

## GATE - ELECTRONICS AND COMMUNICATION ENGINEERING

### SAMPLE THEORY

- **BOOLEAN ALGEBRA**
- **THE Z-TRANSFORM**
- **THE SUPERHETERODYNE RECEIVER**

# VPM CLASSES

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## BOOLEAN ALGEBRA

Boolean algebra can be used to formalize the combinations of binary logic states. The fundamental relations are given in Table of the text. In these relations, A and B are binary quantities, that is, they can be either logical true (T or 1) or logical false (F or 0). Most of these relations are obvious. Here are a few of them: given by

$$AA = A; A + A = A; A + \bar{A} = 1 \quad A\bar{A} = 0; \bar{\bar{A}} = A$$

Recall the text sometimes uses an apostrophe for inversion (A'). We use the standard overbar notation ( $\bar{A}$ ).

We can use algebraic expressions to complete our definitions of the basic logic gates we began above. Note that the Boolean operations of "multiplication" and "addition" are defined by the truth tables for the AND and OR gates given. Using these definitions, we can define all of the logic gates algebraically. The truth tables can also be constructed from these relations, if necessary.

- AND:  $Q = AB$
- OR :  $Q = A + B$
- NAND:  $Q = \overline{AB}$
- NOR:  $Q = \overline{A + B}$
- XOR:  $Q = A \oplus B$
- INV:  $Q = \bar{A}$
- BUF:  $Q = A$

### Combining Gates

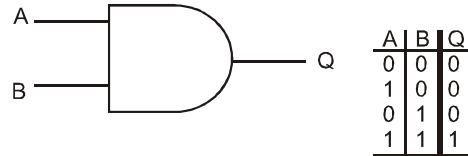
Let's re-express the XOR operation in terms of standard Boolean operations. The following truth table evaluates the expression  $Q = \bar{A}B + A\bar{B}$ .

A	B	$\bar{A}B$	$A\bar{B}$	Q
0	0	0	0	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	0

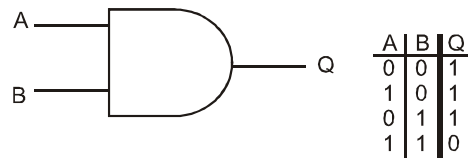
We see that this truth table is identical to the one for the XOR operation. Therefore, we can write

$$A \oplus B = \bar{A}B + A\bar{B} \quad \dots(1)$$

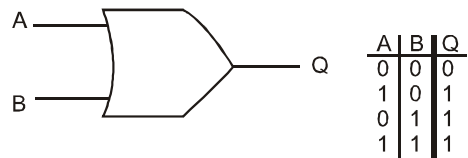
A schematic of this expression in terms of gates is given in Fig. Recall that the open circles at the output or input of a gate represent inversion.



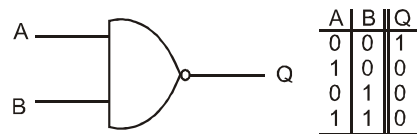
**Fig-1 : AND gate.**



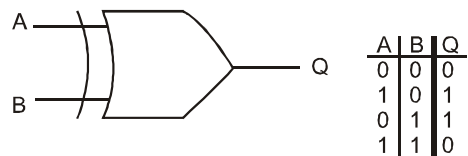
**Fig-2 NAND gate**



**Fig-3 : OR gate.**



**Fig-4 NOR gate**



**Fig-5 : XOR (exclusive OR) gate.**

## 2. Properties of Boolean Algebra

Boolean Algebra is a mathematical system consisting of a set of two or more distinct elements, two binary operators denoted by the symbols (+) and (.) and one unary operator denoted by the symbol either bar (–) or prime ('). They satisfy the commutative, associative distributive, absorption, consensus and idempotency properties of the Boolean Algebra.

**Commutative property** Boolean addition is cumulative, given by

$$A + B = B + A \quad \dots(1a)$$

According to this property, the order of the OR operation conducted on the variables makes no difference. Boolean algebra is also commutative over multiplication, given by

$$A \cdot B = B \cdot A \quad \dots(1b)$$

This means that the order of the AND operation conducted on the variables makes no difference.

**Associative property** The associative property of addition is given by

$$A + (B + C) = (A + B) + C \quad \dots(2a)$$

The OR operation of several variables results in the same, regardless of the grouping of the variables. The associative law of multiplication is given by

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C \quad \dots(2b)$$

According to this law, it makes no difference in what order the variables are grouped during the AND operation of several variables.

