

## GATE 2014: General Instructions during Examination

1. Total duration of the GATE examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Common Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use a non-programmable type calculator, however, sharing of calculators is not allowed.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

 You have not visited the question yet.

 You have not answered the question.

 You have answered the question.

 You have NOT answered the question, but have marked the question for review.

 You have answered the question, but marked it for review.

The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is answered, but marked for review, then the answer will be considered for evaluation unless the status is modified by the candidate.*

### Navigating to a Question :

7. To answer a question, do the following:
  - a. Click on the question number in the Question Palette to go to that question directly.
  - b. Select an answer for a multiple choice type question by clicking on the bubble placed before the 4 choices, namely A, B, C and D. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
  - c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
  - d. Click on **Mark for Review & Next** to save your answer for the current question and also mark it for review, and then go to the next question.

**Caution:** Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

### Answering a Question :

8. Procedure for answering a multiple choice (MCQ) type question:
  - a. Choose one answer from the 4 options (A,B,C,D) given below the question, click on the bubble placed before the chosen option.
  - b. To deselect your chosen answer, click on the bubble of the chosen option again or click on the **Clear Response** button.
  - c. To change your chosen answer, click on the bubble of another option.
  - d. To save your answer, you MUST click on the **Save & Next** button.
9. Procedure for answering a numerical answer type question:
  - a. To enter a number as your answer, use the virtual numerical keypad.
  - b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.
  - c. To clear your answer, click on the **Clear Response** button.
  - d. To save your answer, you MUST click on the **Save & Next** button
10. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ) or entered (for numerical answer type) for a question that is **Marked for Review**, that answer will be considered in the evaluation unless the status is modified by the candidate.*
11. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.
12. Note that ONLY Questions for which answers are *saved* or *marked for review after answering* will be considered for evaluation.

### Choosing a Section :

13. Sections in this question paper are displayed on the top bar of the screen. Questions in a Section can be viewed by clicking on the name of that Section. The Section you are currently viewing will be highlighted.
14. A checkbox is displayed for every optional Section, if any, in the Question Paper. To select the optional Section for answering, click on the checkbox for that Section.
15. If the checkbox for an optional Section is not selected, the **Save & Next** button and the **Mark for Review & Next** button will NOT be enabled for that Section. You will

only be able to see questions in this Section, but you will not be able to answer questions in the Section.

16. After clicking the **Save & Next** button for the last question in a Section, you will automatically be taken to the first question of the next Section in sequence.
17. You can move the mouse cursor over the name of a Section to view the answering status for that Section.

### **Changing the Optional Section :**

18. After answering the chosen optional Section, partially or completely, you can change the optional Section by selecting the checkbox for a new Section that you want to attempt. A warning message will appear along with a table showing the number of questions answered in each of the previously chosen optional Sections and a checkbox against each of these Sections. Click on a checkbox against a Section that you want to reset and then click on the **RESET** button. Note that RESETTING a Section will DELETE all the answers for questions in that Section. Hence, if you think that you may want to select this Section again later, you will have to note down your answers for questions in that Section. If you do not want to reset the Section and want to continue answering the previously chosen optional Section, then click on the **BACK** button.
19. If you deselect the checkbox for an optional Section in the top bar, the following warning message will appear: "Deselecting the checkbox will DELETE all the answers for questions in this Section. Do you want to deselect this Section?" If you want to deselect, click on the **RESET** button. If you do not want to deselect, click on the **BACK** button.
20. You can shuffle between different Sections or change the optional Sections any number of times.

# GATE 2014 Examination

## MA: Mathematics

Duration: 180 minutes

Maximum Marks: 100

### Read the following instructions carefully.

1. To login, enter your Registration Number and password provided to you. Kindly go through the various symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **View All Questions** button in the screen.
3. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory.  
The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.  
The subject specific GATE paper section consists of **55** questions, out of which question numbers 1 to 25 are of 1-mark each, while question numbers 26 to 55 are of 2-mark each.
4. Depending upon the GATE paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Common Data** button that appears at the top, right hand side of the screen.
5. The computer allotted to you at the examination center runs specialized software that permits only one answer to be selected for multiple-choice questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual keyboard and mouse.
6. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
7. In each paper a candidate can answer a total of 65 questions carrying 100 marks.
8. The question paper may consist of questions of **multiple choice type (MCQ)** and **numerical answer type**.
9. Multiple choice type questions will have four choices against A, B, C, D, out of which only **ONE** is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
10. For numerical answer type questions, each question will have a numerical answer and there will not be any choices. **For these questions, the answer should be entered** by using the virtual keyboard that appears on the monitor and the mouse.
11. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of  $\frac{1}{3}$  marks for a 1-mark question and  $\frac{2}{3}$  marks for a 2-mark question.
12. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
13. Non-programmable type Calculator is allowed. Charts, graph sheets, and mathematical tables are **NOT** allowed in the Examination Hall. You must use the Scribble pad provided to you at the examination centre for all your rough work. The Scribble Pad has to be returned at the end of the examination.

