}(1) = 0$. Suppose f. g $\in$ W. Then f(1) = 0 and g(1) = 0. Also, for scalars a and b, we have

$$(af + bg)(1) = af(1) + bg(1) = a0 + b0 = 0$$

Thus af + bg $\in$ W, and hence W is a subspace.

(b) $\hat{0} \in W$, since $\hat{0}(3) = 0 = \hat{0}(1)$. Suppose f, g $\in$ W. Them f(3) = f(1) and g(3) g(1). Thus, for any scalars a and b, we have (af + bg)(3) = af(3) + bg(3) = af(1) + bg(1) = (af + bg) (1)

Thus af + bg $\in$ W, and hence W is a subspace.

(c) 0 $\in$ W, since $\hat{0}(-x) = 0 = -0 = -\hat{0}(x)$. Suppose f . g $\in$ W. Then f(–x) = –f(x) and g(–x) = –g(x).

Also, for scalar a and b,

(af + bg)(–x) = af (–x) + bg(–x) = –af(x) – bg(x) = –(af + bg)(x)

Thus ab + gf $\in$ W, and hence W is a subspace of V.

$f_1 g \in$ **W**

f(4) = 3 + f(2)

and g(4) = 3 + g(2)

If a, b $\in \mathbb{R}$, then

af + bg)(4) = af (4) + bg(4) = a[3 + f(2)] + b[3 + g(2)], from (i)

= af(2) + bg(2) + 3a + 3b = (af + bg) (2) + 3(a + b)

$\neq 3 + (af + bg)(2)$

$\therefore$ $(af + bg)(4) \notin$ **W**

Hence, **W** is not a subspace of **V**.

15.(D) Here $R : 0 \le y \le 2$

$y^2 \le x \le 4$

on changing the order of integration

we get $0 \le x \le 4$

$0 \le y \le \sqrt{x}$

so $\int_0^2 \int_{y^2}^4 \cos(x^{3/2}) dx\, dy = \int_0^4 \int_0^{\sqrt{x}} \cos(x^{3/2}) dx\, dy = \int_0^4 y \cos(x^{3/2})\Big|_0^{\sqrt{x}} dx = \int_0^4 \sqrt{x} \cos(x^{3/2}) dx$

$= \int_0^8 \frac{2}{3} \cos 4\, du = \frac{2}{3} \sin 8$

16.(C) The equations of the curve are $x = t^2,\ y = t - \frac{1}{3}t^3$

$\therefore dx/dt = 2t$ and $dy/dt = (1 - t^2)$

Hence $\frac{ds}{dt} = \sqrt{\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\}} = \sqrt{\{4t^2 + (1-t^2)^2\}} = \sqrt{(1+t^2)^2}$

or $ds/dt = (1 + t^2)$

Also for the loop (putting y = 0) t varies from 0 to . $\sqrt{3}$

The required surface $= 2\pi \int_{t=0}^{\sqrt{3}} y\, ds = 2\pi \int_{t=0}^{\sqrt{3}} y . \frac{ds}{dt} dt$

$= 2\pi \int_{t=0}^{\sqrt{3}} \left(t - \frac{1}{3}t^3\right)(1 + t^2)\, dt$

$= \frac{2\pi}{3} \int_0^{\sqrt{3}} (3t + 2t^3 - t^5) dt = \frac{2\pi}{3}\left[\frac{3}{2}t^2 + \frac{1}{2}t^4 - \frac{1}{6}t^6\right]_0^{\sqrt{3}}$

$= \frac{2}{3}\pi[(9/2) + (9/2) - (9/2)] = 3\pi.$

17.(B) A.E. is $m^2 + 4 = 0$ m = 2i, -2i

C.F. is $y = C_1 \cos 2x + c_2 \sin 2x$

$$\text{P.I.} = \frac{1}{D^2+4}\sin^2 x = \frac{1}{D^2+4}\frac{1}{2}(1-\cos 2x) = \left[\frac{1}{D^2+4}1 - \frac{1}{D^2+4}\cos 2x\right] = \left[\frac{1}{4} - x\frac{1}{2D}\cos 2x\right]$$

$$= \left[\frac{1}{4} - \frac{x}{4}\sin 2x\right] = \frac{1}{8} - \frac{1}{8}\, x \sin 2x$$

Required solution is $y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{8} - \frac{1}{8} x \sin 2x$

18.(A) $M = \iiint_{\Re} r^2 dV = \int_0^{2\pi}\int_0^b\int_0^h r^2 \cdot r dz\, dr\, d\theta = \int_0^{2\pi}\int_0^b r^3 h\, dr\, d\theta =$

$$\int_0^{2\pi} \frac{1}{4} hr^4 \Big]_0^b d\theta = \int_0^{2\pi}\frac{1}{4}hb^4 d\theta = \frac{1}{4}hb^4 \cdot 2\pi = \frac{1}{2}\pi hb^4.$$

19.(B) It is a very well known theorem that if f and g be continuous real – valued function on the metric space M and A be the set of all $x \in M$ s.t. $f(x) < g(x)$ then A is open set.