**Distributive property**

1. The Boolean addition is distributive over Boolean multiplication, given by

$$A + BC = (A + B)(A + C) \quad \dots(3a)$$

This property states that the AND operation (multiplication) of several variables and then the OR operation (addition) of the result with a single variable is equivalent to the OR operation of the single variable with each of the several variables and then the AND operation of the sums.

**Absorption laws**

1.  $A + AB = A$  ... (4a)

**Proof**

$$\begin{aligned} A + AB &= A \cdot 1 + AB \\ &= A(1 + B) \\ &= A \cdot 1 = A \end{aligned}$$

2.  $A \cdot (A + B) = A$  ... (4b)

**Proof**

$$\begin{aligned} A(A + B) &= A \cdot A + A \cdot B \\ &= A + AB \\ &= A(1 + B) \end{aligned}$$

$$= A \cdot 1$$

$$= A$$

$$3. \quad \boxed{A + \bar{A}B = A + B} \quad \dots(5a)$$

**Proof**  $[A + BC = (A + B) (A + C)]$

$$= 1 \cdot (A + B)$$

$$4. \quad \boxed{A \cdot (\bar{A} + B) = AB} \quad \dots(5b)$$

$$A \cdot (\bar{A} + B) = A \cdot \bar{A} + AB$$

$$= AB \quad (Q \quad A\bar{A} = 0)$$

## THE Z-TRANSFORM

The discrete-time Fourier transform (DTFT) of a sequence  $x(n)$  is equal to the sum

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega}$$

However, in order for this series to converge, it is necessary that the signal be absolutely summable. Unfortunately, many of the signals that we would like to consider are not absolutely summable and therefore, do not have a DTFT. Some example include

$$x(n) = u(n) \quad x(n) = (0.5)^n u(-n) \quad x(n) = \sin n\omega_0$$

The z-transform is a generalization of the DTFT that allows one to deal with such sequences and is defined as follows:

**Definition:** The z-transform of a discrete-time signal  $x(n)$  is defined by<sup>1</sup>

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad \dots(1)$$

where  $z = r e^{j\omega}$  is a complex variable. The values of  $z$  for which the sum converges define a region in the z-plane referred to as the *region of convergence* (ROC).

Notationally, if  $x(n)$  has a z-transform  $X(z)$ , we write

$$x(n) \xleftrightarrow{Z} X(z)$$

The z-transform may be viewed as the DTFT of an exponentially weighted sequence. Specifically, note that with  $z = r e^{j\omega}$ ,

$$X(s) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=-\infty}^{\infty} [r^{-n}x(n)]e^{-jn\omega}$$

and we see that  $X(z)$  is the discrete-time Fourier transform of the sequence  $r^{-n}x(n)$ . Furthermore, the ROC is determined by the range of values of  $r$  for which

$$\sum_{n=-\infty}^{\infty} |x(n)r^{-n}| < \infty$$

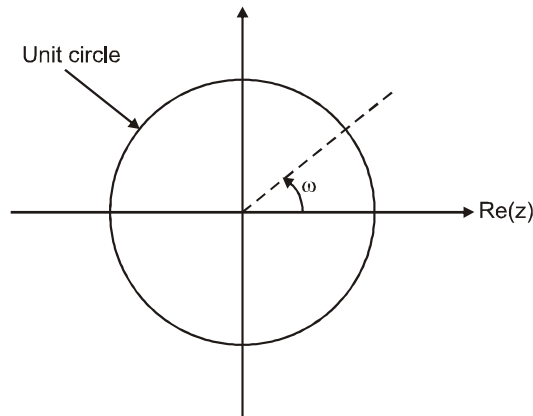
Because the  $z$ -transform is a function of a complex variable, it is convenient to describe it using the complex  $z$ -plane. With

$$z = \text{Re}(z) + j \text{Im}(z) = r e^{j\omega}$$

the axes of the  $z$ -plane are the real and imaginary parts of  $z$  its illustrated in Fig, and the contour corresponding to  $|z| = 1$  is a circle of unit radius referred to as the *unit circle*. The  $z$ -transform evaluated on the unit circle corresponding to the DTFT.

$$X(e^{j\omega}) = X(z) \Big|_{z=e^{j\omega}} \quad \dots(2)$$

More specifically, evaluating  $X(z)$  at points around the unit circle, beginning at  $z = 1$  ( $\omega = 0$ ), through  $z = j$  ( $\omega = \pi/2$ ), to  $z = -1$  ( $\omega = \pi$ ), we obtain the values of  $X(e^{j\omega})$  for  $0 \leq \omega \leq \pi$ . Note that in order for the DTFT of a signal to exist, the unit circle must be within the region of convergence of  $X(z)$ .



