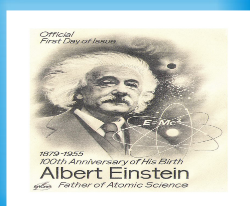


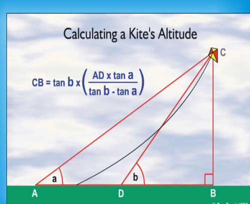


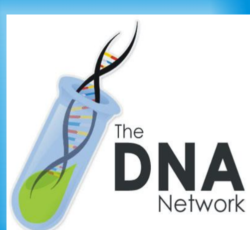


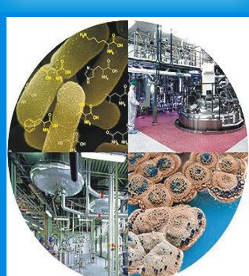

# GATE - CHEMICAL ENGINEERING

## SAMPLE THEORY

- **BERNOULLI'S EQUATION**
- **HEAT TRANSFER: RADIATION**
- **CONTROLLER MODES**

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## BERNOULLI'S EQUATION

It is obtained by Euler's energy equation, $\frac{\partial p}{\rho} + gdz + \nu \partial \nu = 0$

Integrating above equation, we get Bernoulli's equation.

$$\int \frac{dp}{\rho} + \int gdz + \int \nu d\nu = \text{constant}$$

For an incompressible flow, $\rho$ = constant

$\therefore \quad \frac{p}{\rho} + gz + \frac{\nu^2}{2} = \text{constant}$ (Bernoulli's energy equation)

or $\quad \frac{p}{\rho g} + z + \frac{\nu^2}{2g} = \text{constant}$

$\frac{p}{\rho g} = \frac{p}{\gamma}$ = pressure energy per unit weight of fluid or pressure head

$\frac{\nu^2}{2g}$ = kinetic energy per unit weight or kinetic head

z = potential energy per unit weight or potential head

It is applied to the problems of

(i) flow under a sluice gate which is a construction in an open channel

(ii) free liquid jet

(iii) redial flow

(iv) free vortex motion.

**Assumptions in Bernoulli's equation.**

(i) Ideal fluid

(ii) Continuity of flow

(iii) Steady and incompressible flow.

**Flow Through An Orifices.**

**(i) Flow through small orifices.**

Jet of water or fluid has minimum area at vena contract.

Coefficient of contraction, $C_C = \frac{\text{Area of jet at vena contracta}}{\text{Area of opening}}$

$= 0.61$ for $d << H_1$.

Bernoulli's equation gives, $U_t = \sqrt{2gH_1}$

and $U_a = C_v\sqrt{2gH_1}$

Where $U_t$ and $U_a$ are theoretical and actual velocities at vena contract.

The value of $C_v$ is unity if there is no friction. Otherwise

$$C_v = 0.97 \text{ to } 0.99$$

$$Q = \text{area} \times \text{velocity}$$

$$= C_c\alpha\, C_v\sqrt{2gH_1}$$

$$= C_d\alpha\sqrt{2gH}$$

Where $C_d$ = discharge coefficient = $C_c \times C_v$ .

## HEAT TRANSFER: RADIATION

Thermal radiation is the transmission of thermal energy without any physical contact between the bodies involved. Unlike heat transfer by conduction and convection, transport of thermal radiation does not necessarily affect the material medium between the heat source and the receiver.

- **Absorptivity, Reflectivity and Transmissivity**

The total radiant energy ($Q_0$) impinging upon a body would be partially or totally absorbed by it ($Q_a$), reflected from its surface ($Q_r$), or transmitted through if ($Q_t$) in accordance with the characteristics of the body (Fig.). By the conservation of energy principle, the total sum must be equal to the incident radiation. That is :

$$Q_a + Q_r + Q_t = Q_0$$

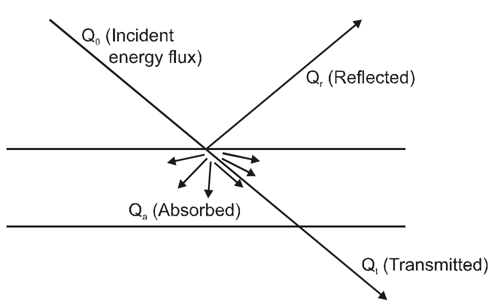


**Fig.1** Absorption, reflection and transmission of radiation

$$\frac{Q_a}{Q_0}+\frac{Q_r}{Q_0}+\frac{Q_t}{Q_0}=\frac{Q_0}{Q_0}$$

- **Planck's Law**

Thermal radiation strikes a surface which has a reflectivity to 0.55 and a transmissivity of 0.032. The absorbed flux as measured indirectly by heating effect works out to be 95 W/$m^2$. Determine the rate of incident flux.