### Declaration by the candidate:

“I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition”.

**Q. 1 – Q. 5 carry one mark each.**

- Q.1 A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.

The word closest in meaning to comprehension is

- (A) understanding (B) meaning (C) concentration (D) stability

- Q.2 Choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest \_\_\_\_\_ was his ability to forgive.

- (A) vice (B) virtues (C) choices (D) strength

- Q.3 Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

- (A) Rajan has decided to work only in a group.  
 (B) Rajan and Sajan were formed into a group against their wishes.  
 (C) Sajan had decided to give in to Rajan's request to work with him.  
 (D) Rajan had believed that Sajan and he would be working together.

- Q.4 If  $y = 5x^2 + 3$ , then the tangent at  $x = 0, y = 3$

- (A) passes through  $x = 0, y = 0$  (B) has a slope of +1  
 (C) is parallel to the  $x$ -axis (D) has a slope of -1

- Q.5 A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

**Q. 6 – Q. 10 carry two marks each.**

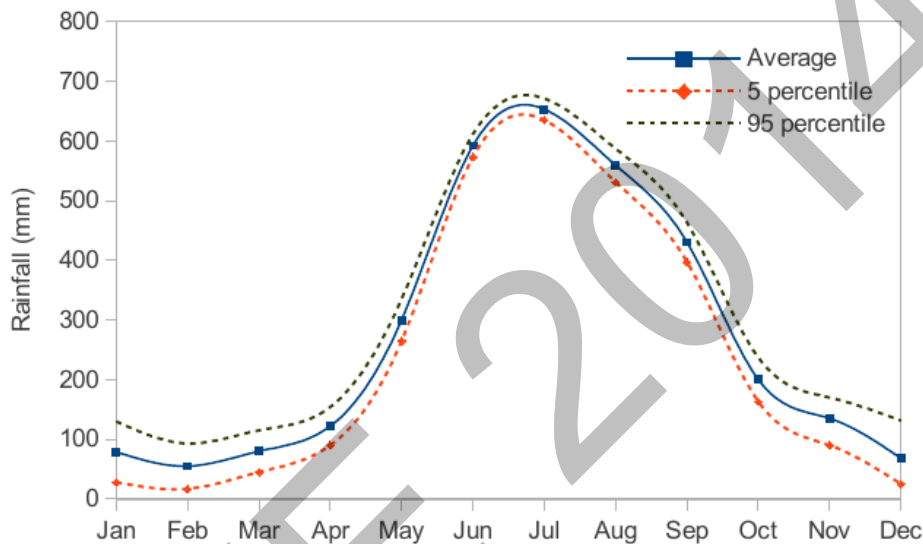
- Q.6 Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

- (A) ALRVX (B) EPVZB (C) ITZDF (D) OYEIK

- Q.7 Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

- Q.8 The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?
- Q.9 One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?
- (A) 3.0 (B) 2.5 (C) 1.5 (D) 1.25
- Q.10 The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? ( $k$  percentile is the value such that  $k$  percent of the data fall below that value)



- (i) On average, it rains more in July than in December  
 (ii) Every year, the amount of rainfall in August is more than that in January  
 (iii) July rainfall can be estimated with better confidence than February rainfall  
 (iv) In August, there is at least 500 mm of rainfall
- (A) (i) and (ii) (B) (i) and (iii)  
 (C) (ii) and (iii) (D) (iii) and (iv)