20.(A) Let $f(x) = \sin x\,(1 + \cos x)$

$\Rightarrow f'(x) = \cos x\,(1 + \cos x) + \sin x\,(-\sin x) = \cos x + \cos^2 x - \sin^2 x$

$= \cos x + \cos 2x$

$f''(x) = -\sin x - 2\sin 2x = -(\sin x + 2\sin 2x)$

for maximum or minimum value of f(x), $f'(x) = 0$

therefore $\cos x + \cos 2x = 0$

$\Rightarrow \cos x = -\cos 2x$

$\Rightarrow \cos x = -\cos(\pi \pm 2x)$

$\Rightarrow x = \pi \pm 2x$

$\Rightarrow \qquad x = \frac{\pi}{3}, -\pi$

Now $f''\left(\frac{\pi}{3}\right) = -2\sin\frac{2\pi}{3} - \sin\frac{\pi}{3} = -\frac{2\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \qquad = -\frac{3\sqrt{3}}{2} = -ve$

Therefore f(x) is maximum at x =π/3.

21.(B) Let

$$g(\alpha) = \int_{\frac{\pi}{2}-\alpha}^{\pi/2} \sin\theta \cos^{-1}(\cos\alpha\cos\theta)\,d\theta$$

Applying the general Leibnity's rule we obtain

$$g'(\alpha) = \int_{\frac{\pi}{2}-\alpha}^{\pi/2} \frac{\sin\alpha\,d\theta}{\sqrt{1-\cos^2\alpha\,\mathrm{cosec}^2\theta}} + \sin\left(\frac{\pi}{2}-\alpha\right)\cos^{-1}\left(\cos\alpha\,\mathrm{cosec}\left(\frac{\pi}{2}-\alpha\right)\right)$$

$$= \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} \frac{\sin\alpha\sin\theta\,d\theta}{\sqrt{\sin^2\theta - \cos^2\theta}} + \cos\alpha\cos^{-1}(1) = \int_0^{\sin\alpha} \frac{\sin\alpha\,dt}{\sqrt{\sin^2\alpha - t^2}} + 0 = \sin\alpha\left[\sin^{-1}\left(\frac{t}{\sin\alpha}\right)\right]_0^{\sin\alpha}$$

$$= .\ \sin\alpha\left[\sin^{-1}\left(\frac{\sin\alpha}{\sin\alpha}\right) - \sin^{-1}0\right] = \frac{\pi}{2}(1-\cos\alpha)$$

Integrating w.r. to α.

$$g(\alpha) = -\frac{\pi}{2}\cos\alpha + C$$

But , $g\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ then c = π/2

hence . $g(\alpha) = \frac{\pi}{2}(1-\cos\alpha)$

22.(C) **2 and – 2**

We want extreme values of f(x,y) = xy subject to the constraints

$$g(x,y)=\frac{x^2}{8}+\frac{y^2}{2}-1=0 \qquad (1)$$

To do so, we first find the value of x,y and λ for which

$$\nabla f = \lambda \nabla g$$

and $\quad g(x,y) = 0$

the gradient eq. in eq (1) gives

$$y\hat{i}+x\hat{j}=\frac{\lambda}{4}x\hat{i}+\lambda y\hat{j}$$

from which we find

$$y=\frac{\lambda}{4}x,\ \ x=\lambda y$$

and . $y=\frac{\lambda}{4}(\lambda y)=\frac{\lambda^2}{4}y$

so y = 0 or λ = ± 2. we now consider these two cases

case I if y = 0 then x = y = 0 But (0, 0) is not on the ellipse therefore y + 0.

case II If y ≠ 0 then λ = ± 2 and x = substituting this in the eq. g(x,y) = 0

given

$$\frac{(\pm 2y)^2}{8}+\frac{y^2}{2}=1$$

$\Rightarrow 4y^2 + 4y^2 = 8 \Rightarrow y = \pm 1$

The function f(x,y) = xy therefore takes on its extrem values on the ellipse at the four points

(± 2, 1), (± 2, – 1)

The extreme values are xy = 2 and xy = – 2.

23.(D) Let L(w) = w

and x,y $\in$ w, $\alpha, \beta \in$ FE

then x,y$\in$ L(w)

x,y are linear combination of members of w.

$\Rightarrow$ $\alpha x + \beta y$ is a linear combination of members of w

$\Rightarrow$ $\alpha x + \beta y \in$ L(w)

$\Rightarrow$ $\alpha x + \beta y \in$ w

$\Rightarrow$ w is a subspace of v.

24.(C) When origin O is inside S. In this case, divergence theorem cannot be applied to the region V enclosed by S, since $F = \frac{r}{r^3}$ has a point to discontinuity at the origin. To remove this difficulty, let us enclose the origin by a small sphere Σ of radius ε.

