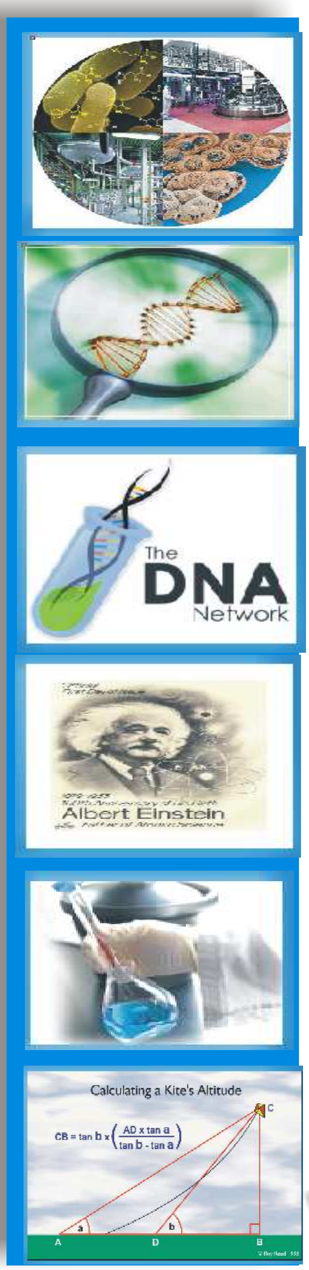

JNU-MCA

SAMPLE THEORY

- NUMERICAL DIFFERENTIATION
- ARITHMETIC OPERATIONS OVER BINARY NUMBERS

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1. Numerical Differentiation

When for different values of the independent variable, the corresponding values of the function are known, then finding the differential coefficient of that function at any particular values of the independent variable is called numerical differentiation. Following some of the important point for proper selection of the interpolation formula.

(i) If the intervals of the variable are equal , then the function can be obtained by Newton Gregory formula.

(ii) If it is required to find the derivative of the function at a point near the beginning of a set of tabular values , Newton's Gregory forward (backward) formula should be used.

(iii) If the derivative at a point near the middle of the table, central difference formula be used.

(iv) If the values of the arguments are unequal spaced, Newton's divided difference formula should be used to represent the function.

Ex.1 Find first and second derivatives at x = 1.1 from the following table :

x	**:**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**
f(x)	**:**	**0**	**.1280**	**.5440**	**1.2960**	**2.4320**	**4.00**

Sol. According to the problem, the variable are equidistant and the value of the derivative of the function at x = 1.1 is desired, therefore here Newton's Gregory forward formula is preferred :

Table-1: Difference Table

x	f(x)	Δ	Δ^2	Δ^3	Δ^4
1.0	0.0000				
		.1280			
1.2	0.1280		.2880		
		.4160		.0480	
1.4	0.5440		.3360		0
		.7520		.0480	
1.6	1.2960		.3840		0
		1.1360		.0480	
1.8	2.4320		.4320		
		1.5680			
2.0	4.000				

Newton's Gregory forward formula is : $f(a + xh) = f(a) + {}^xC_1 \Delta f(a) + {}^xC_2\, \Delta^2 f(a) + {}^xC_3\, \Delta^3 f(a) + \ldots$

$$= f(a) + x\Delta f(a) + \frac{x^2 - x}{2}\, \Delta^2\, f(a) + \frac{x^3 - 3x^2 + 2x}{6}\, \Delta^3\, f(a) + \ldots \qquad \ldots(1)$$

Differentiating (1) twice w.r.t.x,

$$hf'(a + xh) = \Delta f(a) + \frac{2x-1}{2}\, \Delta^2\, f(a) + \frac{3x^2 - 6x + 2}{6}\, \Delta^3\, f(a) \qquad \ldots(2)$$

and $\quad h^2 f''(a + xh) = \Delta^2 f(a) + (x - 1)\, \Delta^3 f(a) \qquad \ldots(3)$

Replacing a = 1, h = .2, x = 1/2 and substituting the desired differences from the tables , in (2) and (3)

$$(.2)\, f'(1.1) = .1280 + 0 + \frac{1}{6}\left(3\times\frac{1}{4} - 6\frac{1}{2} + 2\right)(.480) + 0 \qquad \text{or} \qquad f'(1.1) = .630$$

and $\quad (.2)^2\, f''(1.1) = .2880 + \left(\frac{1}{2} - 1\right)(.0480) + 0 = .2880 - .024 = .264$ or,

$f''(1.1) = 6.60$

Ex.2 Find (.04) from the following table :

x :	.01	.02	.03	.04	.05	.06
f(x) :	.1023	.1047	.1071	.1096	.1122	.1148

Sol. Here we have to determine the first derivative at x = .04 which is situated in the middle of the table and variable are also equidistant. Therefore here the use of Gauss's forward difference formula is preferred.