**Fig 6.** The unit circle in the complex  $z$ -plane.

Many of the signals of interest in digital signal processing have  $z$ -transforms that are rational functions of  $z$  :

$$X(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^q b(k)z^{-k}}{\sum_{k=0}^p a(k)z^{-k}} \quad \dots(3)$$

Factoring the numerator and denominator polynomials, a rational z-transform may be expressed as follows:

$$X(z) = C \frac{\prod_{k=1}^q (1 - \beta_k z^{-1})}{\prod_{k=1}^p (1 - \alpha_k z^{-1})} \quad \dots(4)$$

The roots of the numerator polynomial  $\beta_k$ , are referred to as the zeros of  $X(z)$ , and the roots of the denominator polynomial,  $\alpha_k$ , are referred to as the *poles*. The poles and zeros uniquely define the functional form of a rational z-transform to within a constant. Therefore, they provide a concise representation for  $X(z)$  that of then represented pictorially in terms of a pole-zero plot in the z-plane. With a pole-zero plot, the location of each pole is indicated by an “x” and the location of each zero is indicated by an “o”, with the region of convergence indicated by shading the appropriate region of the z-plane. The region of convergence is, in general, an annulus of the form

$$\alpha < |z| < \beta$$

If  $\alpha = 0$ , the ROC may also include the point  $z = 0$ , and if  $\beta = \infty$ , the ROC may also include infinity. For a rational  $X(z)$ , the region of convergence will contain no poles. Listed below are the properties of the region of convergence:

1. A finite-length sequence has z-transform with a region of convergence that includes the entire z-plane except, possibly,  $z = 0$  and  $z = \infty$ . The point  $z = \infty$  will be included if  $x(n) = 0$  for  $n < 0$ , and the point  $z = 0$  will be included if  $x(n) = 0$  for  $n > 0$ .
2. A right-sided sequence has z-transform with a region of convergence that is the *exterior* of a circle:

$$\text{ROC} : |z| > \alpha$$

3. A left-sided sequence has a z-transform with a region of convergence that is the *interior* of a circle:

$$\text{ROC} : |z| < \beta$$

## 1. Properties

There are a number of important and useful z-transform properties. A few of these properties are described below.

### Linearity

As with the DTFT, the z-transform is *linear* operator. Therefore, if  $x(n)$  has a z-transform  $X(z)$  with a region of convergence  $R_x$ , and if  $y(n)$  has a z-transform  $Y(z)$  with a region of convergence  $R_y$ ,

$$w(n) = ax(n) + by(n) \xrightarrow{Z} W(z) = aX(z) + bY(z)$$

and the ROC of  $w(n)$  will include the intersection of  $R_x$  and  $R_y$ , that is,

$$R_w \text{ contains } R_x \cap R_y$$

However, the region of convergence of  $W(z)$  may be larger. For example, if  $x(n) = u(n)$  and  $y(n) = u(n - 1)$ , the ROC of  $X(z)$  and  $Y(z)$  is  $|z| > 1$ . However, the z-transform of  $w(n) = x(n) - y(n) = \delta(n)$  is the entire z-plane.

### Shifting Property

Shifting a sequence (delaying or advancing) multiplies the z-transform by a power of  $z$ . That is to say, if  $x(n)$  has a z-transform  $X(z)$ ,

$$x(n - n_0) \xrightarrow{Z} z^{-n_0} X(z)$$

Because shifting a sequence does not affect its absolute summability, shifting does not change the region of convergence. Therefore, the z-transforms of  $x(n)$  and  $x(n - n_0)$  have the same region of convergence, with the possible exception of adding or deleting the points  $z = 0$  and  $z = \infty$ .

### Time Reversal

If  $x(n)$  has a z-transform  $X(z)$  with a region of convergence  $R_x$  that is the annulus  $\alpha < |z| < \beta$ , the z-transform of the time-reversed sequence  $x(-n)$  is

$$x(-n) \xrightarrow{Z} X(z^{-1})$$

and has a region of convergence  $1/\beta < |z| < 1/\alpha$ , which is denoted by  $1/R_x$ .