The function F is continuously differentiable at the points of the region v´ enclosed between S and Σ. Therefore applying divergence theorem for this region V´, we have

$$\iint_S \frac{r}{r^3} \cdot n\, dS = \iint_\Sigma \frac{r}{r^3} \cdot d\Sigma$$

$$\iiint_{V'} \text{div}\left(\frac{r}{r^3}\right) dV' = 0 \qquad \text{since div} \cdot \left(\frac{r}{r^3}\right) = 0$$

$$\therefore \iint_S \frac{r}{r^3}\cdot n\,dS = -\iint_\Sigma \frac{r}{r^3}\cdot d\Sigma$$

Now on the sphere Σ, the outward drawn normal n is directed towards the centre. Therefore on Σ, we have

$$n = -\frac{r}{\varepsilon}$$

$$\therefore,\ -\iint_\Sigma \frac{r}{r^3}\cdot n\,d\Sigma = -\iint_\Sigma \frac{r}{\varepsilon^3}\cdot\left(-\frac{r}{\varepsilon}\right)d\Sigma \text{ since on } \Sigma, r = \varepsilon$$

$$= \iint_\Sigma \frac{r^2}{\varepsilon^4}\,d\Sigma = \iint_\Sigma \frac{\varepsilon^2}{\varepsilon^4}\,d\Sigma = \frac{1}{\varepsilon^2}\iint_\Sigma d\Sigma = \frac{1}{\varepsilon^2}4\pi r\varepsilon^2 = 4\pi$$

Hence . $\iint_S \frac{r}{r^3}\cdot n\,dS = 4\pi$

25.(B) Since here $a_n = p(n)$

if let $p(x) = a_0x^N + a_1x^{N-1} + ... + a_N$

Radius of convergence of power series is given by

$$R = \lim_{n\to\infty}\frac{a_n}{a_{n+1}} = \lim_{n\to\infty}\frac{p(n)}{p(n+1)} = \operatorname{Lim}_{n\to\infty}\frac{a_0n^N + a_1n^{N-1} + ...a_N}{a_0(n+1)^N + a_1(n+1)^{N-1} + ... + a_N}$$

$$= \operatorname{Lim}_{n\to\infty}\frac{a_0 + a_1 1/n + ...a_N\frac{1}{N^N}}{a_0\left(1+\frac{1}{n}\right)^N + a_1\frac{1}{n}\left(1+\frac{1}{n}\right)^{N-1} + ...a_n\left(\frac{1}{N^N}\right)} = \frac{a_0}{a_0} = 1$$

26.(D) The definition of lipschitz function is given by

Let $A \subseteq R$ and let $f : A \to \mathbb{R}$ if there exists a constant k > 0 such that

$|f(x) - f(u)| \le k|x - u|$ for all x, u $\in$ A

so we have lipschitz function in our question asked (as it satisfies lipschitz condition)

now we will show uniform continuity of lipschitz function

Since $|f(x) - f(y)| \le c|x - y| \quad \forall x, y \in \mathbb{R}$

The given ε > 0 we can take δ = ε/k if x. y $\in \mathbb{R}$ satisfy |x – y| < δ

Then $|f(x) - f(y)| < K \cdot \frac{\varepsilon}{k} = \varepsilon$

Therefore f is uniformly continuous on R

But for differentiability

Since $|f(x) - f(y)| \le c|x - y|$

$\Rightarrow \frac{|f(x) - f(y)|}{|x - y|} \le c$

$\Rightarrow \left|\frac{f(x) - f(y)}{x - y} - 0\right| \le c$

$\Rightarrow$ f is differentiable at y $\forall x, y \in \mathbb{R}$

$\Rightarrow$ f is differentiable function to 0

27.(A) Since Given system of linear equation

$a_1x = b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

It can be represented as AX = B

here [A] can have rank 3 or less than 3

i.e. $\rho(A) \leq 3$

and [Ab] have rank 3 as we have given

$$\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix} \neq 0$$

$\Rightarrow$ ρ(A/b) = 3

case(I) if ρ(A) = 3

Then ρ(A) = ρ(Ab)

$\Rightarrow$ system have exactly one solution

Case(II) if

ρ(A) < 3

$\Rightarrow$ system have no solution.

$\Rightarrow$ system can have at most one solution.

28.(C) Since $y_1 = 1 + x$ and $y_2 = e^x$ be two solutions of

$y''(x) + P(x)\, y'(x) + Q(x)\, y(x) = 0$ ....(1)

then will satisfy (1)

so if $y_1 = 1 + x$ if $y_2 = e^x$

$y' = 1$ $y_2^1 = e^x$

$y'' = 0$ $\qquad\qquad$ $y_2'' = e^x$

and $\quad P(x) + Q(1 + x) = 0$

$(1 + P + Q)e^x = 0$

$\Rightarrow \quad P + Q(1 + x) = 0$

$P + Q + 1 = 0$

on solving them we get $\quad P = \dfrac{-1-x}{x}$

$Q = \dfrac{1}{x}$

If $P = \dfrac{-1-x}{x}$ and $Q = \dfrac{1}{x}$

Differential equation becomes

$xy'' - (1 + x)y' + 1 = 0$

and solution is of the form

$y = c_1e^x + c_2(1 + x)$

take options $\left.\begin{matrix} y(0)=2=c_1+c_2 \\ y'(0)=1=c_1+c_2 \end{matrix}\right]$ ....(1)

on solving equation set (1)

we get no solution

Thus for the conditions $y(0) = 2$, $y'(0) = 1$

differential equation has no solution

29.(D) In this case the region D will be the region between these two circles and that will only change the limits in the double integral.