The energy emitted by a black surface varies in accordance with wavelength, temperature and surface characteristic of the body.

The laws governing the distribution of radiant energy over wavelength for a black body at a fixed temperature were formulated by Planck. He suggested the following law for the spectral distribution of emissive power :

$$(E_\lambda)_b = 2\pi c^2 h \frac{\lambda^{-5}}{\exp[ch/k\lambda T]-1}$$

$$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left[C_2/\lambda T\right]-1}$$

The quantity $(E_\lambda)_b$ denotes the monochromatic (single wavelength) emissive power, and is defined as the energy emitted by the black surface (in all directions) at a given wavelength $\lambda$ per unit wavelength interval around $\lambda$.

The variation of distribution of the monochromatic emissive power with wavelength is called the **spectral energy distribution**,

(i) The monochromatic emissive power varies across the wavelength spectrum; the distribution is continuous but nonuniform. With increase in wavelength, the monochromatic emissive power increases and attains a certain maximum value.

(a) For shorter wavelengths, the factor $C_2/\lambda T$ becomes very large, IN that case

$$\exp\left[\frac{C_2}{\lambda T}\right] > 1$$

The plack's law then reduces to

$$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left[C_2/\lambda T\right]}$$

Equation is called **Wien's law**, and it is accurate within 1 percent for $\lambda T$ less than 3000 $\mu$K.

(b) For longer wavelengths, the factor $C_2/\lambda T$ is small. In that case exp $(C_2/\lambda T)$ can be expanded in series to give

$$\exp\left[C_2/\lambda T\right] = 1 + \frac{C_2}{\lambda T} + \frac{1}{2!}\left(\frac{C_2}{\lambda T}\right)^2 + ...$$

$$= 1 + \frac{C_2}{\lambda T}$$

The Planck's distribution law then becomes

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{1 + C_2/\lambda T - 1} = \frac{C_1 T}{C_2 \lambda^4}$$

The above identity is known as **Rayleight-Jean's Law**.

- **Total Emissive Power ; Stefan-Boltzman Law**

  The total emissive power E of a surface is defined as the total radiant energy emitted by the surface in all directions over the entire wavelength range per unit surface area per unit time. The basic rate equation for radiation transfer is based on Stefan-Boltzman law which states that the amount of radiant energy emitted per unit time from unit area of black surface is proportional to the fourth power of its absolute temperature.

$$E_b = \sigma_b T^4$$

where $\sigma_b$ is the radiation coefficient of a black body.

$$\sigma_b = 5.67 \times 10^{-8}\ \text{W/m}^2\text{K}^4$$

**radiation coefficient** or the Stefan-Boltzman constant.

- **Kirchoff's Law**

  The ratio of the emissive power E to absorptivity $\alpha$ is same for all bodies, and is equal to the emissive power of a black body at the same temperature. This relationship is known as the Kirchoff's law.

  Kirchoff's law can also be stated as : "The emissivity $\in$ and absorptivity $\alpha$ of a real surface are equal for radiation with identical temperature and wavelength".

- **Wien's Displacement Law**

$$\therefore \quad \lambda_{max} T = \frac{C_2}{4.965}$$

$$= \frac{1.4388 \times 10^{-2}}{4.965} = 2.898 \times 10^{-3} = 0.0029 \text{ mK}$$

The Wien's displacement law may be stated as "the product of absolute temperature and the wavelength at which the emissive power is maximum, is constant". The law suggests that $\lambda_{max}$ is inversely proportional to the absolute temperature and accordingly the maximum spectral intensity of radiation shifts towards the shorter wavelength with rising temperature.

- **Shape Factor**

Consider heat exchange between elementary areas $dA_1$ and $dA_2$ of two black radiating bodies having areas $A_1$ and $A_2$ respectively. The elementary areas are at a distance r apart and the normal to these areas make angles $\theta_1$ and $\theta_2$ with the line joining them. The surface $dA_1$ is at temperature $T_1$ and the surface $dA_2$ is at temperature $T_2$.