**END OF THE QUESTION PAPER**

## Symbols and Notation Used

$\mathbb{R}$	The set of all real numbers
$\mathbb{C}$	The set of all complex numbers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{N}$	The set of all positive integers
$\mathbb{Z}$	The set of all integers
$\mathbb{Z}_n$	The cyclic group of order $n$
$\operatorname{Re}(z)$	Real part of a complex number $z$
$\operatorname{Im}(z)$	Imaginary part of a complex number $z$
$P(E)$	Probability of an event $E$
$E[X]$	Expectation of a random variable $X$
$\operatorname{Var}(X)$	Variance of a random variable $X$
$M_n(\mathbb{R})$	The set of all $n \times n$ matrices with real entries
$GL_2(\mathbb{R})$	The set of all $2 \times 2$ matrices with determinant 1
$C[a, b]$	The set of all real valued continuous functions on the real interval $[a, b]$
$C^n[a, b]$	The set of all real valued $n$ -times continuously differentiable functions on the real interval $[a, b]$
$\ell^p$	The space of all $p$ -summable sequences
$L^p[a, b]$	The space of all $p$ -integrable functions on the interval $[a, b]$
$\langle \cdot, \cdot \rangle$	An inner product
$\dot{\theta}$	Derivative of $\theta$ with respect to $t$

**Q. 1 – Q. 25 carry one mark each.**

Q.1 The function  $f(z) = |z|^2 + i\bar{z} + 1$  is differentiable at

- (A)  $i$                       (B)  $1$                       (C)  $-i$                       (D) no point in  $\mathbb{C}$

Q.2

The radius of convergence of the power series  $\sum_{n=0}^{\infty} 4^{(-1)^n n} z^{2n}$  is \_\_\_\_\_

Q.3 Let  $E_1$  and  $E_2$  be two non empty subsets of a normed linear space  $X$  and let

$E_1 + E_2 := \{x + y \in X : x \in E_1 \text{ and } y \in E_2\}$ . Then which of the following statements is **FALSE**:

- (A) If  $E_1$  and  $E_2$  are convex, then  $E_1 + E_2$  is convex  
 (B) If  $E_1$  or  $E_2$  is open, then  $E_1 + E_2$  is open  
 (C)  $E_1 + E_2$  must be closed if  $E_1$  and  $E_2$  are closed  
 (D) If  $E_1$  is closed and  $E_2$  is compact, then  $E_1 + E_2$  is closed

Q.4

Let  $y(x)$  be the solution to the initial value problem  $\frac{dy}{dx} = \sqrt{y+2x}$  subject to  $y(1.2) = 2$ . Using the Euler method with the step size  $h = 0.05$ , the approximate value of  $y(1.3)$ , correct to two decimal places, is \_\_\_\_\_

Q.5 Let  $\alpha \in \mathbb{R}$ . If  $\alpha x$  is the polynomial which interpolates the function  $f(x) = \sin \pi x$  on  $[-1, 1]$  at all the zeroes of the polynomial  $4x^3 - 3x$ , then  $\alpha$  is \_\_\_\_\_

Q.6 If  $u(x, t)$  is the D'Alembert's solution to the wave equation  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ ,  $x \in \mathbb{R}$ ,  $t > 0$ , with the condition  $u(x, 0) = 0$  and  $\frac{\partial u}{\partial t}(x, 0) = \cos x$ , then  $u\left(0, \frac{\pi}{4}\right)$  is \_\_\_\_\_

Q.7

The solution to the integral equation  $\varphi(x) = x + \int_0^x \sin(x - \xi)\varphi(\xi)d\xi$  is

- (A)  $x^2 + \frac{x^3}{3}$                       (B)  $x - \frac{x^3}{3!}$                       (C)  $x + \frac{x^3}{3!}$                       (D)  $x^2 - \frac{x^3}{3!}$



Q.8

The general solution to the ordinary differential equation  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(4x^2 - \frac{9}{25}\right)y = 0$  in terms of Bessel's functions,  $J_\nu(x)$ , is

(A)  $y(x) = c_1 J_{3/5}(2x) + c_2 J_{-3/5}(2x)$

(B)  $y(x) = c_1 J_{3/10}(x) + c_2 J_{-3/10}(x)$

(C)  $y(x) = c_1 J_{3/5}(x) + c_2 J_{-3/5}(x)$

(D)  $y(x) = c_1 J_{3/10}(2x) + c_2 J_{-3/10}(2x)$

Q.9

The inverse Laplace transform of  $\frac{2s^2 - 4}{(s-3)(s^2 - s - 2)}$  is

(A)  $(1+t)e^{-t} + \frac{7}{2}e^{-3t}$

(B)  $\frac{e^t}{3} + te^{-t} + 2t$

(C)  $\frac{7}{2}e^{3t} - \frac{e^{-t}}{6} - \frac{4}{3}e^{2t}$

(D)  $\frac{7}{2}e^{-3t} - \frac{e^t}{6} - \frac{4}{3}e^{-2t}$

Q.10

If  $X_1, X_2$  is a random sample of size 2 from an  $\mathcal{N}(0,1)$  population, then  $\frac{(X_1 + X_2)^2}{(X_1 - X_2)^2}$  follows