Here is the work for this integral.

$$\oint_C y^3dx - x^3dy = -3\iint_D (x^2+y^2)\,dA = -3\int_0^{2\pi}\int_1^2 r^3drd\theta = -3\int_0^{2\pi}\frac{1}{4}r^4\Big|_1^2 d\theta = -3\int_0^{2\pi}\frac{15}{4}d\theta = -\frac{45\pi}{2}$$

30.(B) Here S_n = Sum of first n terms of the given series

$$S_n = 1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots + \left(\frac{1}{3}\right)^{n-1} = \frac{1-(2/3)^n}{1-2/3} = 3 - 3(2/3)^n$$

31. (A,C) Given eq. can be written as

$$y_2 + \frac{1}{x}y_1 - \frac{1}{x^2}y = e^x \qquad \ldots(1)$$

$\Rightarrow \quad (x^2D^2 + xD - 1)\,y = 0 \qquad \ldots(2)$

where

$$D = \frac{d}{dx}$$

which is a homogeneous eq.

Put $x = e^z$

then $D_1 = D = \frac{d}{dx}$ then eq (2) become

$[D_1(D_1 - 1) + D_1 - 1]\,y = 0$

$[D_1^2 - 1]\,y = 0 \qquad \ldots(3)$

its A E is $m^2 - 1 = 0$

$m = \pm 1$

then solu. of eq (3) is

$y = ae^{z} + be^{-z}$

$y = ae^{z} + b(e^{z})^{-1}$

$y = Ax + Bx^{-1}$ ...(4)

be the complete solution of eq (1) then A and B are function of x which are so chosen that (1) will be satisfied. Differentiating (4) w.r.t x we have

$y_1 = A_1x + A + B_1x^{-1} - Bx^{-2}$ ...(5)

Choose A and B s.t.

$A_1x + B_1x^{-1} = 0$ ...(6)

then by eq (4) we get

$y_1 = A - Bx^{-2}$ ...(7)

Differentiating (7)

$y_2 = A_1 - (B_1x^{-2} - 2Bx^{-3})$ ...(8)

using (4), (7) and (8), (1) reduces to

$A_1 - B_1x^{-2} = e^{x}$ ...(9)

Solving (6) and (9)

$A_1 = \frac{dA}{dx} = \frac{1}{2}\ e^{x}$

$$B_1 = \frac{dB}{dx} = -\frac{1}{2}x^2 e^x$$

Integrating

$$A = \frac{1}{2}\int e^x dx + c_1 = \frac{1}{2}e^x + c_1$$

$$B = -\frac{1}{2}\int x^2 e^x + dx + c_2 = c_2 - \frac{1}{2}x^2 e^x + xe^x - e^x.$$

Substitute the value of A and B in eq. (4) we have the required solution is

$$y = \left[\frac{1}{2}e^x + c_1\right]x + \left[c_2 - \frac{1}{2}x^2 e^x + xe^x - e^x\right]x^{-1}$$

$$\Rightarrow \quad y = c_1 x + c_2 x^{-1} + e^x - x^{-1}e^x$$

$$\Rightarrow \quad y = c_1 x + c_2 \frac{1}{x} + e^x - x^{-1} e^x$$

$$\Rightarrow \quad xy = c_1 x^2 + c_2 + xe^x - e^x$$

32.(C,D) Given differential eq. is

$$(D^2 + 4)\, y = \sec 2x$$

its AE is

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$CF = c_1 \cos 2x + c_2 \sin 2x$$

$$PI = \frac{1}{D^2+4}\sec 2x = \frac{1}{(D+2i)(D-2i)}\sec 2x = \frac{1}{4i}\left[\frac{1}{D-2i} - \frac{1}{D+2i}\right]\sec 2x \qquad ...(1)$$

Now $$\frac{1}{D-2i}\sec 2x = e^{2ix}\int \sec 2x.\, e^{-2ix}dx = e^{2ix}\int \sec 2x\,(\cos 2x - i\sin 2x)\,dx$$

$$= e^{2ix}\int (1 - i\tan 2x)\,dx = e^{2ix}\left[x + \frac{i}{2}\log\cos 2x\right] \qquad ...(2)$$

Similarly

$$\frac{1}{D+2i}\sec ax = + e^{-2ix}\left[x - \frac{i}{2}\log\cos 2x\right] \qquad ...(3)$$

by eq (2) and (3)

eq (1) becomes

$$PI = \frac{1}{4i}\left[e^{2ix}\left(x + \frac{i}{2}\log\cos 2x\right) - e^{-2ix}\left(x - \frac{i}{2}\log\cos 2x\right)\right]$$

$$= x\left(\frac{e^{2ix} - e^{-2ix}}{4i}\right) + \frac{i}{2}\log\cos 2x\left(\frac{e^{2ix} + e^{-2ix}}{4i}\right)$$

$$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\log\cos 2x$$

hence the required sol is

$$y = CF + PI$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\log\cos 2x$$

$$4y = 4c_1\cos 2x + 4c_2\sin 2x + 2x\sin 2x + \cos 2x\log\cos 2x$$

33. (A,C,D) In this case we differentiate from left to right. Here are the derivatives for this part.

$$f_x = \frac{z^3y^2}{x}$$

$$f_{xx} = -\frac{z^3y^2}{x^2}$$

$$f_{xxy} = -\frac{2z^3y}{x^2}$$

$$f_{xxyz} = -\frac{6z^2y}{x^2}$$

$$f_{xxyzz} = -\frac{12zy}{x^2}$$

34. (A,C,D) The Chain Rule will have to be applied three times in this.