## Multiplication by an Exponential

If a sequence  $x(n)$  is multiplied by a complex exponential  $\alpha^n$ .

$$\alpha^n x(n) \xrightarrow{Z} X(\alpha^{-1} z)$$

This corresponds to a scaling of the  $z$ -plane. If the region of convergence of  $X(z)$  is  $r_- < |z| < r_+$ , which will be denoted by  $R_x$ , the region of convergence of  $X(\alpha^{-1} z)$  is  $|\alpha|r_- < |z| < |\alpha|r_+$ , which is denoted by  $|\alpha| R_x$ . As a special case, note that if  $x(n)$  is multiplied by a complex exponential,  $e^{jn\omega_0}$ ,

$$e^{jn\omega_0} x(n) \xrightarrow{Z} X(e^{-j\omega_0} z)$$

which corresponds to a rotation of the  $z$ -plane.

## Convolution Theorem

Perhaps the most important  $z$ -transform property is the convolution theorem, which states that convolution in the time domain is mapped into multiplication in the frequency domain, that is,

$$y(n) = x(n) * h(n) \xrightarrow{Z} Y(z) = X(z)H(z)$$

The region of convergence of  $Y(z)$  includes the intersection of  $R_x$  and  $R_y$ ,

$$R_w \text{ contains } R_x \cap R_y$$

However, the region of convergence of  $Y(z)$  may be larger, if there is a pole-zero cancellation in the product  $X(z)H(z)$ .

## Conjugation

If  $X(z)$  is the  $z$ -transform of  $x(n)$ , the  $z$ -transform of the complex conjugate of  $x(n)$  is

$$x^*(n) \xrightarrow{Z} X^*(z^*)$$

As a corollary, note that if  $x(n)$  is real-valued,  $x(n) = x^*(n)$ , then

$$X(z) = X^*(z^*)$$

## Derivative

If  $X(z)$  is the  $z$ -transform of  $x(n)$ , the  $z$ -transform of  $nx(n)$  is

$$nx(n) \xrightarrow{Z} -z \frac{dX(z)}{dz}$$

## Initial Value Theorem

If  $x(n)$  is equal to zero for  $n < 0$ , the initial value,  $x(0)$ , may be found from  $X(z)$  as follows:

$$x(0) = \lim_{z \rightarrow \infty} X(z)$$

This property is a consequence of the fact that if  $x(n) = 0$  for  $n < 0$ ,

$$X(z) = x(0) + x(1)z^{-1} + x(2)z^{-2} + \dots$$

Therefore, if we let  $z \rightarrow \infty$ , each term in  $X(z)$  goes to zero except the first.

## 2. The Inverse z-Transform

The z-transform is a useful tool in linear systems analysis. However, just as important as techniques for finding the z-transform of a sequence are methods that may be used to invert the z-transform and recover the sequence  $x(n)$  from  $X(z)$ . Three possible approaches are described below.

### Partial Fraction Expansion

For z-transforms that are rational functions of  $z$ ,

$$X(z) = \frac{\sum_{k=0}^q b(k)z^{-k}}{\sum_{k=0}^p a(k)z^{-k}} = C \frac{\prod_{k=1}^q (1 - \beta_k z^{-1})}{\prod_{k=1}^p (1 - \alpha_k z^{-1})}$$

a simple and straightforward approach to find the inverse z-transform is to perform a partial fraction expansion of  $X(z)$ . Assuming that  $p > q$ , and that all the roots in the denominator are simple,  $\alpha_i \neq \alpha_k$  for  $i \neq k$ ,  $X(z)$  may be expanded as follows:

$$X(z) = \sum_{k=1}^p \frac{A_k}{1 - \alpha_k z^{-1}} \quad \dots(5)$$

for some constants  $A_k$  for  $k = 1, 2, \dots, p$ . The coefficients  $A_k$  may be found by multiplying both sides of Eq. (5) by  $(1 - \alpha_k z^{-1})$  and setting  $z = \alpha_k$ . The result is

$$A_k = \left[ (1 - \alpha_k z^{-1}) X(z) \right]_{z=\alpha_k}$$

If  $p \leq q$ , the partial fraction expansion must include a polynomial in  $z^{-1}$  of order  $(p - q)$ . The coefficients of this polynomial may be found by long division (i.e., by dividing the numerator polynomial by the denominator). For multiple-order poles, the expansion must be modified. For example, if  $X(z)$  has a second-order pole at  $z = \alpha_k$ , the expansion will include two terms,