If the surface $dA_2$ subtends a solid angle $d\omega_1$ at the centre of the surface $dA_1$, then radiant energy emitted by $dA_1$ and impinging on (and absorbed by) the surface $dA_2$ is :

$$dQ_{12} = I_{0_1}\, d\omega_1\, dA_1 = I_{n1} \cos\theta_1\, d\omega_1\, dA_1$$

Projected area of $dA_2$ normal to the line joining $dA_1$ and $dA_2$ = $dA_2 \cos\theta_2$

$$\text{solid angle } d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}$$

$$\therefore \quad dQ_{12} = I_{n1} \cos\theta_1 \frac{dA_2 \cos\theta_2}{r^2} dA_2 = I_{n1} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2}$$

But $I_{n1} = \frac{\sigma_1 T_1^4}{\pi}$ and therefore

$$dQ_{12} = \frac{\sigma_1 T_1^4}{\pi} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2} \quad ...(1)$$

Integration of equation over finite areas $A_1$ and $A_2$ gives :

$$Q_{12} = \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_1} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} \quad ...(2)$$

The solution to this equation is simplified by introducing a term and called **radiation shape factor, geometrical factor, configuration factor or view factor.**

"The fraction of the radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections."

The radiation shape factor is represented by the symbol $F_{ij}$ which means the shape factor form a surface, $A_i$ to another surface, Thus the radiation shape factor $F_{12}$ of surface $A_1$ to surface $A_2$ is

$$F_{12} = \frac{\text{direct radiation from surface 1 incident upon surface 2}}{\text{total radiation from emittign surface 1}}$$

$$= \frac{Q_{12}}{\sigma_1 A_1 T_1^4} \frac{1}{\sigma_1 A_1 T_1^4} \times \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_1} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

$$= \frac{1}{A_1 \pi} \int_{A_1} \int_{A_1} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} \quad ...(3)$$

Thus the amount of radiation leaving $A_1$ and striking $A_2$ may be written as :

$$Q_{12} = A_1 F_{12} \sigma_1 T_1^4 \quad ...(4)$$

Similarly the energy leaving $A_2$ and arriving $A_1$ is :

$$Q_{21} = A_2 F_{21} \sigma_2 T_2^4 \quad ...(5)$$

and the net energy exchange from $A_1$ to $A_2$ is :

$$(Q_{12})_{net} = A_1 F_{12} \sigma_1 T_1^4 - A_2 F_{21} \sigma_2 T_2^4$$

When the surface are maintained at the same temperatures, $T_1 = T_2$, there can be no heat exchange.

$$\therefore \quad 0 = (A_1 F_{12} - A_2 F_{21}) \sigma_b T_1^4$$

Since $\sigma_b$ and $T_1$ are both nonzero quantities,

$$A_1 F_{12} - F_2 F_{21} = 0 \quad \text{or} \quad A_1 F_{12} = A_2 F_{21}$$

The above result is known as a **reciprocity theorem.**

**Shape Factor Algebra and Salient Features of The Shape Factor**

The interrelation between various shape factors is called shape factor algebra

The following facts and properties will be useful of the calculation of shape factors of specific geometries and for the analysis of radiant heat exchange between surfaces :

**Radition between Solid Surface**

$$Q_{net} = f_{12} f_{12}\, \sigma_b (T_1^4 - T_2^4) \quad ...(6)$$

$f_{12}f_{12} \rightarrow$ Interchange factor = $\varepsilon_1$

$F_{12} \rightarrow$ geometric factor or shape factor

It is shown as follows

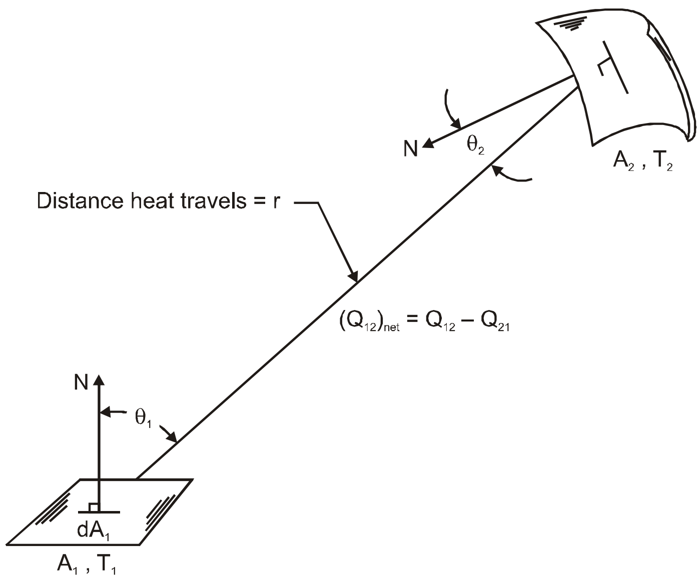


**Fig 2.** Radiant heat exchange between two black surface

# CONTROLLER MODES

## 1. Proportional Controller

The proportional controller is a device that produces a control signal which is proportional to the input error signal e(t). The error signal is the differenece between the reference input signal and the feedback signal obtained from the output.