$$p'(x) = 2(\sin(h(2x))) \cdot \cos(h(2x)) \cdot h'(2x) \cdot 2$$

Because $2x = 2\left(\frac{\pi}{2}\right) = p$, you can write:

$$p'\left(\frac{\pi}{2}\right) = 2(\sin(h(\pi))) \cdot \cos(h(\pi)) \cdot h'(\pi) \cdot 2$$

$$p'\left(\frac{\pi}{2}\right) = 2(\sin)\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{4}\right) \cdot (-1) \cdot 2$$

$$p'\left(\frac{\pi}{2}\right) = 2 \cdot \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}(-2)$$

$$p'\left(\frac{\pi}{2}\right) = -2$$

35. (B,D) $\begin{vmatrix} u_x & u_y \\ u_x & u_y \end{vmatrix} = \begin{vmatrix} 2x/y & -x^2/y^2 \\ y & x \end{vmatrix} = 3x^2/y$. Therefore,

dudv = $(3x^2/y)$ dxdy = 3u dxdy

$\Rightarrow \quad dxdy = \frac{1}{3u} dudv$

$\Rightarrow \quad 6\,dxdy = \frac{2}{u} dudv$

36.(A,D) $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \\ -1 & 0 & -2 \\ 3 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$

$\begin{bmatrix} 12-1+6 & 16+0+8 & 20-2+14 \\ -5+9 & 0+0+12 & 0-10+21 \end{bmatrix} = \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$

$\begin{bmatrix} 17 & 24 & 32 \\ 4 & 12 & 11 \end{bmatrix} - \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix} = 0$

$\begin{bmatrix} 17-8x-3y & 24-6z & 0 \\ 0 & 0 & 11-26x+5y \end{bmatrix} = 0$

$\Rightarrow \quad 24 – 6z = 0$

$17 – 8x – 3y = 0$

$11 – 26x – 5y = 0$

on solving the above eq.we get

x = 1, y = 3

Hence x = 1, y = 3, z = 4

37.(A,C,D) Given $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 3 & 1 & 2 \\ -4 & 1 & 0 \end{bmatrix}$

$$B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$$

$$B' = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 2 & 3 & 4 \\ -9 & 4 & 2 \\ -10 & 6 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -9 & -10 \\ 3 & 4 & 6 \\ 4 & 2 & 4 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 2 & 3 & 4 \\ -9 & 4 & 2 \\ -10 & 6 & 4 \end{bmatrix}$$

$\Rightarrow (AB)' = B'A'$

38.(A,B,D) Let $S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$

$\alpha_1 = (1, 2, 3)$, $\alpha_2 = (2, 1, -1)$

$\alpha_3 = (1, -1, -4)$, $\alpha_4 = (4, 2, -2)$

given

$\alpha_4 = 2\,\alpha_2$ so that the

vectors α_2 and α_4 are linearly dependent

If $S_1 = \{\alpha_1, \alpha_2, \alpha_3\}$ then by subspace of R^3 spanned by S_1 is the same as that spanned by S.

There is no real number c s.t.

$\alpha_1 = c\alpha_2$ therefore the vectors α_1 and α_2 are linearly independent.

Now,examine whether the vector α_3 lie in the subspace of R^3 spanned by the vector α_1 and α_2 or not

Let $\alpha_3 = a\alpha_1 + b\alpha_2$.

where $a, b \in R$

then $(1, -1, -4) = a(1, 2, 3) + b(2, 1, -1)$

$a + 2b = 1$ ... (i)

$2a + b = -1$ ... (ii)

$3a - b = -4$ ... (iii)

Solving the eq. (i) and (ii) we get a = –1, b = 1 these values of a and b also satisfy the eq. (iii).

so $\alpha_3 = -\alpha_1 + \alpha_2$.

thus the vector α_3 has be expressed as a linear combination of α_1 and α_2 so that the subspace of R^3 spanned by the vectors α_1, α_2 and α_3.

Hence $T = \{\alpha_1, \alpha_2\}$ is a linear independent subset of S which spans the same subspace of R^3 as a is spanned by S.

39.(B,C,D) $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & \alpha \end{bmatrix}$

$$\Rightarrow \bar{A} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ i & \alpha \end{bmatrix}$$

$$\therefore (\bar{A})' = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & \alpha \end{bmatrix} = A^{\theta}$$

$$\Rightarrow \quad AA^{\theta} = 1$$

$$\Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & \alpha \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\begin{bmatrix} 2 & i(1+\alpha) \\ -i(1+\alpha) & 1+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 1+\alpha = 0 \quad \Rightarrow \quad \alpha = -1$$

40.(A,C,D) The set

$$A = \left\{ w_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ w_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i,\ w_3 = 1 \right\}$$

of the cube roots of 1 forms a group w.r.t. multiplication set of complex number c. since

(I) The product of any two elements of the set is an element of the set.

$$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1$$

(II) The associative law holds in c and hence in A.

(III) w_3 is the identity element.

(iv) The inverse of w_1, w_2 and w_3 are w_2, w_1 and w_3 respectively therefore A is group w.r.t. multiplication.