$$\frac{B_1}{1 - \alpha_k z^{-1}} + \frac{B_2}{(1 - \alpha_k z^{-1})^2}$$

where  $B_1$  and  $B_2$  are given by

$$B_1 = \alpha_k \left[ \frac{d}{dz} (1 - \alpha_k z^{-1})^2 X(z) \right]_{z=\alpha_k}$$

$$B_2 = \left[ (1 - \alpha_k z^{-1})^2 X(z) \right]_{z=\alpha_k}$$

## Power Series

The z-transform is a power series expansion,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \dots + x(-2)z^2 + x(-1)z + x(0) + x(1)z^{-1} + x(2)z^{-2} + \dots$$

where the sequence values  $x(n)$  are the coefficients of  $z^{-n}$  in the expansion. Therefore, it we can find the power series expansion for  $X(z)$ , the sequence values  $x(n)$  may be founded by simple picking off the coefficients of  $z^{-n}$ .

## Contour Integration

Another approach that may be used to find the inverse z-transform of  $X(z)$  is to use contour integration. This procedure relies on Cauchy's integral theorem, which states that if  $C$  is a closed contour that encircles the origin in a counterclockwise direction,

$$\frac{1}{2\pi j} \oint_C z^{-k} dz = \begin{cases} 1 & k = 1 \\ 0 & k \neq 1 \end{cases}$$

With

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Cauchy's integral theorem may be used to show that the coefficients  $x(n)$  may be found from  $X(z)$  as follows.

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz$$

where  $C$  is a closed contour within the region of convergence of  $X(z)$  that encircles the origin in a counterclockwise direction. Contour integrals of this form may often be evaluated with the help of Cauchy's residue theorem,

$$x(n) = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz = \sum [\text{residues of } X(z) z^{n-1} \text{ at the poles inside } C]$$

If  $X(z)$  is a rational function of  $z$  with a first-order pole at  $z = \alpha_k$ ,

$$\text{Res}[X(z)z^{n-1} \text{ at } z = \alpha_k] = \left[ (1 - \alpha_k z^{-1}) X(z) z^{n-1} \right]_{z = \alpha_k}$$

Contour integration is particularly useful if only a few values of  $x(n)$  are needed.

### 3. The One-Sided z-Transform

The z-transform defined in Sec. in the two-sided, or bilateral, z-transform. The one-sided, or unilateral, z-transform is defined by

$$X_1(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \quad \dots(6)$$

The primary use of the one-sided z-transform is to solve linear constant coefficient difference equations that have initial conditions. Most of the properties of the one-sided z-transform are the same as those for the two-sided z-transform. One that is different, however, is the shift property. Specifically, if  $x(n)$  has a one-sided z-transform

$$x(n-1) \xleftrightarrow{Z} z^{-1} X_1(z) + x(-1)$$

It is this property that makes the one-sided z-transform useful for solving difference equations with initial conditions.

## THE SUPER HETERODYNE RECEIVER

In a broadcasting system, irrespective of whether it is based on amplitude modulation or frequency modulation, the receiver not only has the task of demodulating the incoming modulated signal, but it is also required to perform some other system functions :

- Carrier - frequency tuning, the purpose of which is to select the desired signal (i.e., desired radio or TV station).
- Filtering, which is required to separate the desired signal from other modulated signals that may be picked up along the way.
- Amplification, which is intended to compensate for the loss of signal power incurred in the course of transmission.

The super heterodyne receiver, or superheat as it is often referred to, is a special type of receiver that fulfills all three functions, particularly the first two, in an elegant and practical fashion. Specifically, It overcomes the difficulty of having to build a tunable high - (and

variable-) Q filter. Indeed, practically all radio and TV receivers now being made are of the super heterodyne type.

	AM Radio	FM Radio
RF carrier range	0.535 – 1.605 MHz	88 – 108 MHz
Mid – band frequency of IF section	0.455 MHz	10.7 MHz
If bandwidth	10 kHz	200 kHz

**Table: Typical Frequency Parameters of AM and FM Radio Receivers**

Basically, the receiver consists of a radio - frequency (RF\_) section , a mixer and a local oscillator, an intermediate frequency (IF) section, demodulator, and power amplifier. Typical frequency parameters of commercial AM and FM radio receivers are listed in the above Table Figure 16 shows the block diagram of a super heterodyne receiver for amplitude modulation using an envelope detector for demodulation.