Figure4. shows the block diagram of a proportional control system. The actuating signal is proporitonal to the error signal, hecne the name proportional control.

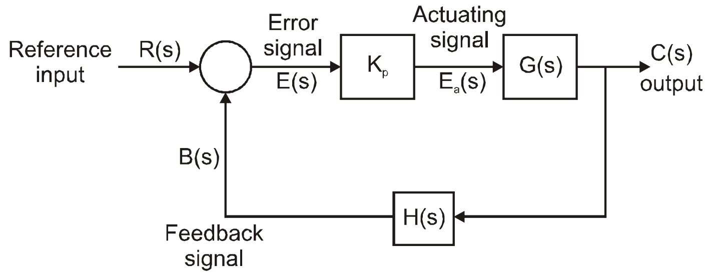


**Fig 3.** Block diagram of proportional (P) control system.

## 2. Proportional Derivative Controller

In Proportional plus Derivative (PD) controllers, the actuating signal $e_a(t)$ is proportional to the error signal e(t) and also proportional derivative of the error signal. Thus, the actuating signal for proportional plus derivative control is given by

$$e_a(t) = K_p e(t) + K_D \frac{d}{dt} e(t) \qquad ...(7)$$

Taking the Laplace transform of both the sides of Eq. (7), we get

$$E_a(s) = K_p E(s) + K_D sE(s)$$

or

$$E_a(s) = (K_p + sK_D) E(s)$$

Fig shows the block diagram of a PD control for a second-order system.

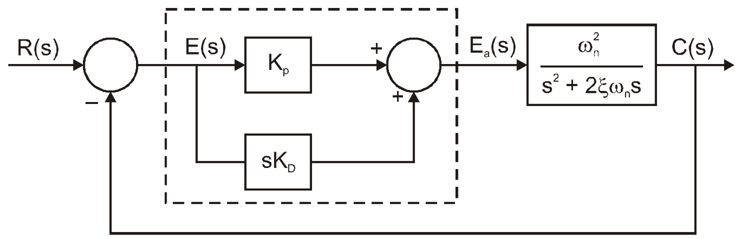


**Fig.4** Block diagram of a PD control system.

From Fig open-loop transfer function is

$$G(s) = \frac{C(s)}{E(s)}$$

$$= (K_p + sK_D) \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} \quad ...(8)$$

and $H(s) = 1$ ...(9)

Therefore, closed-loop transfer function of the system si given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{(K_p + sK_D)\,\omega_n^2}{s^2 + (2\zeta\omega_n + K_D\omega_n^2)s + \omega_n^2 K_p} \quad ...(10)$$

The characterstics equation fo the system given by the denominator of Eq. (10) is

$$s^2 + (2\zeta\omega_n + K_D\omega_n^2)s + K_p\omega_n^2 = 0 \quad ...(11)$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad ...(12)$$

We shall compare (10) with standard Eq. (11)

Now, $2\zeta\omega_n + K_\Delta\omega_n^2 = 2\left(\zeta + \frac{1}{2}K_D\omega_n\right)\omega_n = 2\,\zeta'\,\omega_n$

where $\zeta' = \left(\zeta + \frac{1}{2}k_D\omega_n\right)$ ...(13)