41. **– 2**

For solenoidal vector, $\vec{\nabla}\cdot\vec{V} = 0$ ...(1)

$\Rightarrow \vec{\nabla}.\vec{V} = \frac{\partial}{\partial x}(x+3y)+\frac{\partial}{\partial y}(y-2z)+\frac{\partial}{\partial z}(x+\alpha z) = 1 + 1 + \alpha = 2+\alpha$

Using (1), we get $2 + \alpha = 0 \Rightarrow \alpha = -2$

42. **0**

u is harmonic on $\left\{(x,y)\,\middle|\,x^2+y^2 \le 1\right\}$

We know that $\int_s\int \frac{\partial \vec{u}}{\partial n}\, d\vec{s} = \iint_s \left(\frac{\partial u}{\partial n}\hat{n}\right).\hat{n}\, ds$

$= \iint_s (\Delta u).\hat{n}\, ds = \iiint_v \nabla.(\nabla u)\, dv$, by divergence theorem .

$= \iiint_v \nabla^2 u\, dv = 0$. Since $\nabla^2 u = 0$ in v , u is harmonic inv.

43. **1.428**

Since $\mathbf{r} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ is given by $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $x = t$, $y = t^2$, $z = t^3$

and $\frac{d\mathbf{r}}{dt} = \hat{i} + 2t\hat{j} + 3t^2\hat{k}.$

$$\int_C \mathbf{F}.d\mathbf{r} = \int_{-1}^{1}\left(\mathbf{F}.\frac{d\mathbf{r}}{dt}\right)dt \qquad ....(1)$$

and $\mathbf{F} = xy\hat{i} + yz\hat{j} + zx\hat{k} = t^3\hat{i} + t^5\hat{j} + t^4\hat{k}.$

$\therefore$ $\mathbf{F}.\frac{d\mathbf{r}}{dt} = \left(t^3\hat{i} + t^5\hat{j} + t^4\hat{k}\right).\left(\hat{i} + 2t\hat{j} + 3t^2\hat{k}\right) = t^3 + 2t^6 + 3t^6 = t^3 + 5t^6.$

$\therefore$ From (1), $\int_C \mathbf{F}.d\mathbf{r} = \int_{-1}^{1}(t^3 + 5t^6)dt = \left[\frac{t^4}{4} + \frac{5t^7}{7}\right]_{-1}^{1} = \frac{1}{4} - \frac{1}{4} + \frac{10}{7} = \frac{10}{7} = 1.428$

44. **31**

We have $b^2 = aba^{-1}$

$\Rightarrow b^4 = (aba^{-1})(aba^{-1}) = ab(a^{-1}a)^{nr}b^2\, ab^{mr1} = ab^2a^{-1}$

$= a(aba^{-1})a^{-1}$

$\Rightarrow b^4 = a^2ba^{-2}$

$\Rightarrow b^8 = (a^2ba^{-2})(a^2ba^{-2}) = a^2b^2a^{-2} = a(aba^{-1})a^{-2} = a^3ba^{-3}$

$\Rightarrow b^{16} = a^4ba^{-4}$ (as above)

$\Rightarrow b^{32} = a^4ba^{-5} = b$ as $a^5 = e$

$\Rightarrow b^{31} = e$ 31 is a multiple of 0(b)

Since 31 is a prime number, it is the least +ve integer such that

$b^{31} = e$

$\Rightarrow$ $O(b) = 31$.

We are, of course, taking $b \neq e$.

45. **6**

Let H be any subgroup of order 12 in S_4

Let $H \neq A_4$

and let if possible that H contains and odd permutation thus H has 6 odd and 6 even permutation

$\Rightarrow$ $H \cap A_4$ is a subgroup of A_4 of order 6

$\Rightarrow$ A_4 has subgroup of order 6 contradiction by a well know theorem

Hence A_4 is the only subgroup of S_4 of order 12.

46. **0**

$$L = \lim_{n\to\infty}\left|\frac{(n+1)!(2x+1)^{n+1}}{n!(2x+1)^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)n!(2x+1)}{n!}\right| = |2x+1|\lim_{n\to\infty}(n+1)$$

At this point we need to be careful. The limit is infinite, but there is that term with the x's in front of the limit. We'll have $L = \infty > 1$ provided $x \neq -\frac{1}{2}$

So, this power series will only converge if $x = . -\frac{1}{2}$ We know that every power series well converge for x = a and in this case $a = -\frac{1}{2}$. Remember that we get a from $(x - a)^n$, and coefficient of the x must be a one only

In this case we say the radius of convergence is R = 0 and the interval of convergence is x $= -\frac{1}{2}$, and it is really true that we really did mean interval of convergence even though it is only a point.

47. 3

Here $\iint_s \mathbf{f}\cdot\mathbf{n}\,dS = \iiint_V \nabla\cdot f\,dV$, (By divergence theorem). $= \iiint_V (2x+2y+2z)\,dV$

$$= \int_{x=0}^{1}\int_{y=0}^{1}\int_{x=0}^{1} 2\left(x+y+z^2\right)dz\,dy\,dx = 3.$$

48. 1

Since let $L = \lim_{R\to\infty}\frac{\int_R^\infty r^n\,e^{-r^2/2}dr}{R^{n-1}\,e^{-R^2/2}}$ $\left(\frac{0}{0}\text{form}\right)$

By L hospital rule

$$L = \lim_{R\to\infty}\frac{\infty^n e^{-\infty^2/2}(0) - R^n e^{-R^2/2}dR}{dR(n-1)R^{n-2}e^{-R^2/2} - R^{n-1}e^{-R^2/2}} = \lim_{R\to\infty}-\frac{R^2}{(n-1)-R^2} = \lim_{R\to\infty}-\frac{-1}{\frac{(n-1)}{R^2}-1} = 1$$

