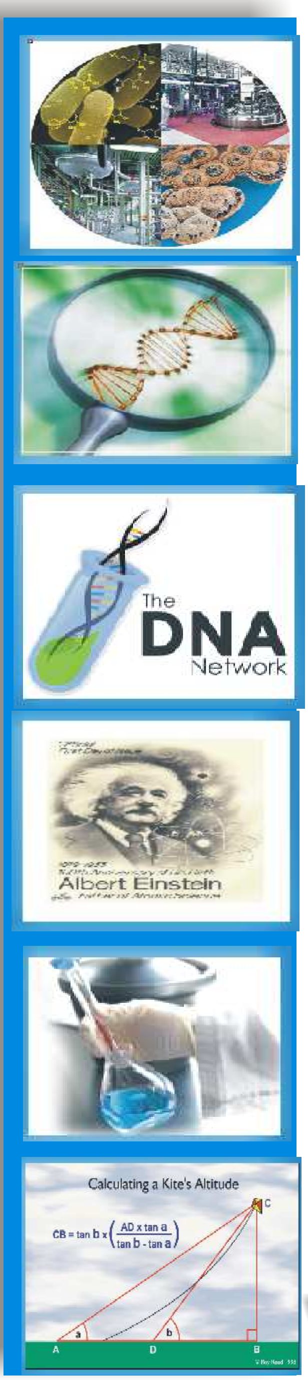

JMU-MCA

SAMPLE THEORY

- ADJOINT OF A MATRIX
- BINARY NUMBER SYSTEM

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1. ADJOINT OF A SQUARE MATRIX

Let A = $[a_{ij}]$ be a square matrix of order n and let C_{ij} be cofactor of a_{ij} in A. Then the transpose of the matrix of cofactors of elements of A is called the adjoint of A and is denoted by adj A

Thus, adj A = $[C_{ij}]^T \Rightarrow (\text{adj } A)_{ij} = C_{ji}$ = cofactor of a_{ij} in A.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$,

then $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$,

where C_{ij} denotes the cofactor of a_{ij} in A.

Example:1 Find the adjoint of matrix $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$,

Sol. $C_{11} = s, C_{12} = -r, C_{21} = -q, C_{22} = p$

$\therefore \quad \text{adj } A = \begin{bmatrix} s & -r \\ -q & p \end{bmatrix}^T = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$

Properties of adjoint matrix : If A, B are square matrices of order n and I_n is corresponding unit matrix, then

(i) $A(\text{adj } A) = |A| I_n = (\text{adj } A)A$

(Thus A (adj A) is always a scalar matrix)

(ii) $|\text{adj } A| = |A|^{n-1}$

(iii) $\text{adj}(\text{adj } A) = |A|^{n-2} A, |A| \neq 0$.

(iv) $|\text{adj (adj A)}|\ |A|^{(n-1)^2}$

(v) $\text{adj}(A^T) = (\text{adj } A)^T$

(vi) adj(AB) = (adj B) (adj A)

(vii) $\text{adj}(A^m) = (\text{adj } A)^m$, $m \in N$

(viii) $\text{adj}(kA) = k^{n-1}(\text{adj } A)$, $k \in R$

(ix) $\text{adj}(I_n) = I_n$

(x) adj(O) = O

(xi) A is symmetric matrix ⇒ adj A is also symmetric matrix.

(xii) A is diagonal matrix ⇒ adj A is also diagonal matrix.

(xiii) A is triangular matrix ⇒ adj A is also triangular matrix

(xiv) A is singular ⇒ |adj A| = 0

Ex.2 Find the adjoint of matrix A, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$$

Verify A(adj A) = (adj A) A = |A| I.

Sol. Writing the given matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$, we have

A_{11} = co-factor of a_{11} = –5 ; A_{12} = co-factor of a_{12} = –3

A_{21} = co-factor of a_{21} = –2 ; A_{22} = co-factor of a_{22} = 1

Hence $$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$$ **Ans.**

We have $|A| = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -5 - 6 = -11.$ ...(1)

$$A\,(\text{adj } A) = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5-6 & -2+2 \\ -15+15 & -6-5 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$$

$$= -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|\,I, \text{ by (1)} \qquad .....(2)$$

$$(\text{adj } A)\,A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -5-6 & -10+10 \\ -3+3 & -6-5 \end{bmatrix} = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$$

$$= -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = |A|\,I.$$

Hence $A(\text{adj } A) = (\text{adj } A)\,A = |A|\,I.$ Verified.

2. BINARY NUMBER SYSTEM

The coefficient of binary number have two possible values: 0 and 1. Each coefficient of binary number is multiplied by powers of 2's.

example:1 Find out the decimal equivalent of 11011.11.

Sol. The decimal equivalent can be obtained by multiplying each place of binary number with appropriate power of two as shown:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75.$$

In general for a base-r system, the decimal equivalent of it can be obtained by multiplying with power's of r.

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} \ldots\ldots + a_1 r + a_0 r^0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + a_{-3} \cdot r^{-3} \ldots\ldots a_{-m} r^{-m}$$

where coefficient a_j range in value from 0 to r–1.

- **BINARY TO OCTAL AND HEXADECIMAL BASE COVERSION:**

The conversion from and to binary, octal, and hexadecimal plays an important part in digital computers. Since $2^3 = 8$ and $2^4 = 16$, each octal digit corresponds to three binary digits and each hexadecimal digit corresponds to four binary digits. The conversion from binary to octal is easily accomplished by partitioning the binary number into groups of three digits each,starting from the binary point and proceeding to the left and to the right. The corresponding octal digit is then assigned to each group.

The following example illustrates the procedure.

Example:1 Convert the $(10110001101011.111100000110)_2$ in octal equivalent.

Sol. $$\left(\underbrace{10}_{2}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{101}_{5}\ \underbrace{011}_{3} \cdot \underbrace{111}_{7}\ \underbrace{100}_{4}\ \underbrace{000}_{0}\ \underbrace{110}_{6}\right)_2 = (26153.7406)_8$$

Example:2 Convert the $(10110001101011.111100000110)_2$ in hexadecimal equivalent.

Sol. Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of four digits.

$$\left(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B} \cdot \underbrace{1111}_{F}\ \underbrace{0010}_{2}\right)_2 = (2C6B.F2)_{16}$$