(A)  $\chi_{(2)}^2$

(B)  $F_{2,2}$

(C)  $F_{2,1}$

(D)  $F_{1,1}$

Q.11

Let  $Z \sim \mathcal{N}(0,1)$  be a random variable. Then the value of  $E[\max\{Z, 0\}]$  is

(A)  $\frac{1}{\sqrt{\pi}}$

(B)  $\sqrt{\frac{2}{\pi}}$

(C)  $\frac{1}{\sqrt{2\pi}}$

(D)  $\frac{1}{\pi}$

Q.12

The number of non-isomorphic groups of order 10 is \_\_\_\_\_

- Q.13 Let  $a, b, c, d$  be real numbers with  $a < c < d < b$ . Consider the ring  $C[a, b]$  with pointwise addition and multiplication. If  $S = \{f \in C[a, b] : f(x) = 0 \text{ for all } x \in [c, d]\}$ , then
- (A)  $S$  is NOT an ideal of  $C[a, b]$
- (B)  $S$  is an ideal of  $C[a, b]$  but NOT a prime ideal of  $C[a, b]$
- (C)  $S$  is a prime ideal of  $C[a, b]$  but NOT a maximal ideal of  $C[a, b]$
- (D)  $S$  is a maximal ideal of  $C[a, b]$
- Q.14 Let  $R$  be a ring. If  $R[x]$  is a principal ideal domain, then  $R$  is necessarily a
- (A) Unique Factorization Domain
- (B) Principal Ideal Domain
- (C) Euclidean Domain
- (D) Field
- Q.15 Consider the group homomorphism  $\varphi: M_2(\mathbb{R}) \rightarrow \mathbb{R}$  given by  $\varphi(A) = \text{trace}(A)$ . The kernel of  $\varphi$  is isomorphic to which of the following groups?
- (A)  $M_2(\mathbb{R}) / \{A \in M_2(\mathbb{R}) : \varphi(A) = 0\}$
- (B)  $\mathbb{R}^2$
- (C)  $\mathbb{R}^3$
- (D)  $GL_2(\mathbb{R})$
- Q.16 Let  $X$  be a set with at least two elements. Let  $\tau$  and  $\tau'$  be two topologies on  $X$  such that  $\tau' \neq \{\emptyset, X\}$ . Which of the following conditions is necessary for the identity function  $id: (X, \tau) \rightarrow (X, \tau')$  to be continuous?
- (A)  $\tau \subseteq \tau'$
- (B)  $\tau' \subseteq \tau$
- (C) no conditions on  $\tau$  and  $\tau'$
- (D)  $\tau \cap \tau' = \{\emptyset, X\}$
- Q.17 Let  $A \in M_3(\mathbb{R})$  be such that  $\det(A - I) = 0$ , where  $I$  denotes the  $3 \times 3$  identity matrix. If the  $\text{trace}(A) = 13$  and  $\det(A) = 32$ , then the sum of squares of the eigenvalues of  $A$  is \_\_\_\_\_
- Q.18 Let  $V$  denote the vector space  $C^5[a, b]$  over  $\mathbb{R}$  and  $W = \left\{ f \in V : \frac{d^4 f}{dt^4} + 2 \frac{d^2 f}{dt^2} - f = 0 \right\}$ . Then
- (A)  $\dim(V) = \infty$  and  $\dim(W) = \infty$
- (B)  $\dim(V) = \infty$  and  $\dim(W) = 4$
- (C)  $\dim(V) = 6$  and  $\dim(W) = 5$
- (D)  $\dim(V) = 5$  and  $\dim(W) = 4$

Q.19 Let  $V$  be a real inner product space of dimension  $10$ . Let  $x, y \in V$  be non-zero vectors such that  $\langle x, y \rangle = 0$ . Then the dimension of  $\{x\}^\perp \cap \{y\}^\perp$  is \_\_\_\_\_

Q.20 Consider the following linear programming problem:

$$\text{Minimize } x_1 + x_2$$

Subject to:

$$2x_1 + x_2 \geq 8$$

$$2x_1 + 5x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

The optimal value to this problem is \_\_\_\_\_

Q.21 Let

$$f(x) := \begin{cases} -3\pi & \text{if } -\pi < x \leq 0 \\ 3\pi & \text{if } 0 < x < \pi \end{cases}$$

be a periodic function of period  $2\pi$ . The coefficient of  $\sin 3x$  in the Fourier series expansion of  $f(x)$  on the interval  $[-\pi, \pi]$  is \_\_\_\_\_