49. –1

Analytic Method : since

$$f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$$

Replacing x by 2x and y by 0, then

$$f(x) = \frac{f(2x)+f(0)}{2}$$

$\Rightarrow$ f(2x) + f(0) = 2f(x)

$\Rightarrow \quad f(2x) - 2f(x) = -f(0)$

$\because \quad f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ .... (1)

$$= \lim_{h\to 0}\frac{f\left(\frac{2x+2h}{2}\right)-f(x)}{h} = \lim_{h\to 0}\left\{\frac{\frac{f(2x)+f(2h)}{2}-f(x)}{h}\right\} = \lim_{h\to 0}\left\{\frac{f(2x)+f(2h)-2f(x)}{2h}\right\}$$

$$= \lim_{h\to 0}\left\{\frac{f(2h)-f(0)}{2h}\right\}$$ {from (1)}

$= f'(0) = -1 \; x \in R$ (given)

Integrating, we get $f(x) = -x + c$

Putting $x = 0$, then $f(0) = 0 + c = 1$ (given)

$\therefore \quad c = 1$

then $\quad f(x) = 1 - x$

$\therefore \quad f(2) = 1 - 2 = -1$

50. 0

$u = \log_e (x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2+y^2} + \frac{1}{1+\frac{y^2}{x^2}}\cdot\left(-\frac{y}{x^2}\right) = \frac{2x-y}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2).2-(2x-y)2x}{(x^2+y^2)^2} = \frac{2y^2-2x^2+2xy}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2} + \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{2y+x}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\left(x^2+y^2\right).(2y+x)2y}{\left(x^2+y^2\right)^2} = \frac{2x^2-2y^2-2xy}{\left(x^2+y^2\right)^2}$$

$$\therefore \qquad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

51. **0.208**

Given Integral

$$I = \int_C \frac{e^y}{x} dx + \left(e^{\frac{y}{n}} x + x \right) dy$$

By Green's theorem

$$\oint (M dx + N dy) = \iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\, dy$$

using Green's Theorem integral becomes = $\iint_C \left[\left(\frac{e^y}{x} + 1 \right) - \frac{e^y}{x} \right] dx\, dy$

$$= \int_{\frac{1}{2}}^{1} \int_{2}^{1+x^2} dx\, dy = \int_{-1}^{\frac{1}{2}} (x^2 - 1) dx = \left[\frac{x^3}{3} - x \right]_{-1}^{\frac{1}{3}} = \frac{1}{24} - \frac{1}{2} + \frac{1}{3} - 1 = \frac{5}{24} = 0.208$$

52. **5**

$$\int_C \mathbf{A}.d\mathbf{r} = \int_C \left[\left(3x^2 + 6y \right) \mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k} \right].(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) = \int_C \left(3x^2 + 6y \right) dx - 14yzdy + 20xz^2 dz$$

If x - t, $y = t^2$, $z = t^3$, points (0, 0, 0) and (1, 1, 1) correspond to t = 0 and t = 1 respectively.

Then

$$\int_C \mathbf{A.dr} = \int_{t=0}^{1}\left(3t^2+6t^2\right)dt - 14\left(t^2\right)\left(t^3\right)d\left(t^2\right) + 20(t)\left(t^3\right)^2 d\left(t^3\right) = \int_{t=0}^{1} 9t^2dt - 28t^6dt + 60t^9dt$$

$$= \int_{t=0}^{1}\left(9t^2 - 28t^6 + 60t^9\right)dt = 3t^3 - 4t^7 + 6t^{10}\Big|_0^1 = 5$$

53. 0

we have $\vec{V} = \hat{i}\dfrac{x}{x^2+y^2} + \hat{j}\dfrac{y}{x^2+y^2}$

Since C : $x^2 + y^2 = 1$

take $x = \cos\theta$ and $y = \sin\theta$

Then $\vec{v} = i\cos\theta + j\sin\theta$

$d\vec{r} = \left(-\sin\theta\,\hat{i} + \hat{j}\cos\theta\right)d\theta$

so $\int_C \vec{v}.d\vec{r} = \int\left(-\cos\theta\sin\theta + \sin\theta\cos\theta\right)d\theta = 0$

$\Rightarrow$ for every closed path $\int_C \vec{v}.d\vec{r} = 0$

54. - 3.5

The matrix form of the given system of equations is

$$\begin{bmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} .$$

The given system of equations will have a unique solution if and only if the coefficient matrix is non-singular.

Performing $R_1 \leftrightarrow R_3$, we get

$$\begin{bmatrix} 1 & 2 & -\lambda \\ 2 & 1 & 1 \\ 3 & -1 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

Performing $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, we get

$$\begin{bmatrix} 1 & 2 & -\lambda \\ 0 & -3 & 1+2\lambda \\ 3 & -7 & 4\lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix}. \quad \ldots(i)$$

Therefore the coefficient matrix will be non-singular if and only if

$$-12\lambda + 7 + 14\lambda \neq 0$$

i.e. if and only if $\lambda \neq . -\frac{7}{2}$

Thus the given system will have a unique solution if $\lambda \neq . -\frac{7}{2}$

In case $\lambda =, -\frac{7}{2}$ the equation (i) becomes

$$\begin{bmatrix} 1 & 2 & \frac{7}{2} \\ 0 & -3 & -6 \\ 0 & -7 & -14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix}$$

Performing $R_3 \to R_3 R_2$, we get

$$\begin{bmatrix} 1 & 2 & \frac{7}{2} \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -\frac{16}{3} \end{bmatrix},$$

showing that given equations are inconsistent in this case.