The incoming amplitude - modulated wave is picked up by the receiving antenna and amplified in the RF section that is tuned to the carrier frequency of the incoming wave. The combination of mixer and local oscillator (of adjustable frequency) provides a heterodyning function, whereby the incoming signal is converted to a predetermined fixed intermediate frequency, usually lower than the incoming carrier frequency. This frequency translation is achieved without disturbing the relation of the sidebands to the carrier. The result of the heterodyning is to produce an intermediate - frequency carrier defined by :

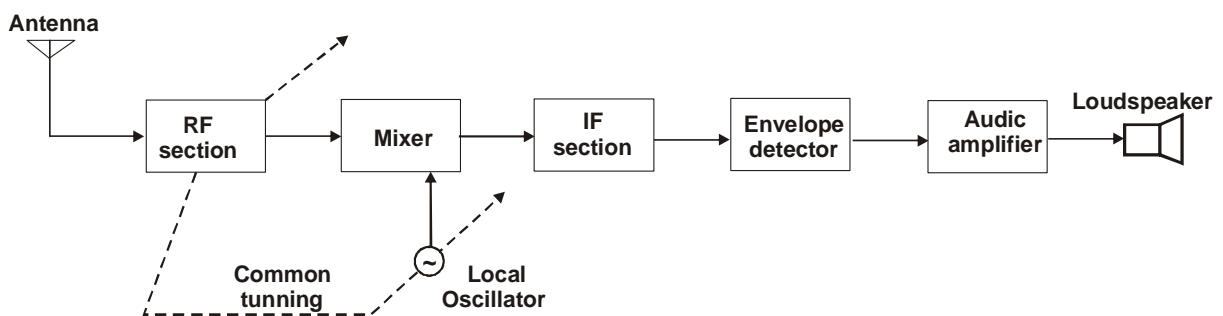
$$f_{IF} = f_{RF} - F_{LO} \quad (62)$$

where  $f_{LO}$  is the frequency of the local oscillator and  $f_{RF}$  is the carrier frequency of the incoming RF signal. We refer to  $f_{IF}$  as the intermediate frequency (IF), because the signal is neither at the original input frequency nor at the final baseband frequency. The mixer -local oscillator combination is sometimes referred to as the first detector, in which case the demodulator is called the second detector.

The IF section consists of one or more stages of tuned amplification, with a bandwidth corresponding to that required for the particular type of signal that the receiver is intended to handle. This section provides most of the amplification and selectivity in the receiver. The output of the IF section is applied to a demodulator, the purpose of which is to recover the

baseband signal. If coherent detection is used, then a coherent signal source must be provided in the receiver. The final operation in the receiver is the power amplification of the recovered message signal.

In a super heterodyne receiver the mixer will develop an intermediate frequency output when the input signal frequency is greater or less than the local oscillator frequency by an amount equal to the intermediate frequency. That is, there are two input frequencies, namely,  $|f_{LO} \pm f_{IF}|$ , which will result in  $f_{IF}$  at the mixer output. This introduces the possibility of simultaneous reception of two signals differing in frequency by twice the intermediate frequency. For example, a receiver tuned to 1 MHz and having an IF of 0.455 MHz is subject to an image interference at 1.910 MHz ; indeed any receiver with this value of IF, when tuned to any station, is subject to image interference at a frequency of 0.910 MHz higher than the desired station. Since the function of the mixer is to produce the difference between two applied frequencies, it is incapable to distinguishing between the desired signal and its image in that it produces of IF output from either one of them. The only practical cure for image interference is to employ highly selective stages in the RF section (i.e. between the antenna and the mixer) in order to favor the desired signal and discriminate against the underside or image signal. The effectiveness of suppressing unwanted image signals increases as the number of selective stages in the radio - frequency section increases, and as the ratio of intermediate signal frequency increases.



**Figure 7: Basic elements of an AM receiver of superheterodyne type.**

In an FM system, the message information is transmitted by variations of the instantaneous frequency, of sinusoidal carrier wave, and its amplitude is maintained constant. Therefore, any variation of the carrier amplitude at the receiver input must result from noise or



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interference. An amplitude limiter, following the IF section, is used to remove amplitude variations by clipping the modulated wave at the IF section output almost to the zero axis. The resulting rectangular wave is rounded off by a band - pass filter that suppresses harmonics of the carrier frequency. Thus the filter output is again sinusoidal, with an amplitude that is practically independent of the carrier amplitude at the receiver input.