Q.22 For the sequence of functions

$$f_n(x) = \frac{1}{x^2} \sin\left(\frac{1}{nx}\right), \quad x \in [1, \infty),$$

consider the following quantities expressed in terms of Lebesgue integrals

$$\text{I: } \lim_{n \rightarrow \infty} \int_1^\infty f_n(x) dx \quad \text{II: } \int_1^\infty \lim_{n \rightarrow \infty} f_n(x) dx.$$

Which of the following is **TRUE**?

- (A) The limit in **I** does not exist
- (B) The integrand in **II** is not integrable on  $[1, \infty)$
- (C) Quantities **I** and **II** are well-defined, but **I**  $\neq$  **II**
- (D) Quantities **I** and **II** are well-defined and **I** = **II**

Q.23 Which of the following statements about the spaces  $\ell^p$  and  $L^p[0, 1]$  is **TRUE**?

- (A)  $\ell^3 \subset \ell^7$  and  $L^6[0, 1] \subset L^9[0, 1]$
- (B)  $\ell^3 \subset \ell^7$  and  $L^9[0, 1] \subset L^6[0, 1]$
- (C)  $\ell^7 \subset \ell^3$  and  $L^6[0, 1] \subset L^9[0, 1]$
- (D)  $\ell^7 \subset \ell^3$  and  $L^9[0, 1] \subset L^6[0, 1]$

Q.24 The maximum modulus of  $e^{z^2}$  on the set  $S = \{z \in \mathbb{C} : 0 \leq \operatorname{Re}(z) \leq 1, 0 \leq \operatorname{Im}(z) \leq 1\}$  is

- (A)  $2/e$
- (B)  $e$
- (C)  $e + 1$
- (D)  $e^2$

- Q.25 Let  $d_1, d_2$  and  $d_3$  be metrics on a set  $X$  with at least two elements. Which of the following is **NOT** a metric on  $X$ ?
- (A)  $\min\{d_1, 2\}$  (B)  $\max\{d_2, 2\}$
- (C)  $\frac{d_3}{1+d_3}$  (D)  $\frac{d_1+d_2+d_3}{3}$

**Q. 26 – Q. 55 carry two marks each.**

- Q.26 Let  $\Omega = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$  and let  $C$  be a smooth curve lying in  $\Omega$  with initial point  $-1 + 2i$  and final point  $1 + 2i$ . The value of  $\int_C \frac{1+2z}{1+z} dz$  is
- (A)  $4 - \frac{1}{2} \ln 2 + i \frac{\pi}{4}$  (B)  $-4 + \frac{1}{2} \ln 2 + i \frac{\pi}{4}$
- (C)  $4 + \frac{1}{2} \ln 2 - i \frac{\pi}{4}$  (D)  $4 - \frac{1}{2} \ln 2 + i \frac{\pi}{2}$

- Q.27 If  $a \in \mathbb{C}$  with  $|a| < 1$ , then the value of

$$\frac{(1-|a|^2)}{\pi} \int_{\Gamma} \frac{|dz|}{|z+a|^2},$$

where  $\Gamma$  is the simple closed curve  $|z| = 1$  taken with the positive orientation, is \_\_\_\_\_

- Q.28 Consider  $C[-1,1]$  equipped with the supremum norm given by  $\|f\|_{\infty} = \sup\{|f(t)| : t \in [-1, 1]\}$  for  $f \in C[-1, 1]$ . Define a linear functional  $T$  on  $C[-1,1]$  by  $T(f) = \int_{-1}^0 f(t) dt - \int_0^1 f(t) dt$  for all  $f \in C[-1,1]$ . Then the value of  $\|T\|$  is \_\_\_\_\_

- Q.29 Consider the vector space  $C[0,1]$  over  $\mathbb{R}$ . Consider the following statements:

**P:** If the set  $\{t f_1, t^2 f_2, t^3 f_3\}$  is linearly independent, then the set  $\{f_1, f_2, f_3\}$  is linearly independent, where  $f_1, f_2, f_3 \in C[0,1]$  and  $t^n$  represents the polynomial function  $t \mapsto t^n, n \in \mathbb{N}$

**Q:** If  $F: C[0,1] \rightarrow \mathbb{R}$  is given by  $F(x) = \int_0^1 x(t^2) dt$  for each  $x \in C[0,1]$ , then  $F$  is a linear map.