Take new variable $u = \frac{x - .04}{h}$

Difference Table

x	u	$f(x) = y_u$	Δy_u	$\Delta^2 y_u$	$\Delta^3 y_u$	$\Delta^4 y_u$	$\Delta^5 y_u$
.01	−3	.1023					
			.0024				
.02	−2	.1047		0			
			.0024		.0001		
.03	−1	.1071		.0001		−.0001	
			.0025		0		0
.04	0	.1096		.0001		−.0001	
			.0026		−.0001		
.05	1	.1122		0			
			.0026				
.06	2	.1148					

Table-2

Gauss's forward interpolation formula of new variable u is :

$$f(u) = f(0) + u\,\Delta f(0) + \frac{u(u-1)}{2!}\Delta^2 f(-1) + \frac{u(u+1)(u-1)}{3!}\Delta^3 f(-1)$$

$$+ \frac{u(u+1)(u-1)(u-2)}{4!}\Delta^4 f(-2) + \ldots.. \qquad \ldots(1)$$

Now $u = \frac{x - .04}{h}$ $\quad \therefore \frac{du}{dx} = \frac{1}{h}$

Also $\frac{d}{dx}\{f(x)\} = \frac{d}{du}[f(u)].\frac{du}{dx} = \frac{1}{h} f'(u)$

Differentiate (1) wrt u and replace u = 0 and substitute the desired differences from the table , we get

$f'(0) = .0026 - \frac{1}{2}(.0001) - \frac{1}{6}(0) + \frac{1}{12}(-.0001) = .00254$ [on simplifying]

$\therefore \frac{d}{dx} f(x = .04) = \frac{1}{h} f'(u = 0) = \frac{.00254}{.01} = .254$

2. ARITHMETIC OPERATIONS OVER BINARY NUMBERS

Addition-

[redacted]

Subtraction-

$$\begin{array}{lr} \text{minuend} & 101101 \\ \text{subtrahend} & -\ 100111 \\ \hline \text{sum} & 000110 \\ \hline \end{array} \quad \text{Ans.}$$

Multiplication-

$$\begin{array}{lr} \text{multiplicand} & 1011 \\ \text{multiplier} & \times\ \ 101 \\ \hline & 1011 \\ & 0000\times \\ & 1011\times\times \\ \hline & 110111 \\ \hline \end{array} \quad \text{Ans.}$$

SUBTRACTION WITH r's COMPLEMENT

The subtraction of two positive numbers (M – N), both base r, may be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N.

2. Inspect the result obtained in step 1 for an end carry:

(a) If an end carry occurs, discard it.

(b) If an end carry does not occur, take the r's complement of the number obtained in step 1 and place a negative sign in front.

SUBTRACTION WITH (r–1)'s COMPLEMENT

The procedure for subtraction with the (r – 1)'s complement is exactly the same as the one used with the r's complement except for one variation, called "end-around carry," as shown below. The subtraction of M – N, both positive numbers in base r, may be calculated in the following manner:

1. Add the minuend M to the (r – 1)'s complement of the subtrahend N.
2. Inspect the result obtained in step 1 for an end carry.

(a) If an end carry occurs, add 1 to the least significant digit (end - around carry).

(b) If an end carry does not occur, take the (r – 1)'s complement of the number obtained in step 1 and place a negative sign in front.

The following examples illustrate the procedure.

Ex.3 Subtract $(3250 – 72532)_{10}$.

Sol. **M = 03250** **03250**

N = 72532

10's complement of N = 27468 + 27468

no carry 30718

answer: –69282 = –(10's complement of 30718)

Ex.4 Use 2's complement to perform M – N with the given binary numbers

(a) M = 1010100 (b) M = 1000100

N = 1000100 N = 1010100

Sol. (a) M = 1010100 1010100

N = 1000100

2's complement of N = 0111100

$$\begin{array}{r} + \\ 0111100 \\ \hline \text{end carry} \rightarrow 1\ \ 0010000 \end{array}$$

answer 10000

(b) M = 1000100 1000100

N = 1010100

2's complement of N = 0101100

$$\begin{array}{r} + \\ 0101100 \\ \hline \text{no carry}\ \ 1110000 \end{array}$$

answer –10000 = –(2's complement of 1110000)