Thus if $\lambda = -\frac{7}{2} = -3.5$, no solution exists.

55. **0.033**

Let $I = \int_0^1 \frac{x^5 + x^4 + x^2}{x}\sqrt{4x^5 + 5x^4 + 10x^2}\,dx = \int_0^1 (x^4 + x^3 + x)\sqrt{4x^5 + 5x^4 + 10x^2}\,dx$

Put $4x^5 + 5x^4 + 10x^2 = t$

$(20x^4 + 20x^3 + 20x)\,dx = dt$

$20(x^4 + x^3 + x)dx = dt = \int_0^{19} \frac{\sqrt{t}}{20}dt = \frac{1}{20}\left[\frac{t^{3/2}}{\frac{3}{2}}\right]_0^{19} = \frac{1}{30}\left[t^{3/2}\right]_0^{19} = \frac{1}{30}(19)^{3/2}$

but given $\int_0^1 \frac{x^5 + x^4 + x^2}{x}\sqrt{4x^5 + 5x^4 + 10x^2}\,dx = \alpha\,(19)^{3/2}$

$\Rightarrow \frac{1}{30}(19)^{3/2} = \alpha(19)^{3/2}$

$\alpha = \frac{1}{30} = 0.033$

56. **1.5**

$av(f) = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2\left(3 - \frac{3}{2}x\right)dx = \frac{1}{2}\left(\int_0^2 3\,dx - \frac{3}{2}\int_0^2 x\,dx\right)$

$= \frac{1}{2}\left[3(2-0) - \frac{3}{2}\left(\frac{2^2}{2} - \frac{0^2}{2}\right)\right] = \frac{1}{2}[6-3] = \frac{3}{2} = 1.5$

57. **15.466**

Since given $f(x) = x^4 - 4x^2 + 6$ and $f(-x) = x^4 - 4x^2 + 6$

So $f(x) = f(-x)$

it is even function on the symmetric interval [–2, 2] ,so

$$\int_{-2}^{2}(x^4-4x^2+6)dx = 2\int_0^2(x^4-4x^2+6)dx = 2\left[\frac{x^5}{5}-\frac{4x^3}{3}+6x\right]_0^2 = 2\left(\frac{32}{5}-\frac{32}{3}+12\right)$$

$$= 2\left(\frac{96-160+180}{15}\right) = \frac{232}{15} = 15.466$$

58. 0.75

Let $I = \int_0^{\pi/6}\cos^{-3}2\theta\,\sin 2\theta\,d\theta = \int_0^{\pi/6}\frac{\sin 2\theta}{\cos^3 2\theta}d\theta$

Put $\cos 2\theta = t$ and $-2\sin 2\theta\,d\theta = dt$

where $\theta = 0 \Rightarrow t = 1$

$\theta = \frac{\pi}{6} \Rightarrow t = \frac{1}{2}$

$$\int_1^{1/2}\frac{-dt/2}{t^3} = -\frac{1}{2}\int_1^{1/2}t^{-3}\,dt = -\frac{1}{2}\left[\frac{t^{-3+1}}{(-3+1)}\right]_1^{1/2} = -\frac{1}{2}\left(\frac{t^{-2}}{-2}\right)_1^{1/2} = \frac{1}{4}\left(\frac{1}{t^2}\right)_1^{1/2} = \frac{1}{4}\left[\frac{1}{\left(\frac{1}{2}\right)^2}-1\right] = \frac{1}{4}[4-1] = \frac{3}{4} =$$

0.75

59. 1.6

$$\int_{-2}^{5}\left[\frac{f(x)+g(x)}{5}\right]dx = \frac{1}{5}\int_{-2}^{5}[f(x)+g(x)]dx$$

by property $\left[\int_a^b kf(x)dx = k\int_a^b f(x)dx\right] = \frac{1}{5}\left[\int_{-2}^{5}f(x)\,dx + \int_{-2}^{5}g(x)dx\right]$ .....(1)

by property $\left[\int_a^b[f(x)\pm g(x)]dx\right] = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

given $\int_{-2}^{5} f(x)dx = 6$

$\int_{-2}^{5} g(x)dx = 2$

then by equation (1) , $= \frac{1}{5}(6+2) = \frac{8}{5}$

$\therefore \int_{-2}^{5} \left(\frac{f(x)+g(x)}{5}\right)dx = \frac{8}{5}$ = 1.6

60. 18

given Integral is

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz\ dy\ dx = \int_0^3 \int_0^{\sqrt{9-x^2}} [z]_0^{\sqrt{9-x^2}} dy\,dx = \int_0^3 \int_0^{\sqrt{9-x^2}} \left[\sqrt{9-x^2}\right] dy\,dx = \int_0^3 \left[\sqrt{9-x^2}\cdot y\right]_0^{\sqrt{9-x^2}} dx$$

$$= \int_0^3 \left(9-x^2\right)dx = \left(9x - \frac{x^3}{3}\right)_0^3 = 27 - \frac{27}{3} = 18$$