Which of the above statements hold **TRUE**?

- (A) Only **P** (B) Only **Q** (C) Both **P** and **Q** (D) Neither **P** nor **Q**

Q.30 Using the Newton-Raphson method with the initial guess  $x^{(0)} = 6$ , the approximate value of the real root of  $x \log_{10} x = 4.77$ , after the second iteration, is \_\_\_\_\_

Q.31 Let the following discrete data be obtained from a curve  $y = y(x)$ :

$$x: \quad 0 \quad 0.25 \quad 0.5 \quad 0.75 \quad 1.0$$

$$y: \quad 1 \quad 0.9896 \quad 0.9589 \quad 0.9089 \quad 0.8415$$

Let  $S$  be the solid of revolution obtained by rotating the above curve about the  $x$ -axis between  $x = 0$  and  $x = 1$  and let  $V$  denote its volume. The approximate value of  $V$ , obtained using

Simpson's  $\frac{1}{3}$  rule, is \_\_\_\_\_

Q.32 The integral surface of the first order partial differential equation

$$2y(z-3)\frac{\partial z}{\partial x} + (2x-z)\frac{\partial z}{\partial y} = y(2x-3)$$

passing through the curve  $x^2 + y^2 = 2x$ ,  $z = 0$  is

(A)  $x^2 + y^2 - z^2 - 2x + 4z = 0$

(B)  $x^2 + y^2 - z^2 - 2x + 8z = 0$

(C)  $x^2 + y^2 + z^2 - 2x + 16z = 0$

(D)  $x^2 + y^2 + z^2 - 2x + 8z = 0$

Q.33

The boundary value problem,  $\frac{d^2\phi}{dx^2} + \lambda\phi = x$ ;  $\phi(0) = 0$  and  $\frac{d\phi}{dx}(1) = 0$ , is converted into the

integral equation  $\phi(x) = g(x) + \lambda \int_0^1 k(x, \xi)\phi(\xi)d\xi$ , where the kernel  $k(x, \xi) = \begin{cases} \xi, & 0 < \xi < x \\ x, & x < \xi < 1 \end{cases}$

Then  $g\left(\frac{2}{3}\right)$  is \_\_\_\_\_

Q.34

If  $y_1(x) = x$  is a solution to the differential equation  $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ , then its general solution is

(A)  $y(x) = c_1x + c_2(x \ln|1+x^2| - 1)$

(B)  $y(x) = c_1x + c_2\left(\ln\left|\frac{1-x}{1+x}\right| + 1\right)$

(C)  $y(x) = c_1x + c_2\left(\frac{x}{2}\ln|1-x^2| + 1\right)$

(D)  $y(x) = c_1x + c_2\left(\frac{x}{2}\ln\left|\frac{1+x}{1-x}\right| - 1\right)$

Q.35 The solution to the initial value problem  $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{-t} \sin t$ ,  $y(0) = 0$  and  $\frac{dy}{dt}(0) = 3$ , is

(A)  $y(t) = e^t (\sin t + \sin 2t)$

(B)  $y(t) = e^{-t} (\sin t + \sin 2t)$

(C)  $y(t) = 3e^t \sin t$

(D)  $y(t) = 3e^{-t} \sin t$

Q.36 The time to failure, in months, of light bulbs manufactured at two plants A and B obey the exponential distribution with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, the probability that it was manufactured at plant A is \_\_\_\_\_

Q.37 Let  $X, Y$  be continuous random variables with joint density function

$$f_{X,Y}(x, y) = \begin{cases} e^{-y}(1 - e^{-x}) & \text{if } 0 < x < y < \infty \\ e^{-x}(1 - e^{-y}) & \text{if } 0 < y \leq x < \infty \end{cases}$$

The value of  $E[X + Y]$  is \_\_\_\_\_

Q.38 Let  $X = [0, 1) \cup (1, 2)$  be the subspace of  $\mathbb{R}$ , where  $\mathbb{R}$  is equipped with the usual topology. Which of the following is **FALSE**?

(A) There exists a non-constant continuous function  $f: X \rightarrow \mathcal{Q}$

(B)  $X$  is homeomorphic to  $(-\infty, -3) \cup [0, \infty)$

(C) There exists an onto continuous function  $f: [0, 1] \rightarrow \bar{X}$ , where  $\bar{X}$  is the closure of  $X$  in  $\mathbb{R}$

(D) There exists an onto continuous function  $f: [0, 1] \rightarrow X$

Q.39 Let  $X = \begin{bmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{bmatrix}$ . A matrix  $P$  such that  $P^{-1}XP$  is a diagonal matrix, is

(A)  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(D)  $\begin{bmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

- Q.40 Using the Gauss-Seidel iteration method with the initial guess  $\{x_1^{(0)} = 3.5, x_2^{(0)} = 2.25, x_3^{(0)} = 1.625\}$ , the second approximation  $\{x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\}$  for the solution to the system of equations

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1, \end{aligned}$$

is

- (A)  $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.1563$   
 (B)  $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.3125, x_3^{(2)} = 2.6563$   
 (C)  $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.6563$   
 (D)  $x_1^{(2)} = 5.4991, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.1563$
- Q.41 The fourth order Runge-Kutta method given by

$$u_{j+1} = u_j + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4], \quad j = 0, 1, 2, \dots,$$

is used to solve the initial value problem  $\frac{du}{dt} = u, \quad u(0) = \alpha$ .

If  $u(1) = 1$  is obtained by taking the step size  $h = 1$ , then the value of  $K_4$  is \_\_\_\_\_

- Q.42 A particle P of mass  $m$  moves along the cycloid  $x = (\theta - \sin \theta)$  and  $y = (1 + \cos \theta)$ ,  $0 \leq \theta \leq 2\pi$ . Let  $g$  denote the acceleration due to gravity. Neglecting the frictional force, the Lagrangian associated with the motion of the particle P is:
- (A)  $m(1 - \cos \theta) \dot{\theta}^2 - mg(1 + \cos \theta)$   
 (B)  $m(1 + \cos \theta) \dot{\theta}^2 + mg(1 + \cos \theta)$   
 (C)  $m(1 + \cos \theta) \dot{\theta}^2 + mg(1 - \cos \theta)$   
 (D)  $m(\theta - \sin \theta) \dot{\theta}^2 - mg(1 + \cos \theta)$

- Q.43 Suppose that  $X$  is a population random variable with probability density function

$$f(x; \theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where  $\theta$  is a parameter. In order to test the null hypothesis  $H_0: \theta = 2$ , against the alternative hypothesis  $H_1: \theta = 3$ , the following test is used: Reject the null hypothesis if  $X_1 \geq \frac{1}{2}$  and accept otherwise, where  $X_1$  is a random sample of size 1 drawn from the above population. Then the power of the test is \_\_\_\_\_

- Q.44 Suppose that  $X_1, X_2, \dots, X_n$  is a random sample of size  $n$  drawn from a population with probability density function

$$f(x; \theta) = \begin{cases} \frac{x}{\theta^2} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where  $\theta$  is a parameter such that  $\theta > 0$ . The maximum likelihood estimator of  $\theta$  is

(A)  $\frac{\sum_{i=1}^n X_i}{n}$

(B)  $\frac{\sum_{i=1}^n X_i}{n-1}$

(C)  $\frac{\sum_{i=1}^n X_i}{2n}$

(D)  $\frac{2\sum_{i=1}^n X_i}{n}$

- Q.45 Let  $\vec{F}$  be a vector field defined on  $\mathbb{R}^2 \setminus \{(0,0)\}$  by  $\vec{F}(x, y) = \frac{y}{x^2 + y^2} \hat{i} - \frac{x}{x^2 + y^2} \hat{j}$ . Let  $\gamma, \alpha: [0, 1] \rightarrow \mathbb{R}^2$  be defined by

$$\gamma(t) = (8 \cos 2\pi t, 17 \sin 2\pi t) \text{ and } \alpha(t) = (26 \cos 2\pi t, -10 \sin 2\pi t).$$

If  $3 \int_{\alpha} \vec{F} \cdot d\vec{r} - 4 \int_{\gamma} \vec{F} \cdot d\vec{r} = 2m\pi$ , then  $m$  is \_\_\_\_\_

- Q.46 Let  $g: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be defined by  $g(x, y, z) = (3y + 4z, 2x - 3z, x + 3y)$  and let  $S = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$ . If

$$\iiint_{g(S)} (2x + y - 2z) dx dy dz = \alpha \iiint_S z dx dy dz,$$

then  $\alpha$  is \_\_\_\_\_

- Q.47 Let  $T_1, T_2: \mathbb{R}^5 \rightarrow \mathbb{R}^3$  be linear transformations such that  $\text{rank}(T_1) = 3$  and  $\text{nullity}(T_2) = 3$ . Let  $T_3: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transformation such that  $T_3 \circ T_1 = T_2$ . Then  $\text{rank}(T_3)$  is \_\_\_\_\_

- Q.48 Let  $\mathbb{F}_3$  be the field of 3 elements and let  $\mathbb{F}_3 \times \mathbb{F}_3$  be the vector space over  $\mathbb{F}_3$ . The number of distinct linearly dependent sets of the form  $\{u, v\}$ , where  $u, v \in \mathbb{F}_3 \times \mathbb{F}_3 \setminus \{(0,0)\}$  and  $u \neq v$  is \_\_\_\_\_

- Q.49 Let  $\mathbb{F}_{125}$  be the field of 125 elements. The number of non-zero elements  $\alpha \in \mathbb{F}_{125}$  such that  $\alpha^5 = \alpha$  is \_\_\_\_\_

- Q.50 The value of  $\iint_R xy dx dy$ , where  $R$  is the region in the first quadrant bounded by the curves  $y = x^2$ ,  $y + x = 2$  and  $x = 0$  is \_\_\_\_\_



Q.51 Consider the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0,$$

with the boundary conditions  $u(0, t) = 0$ ,  $u(\pi, t) = 0$  for  $t > 0$ , and the initial condition  $u(x, 0) = \sin x$ . Then  $u\left(\frac{\pi}{2}, 1\right)$  is \_\_\_\_\_

Q.52 Consider the partial order in  $\mathbb{R}^2$  given by the relation  $(x_1, y_1) < (x_2, y_2)$  EITHER if  $x_1 < x_2$  OR if  $x_1 = x_2$  and  $y_1 < y_2$ . Then in the order topology on  $\mathbb{R}^2$  defined by the above order

- (A)  $[0, 1] \times \{1\}$  is compact but  $[0, 1] \times [0, 1]$  is NOT compact  
 (B)  $[0, 1] \times [0, 1]$  is compact but  $[0, 1] \times \{1\}$  is NOT compact  
 (C) both  $[0, 1] \times [0, 1]$  and  $[0, 1] \times \{1\}$  are compact  
 (D) both  $[0, 1] \times [0, 1]$  and  $[0, 1] \times \{1\}$  are NOT compact.

Q.53 Consider the following linear programming problem:

Minimize:  $x_1 + x_2 + 2x_3$

Subject to

$$x_1 + 2x_2 \geq 4$$

$$x_2 + 7x_3 \leq 5$$

$$x_1 - 3x_2 + 5x_3 = 6$$

$$x_1, x_2 \geq 0, \quad x_3 \text{ is unrestricted}$$

The dual to this problem is:

Maximize:  $4y_1 + 5y_2 + 6y_3$

Subject to

$$y_1 + y_3 \leq 1$$

$$2y_1 + y_2 - 3y_3 \leq 1$$

$$7y_2 + 5y_3 = 2$$

and further subject to:

- (A)  $y_1 \geq 0$ ,  $y_2 \leq 0$  and  $y_3$  is unrestricted  
 (B)  $y_1 \geq 0$ ,  $y_2 \geq 0$  and  $y_3$  is unrestricted  
 (C)  $y_1 \geq 0$ ,  $y_3 \leq 0$  and  $y_2$  is unrestricted  
 (D)  $y_3 \geq 0$ ,  $y_2 \leq 0$  and  $y_1$  is unrestricted

Q.54 Let  $X = C^1[0,1]$ . For each  $f \in X$ , define

$$p_1(f) := \sup \{|f(t)| : t \in [0,1]\}$$

$$p_2(f) := \sup \{|f'(t)| : t \in [0,1]\}$$

$$p_3(f) := p_1(f) + p_2(f).$$

Which of the following statements is **TRUE**?

- (A)  $(X, p_1)$  is a Banach space
- (B)  $(X, p_2)$  is a Banach space
- (C)  $(X, p_3)$  is NOT a Banach space
- (D)  $(X, p_3)$  does NOT have denumerable basis

Q.55 If the power series  $\sum_{n=0}^{\infty} a_n(z + 3 - i)^n$  converges at  $5i$  and diverges at  $-3i$ , then the power series

- (A) converges at  $-2 + 5i$  and diverges at  $2 - 3i$
- (B) converges at  $2 - 3i$  and diverges at  $-2 + 5i$
- (C) converges at both  $2 - 3i$  and  $-2 + 5i$
- (D) diverges at both  $2 - 3i$  and  $-2 + 5i$

**END OF THE QUESTION PAPER**