Equation (13) shows that effective damping has increased using PD control. This makes the system response slower with less overshoots increasing delay time. Proportional derivative control will not affect the steady-state error of the system.This mode of control may be represented by

$$p = K_c \in + K_c\tau_D \frac{d\in}{dt} + p_s \quad ...(14)$$

where $K_c$ = gain

$\tau_D$ = derivative time, min

$p_s$ = constant

In this case, we have added to the proportional term another term, $K_c\,\tau_D\; d\in/d\tau$, which is proportional to the derivative of the error. The values of $K_c$ and $\tau_D$ may be varied separately be knobs on the controller. Other terms that are used to describe the derivative action are rate control and anticipatory control. This response is obtained by introducing the inner function $\in(t) = At$ into Eq. (14) to obtain

$$p(t) = AK_c t + AK_c\tau_D + p_s$$

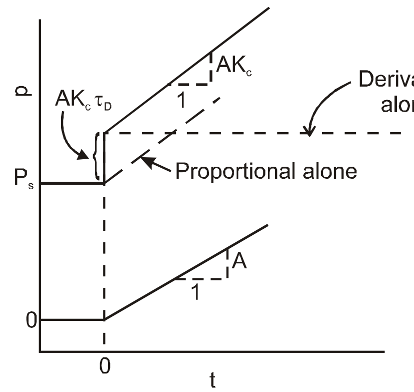


**Fig.5** Response of a PD controller to a linear input in error

Notice that p changes suddenly by an amoun $AK_c\tau_D$ as a result of the derviative action and then cagnes linearly at a rate $AK_c$. The effect of derivative action in this case is to anticipate the linear change in error by adding additional output $AK_c\tau_D$ to the proportional action.

## 3. Proportional Iniegral Controller

In proportional plus Integral controllers, the actuating signal consists of proportinoal error signal added to the integral of the error signal. The actuating signal in time domain is given by

$$e_a(t) = K_p e(t) + K_I \int_0^t e(\tau)\, d\tau \qquad ...(15)$$

Here the constants $K_P$ and $K_I$ are proportional and integral gains known as **controller parameters.**

By taking the Laplace transform of both sides of Eq. (15), we get

$$E_a(s) = K_P E(s) + K_I \frac{K(s)}{s}$$

or

$$E_a(s) = \left(K_P + \frac{K_I}{s}\right) E(s) \qquad ...(16)$$

Figure shows the block diagram of a proportional plus integral (or PI) control system of a csecond-order system.

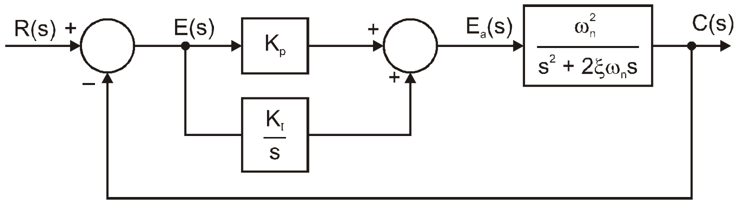


**Fig.6** Block diagram of a PI control system.

From Fig the open-loop transfer function

$$G(s) = \frac{C(s)}{E(s)} = \frac{\omega_n^2\left(K_P + \frac{K_I}{s}\right)}{s^2 + 2\zeta\omega_n s}$$

or $$G(s) = \frac{(K_P s + K_1)\omega_n^2}{s^2(s + 2\zeta\omega_n)} \quad ...(17)$$

and $$H(S) = 1$$

Therefore, closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\frac{(K_P s + K_I)\omega_n^2}{s^2(s + 2\zeta\omega_n)}}{1 + \frac{(K_P s + K_I)\omega_n^2}{s^2(s + 2\zeta\omega_n)}}$$

or $$\frac{C(s)}{R(s)} = \frac{(K_P s + K_I)\omega_n^2}{s^2 + 2\zeta\omega_n s^2 + K_P\omega_n^2 s + K_I\omega_n^2} \quad ...(18)$$

The characterstics equation is

$$s^3 + 2\zeta\omega_n s^2 + K_P\omega_n^2 s + K_I\omega_n^2 = 0 \quad ...(19)$$

Equation (19) is third-order equation.

Thus, a second-order system has been changed to a third-order system by adding an integral control in the system.

## 4. Proportional-Integral and Derivative Controller

A PID controller (Proportional plus Integral plus Derivative Controller) produces an output signal consisting of three terms – one proportional to error signal, another one proportional to integral of error signal and third one proportional to derivative of error signal.

The combination of proportional control action, integral control action and derivative control action is called PID control action. The combined action has the advantage of each of the three individual control actions.

The Proportional controller stabilizes the gain but produces a steady-state error. The integral controller reduces or eliminates the steady-state error. The derivative controller reduces the rate of change of error. The main advantages of PID controllers are higher stability, no offset and reduced overshoot.

PID controllers are commonly used in process control industries.

The actuating signal or outupt signal from a PID controlle rin time domain is given by

$$e_a(t) = K_P e(t) + K_I \int_0^t e(\tau)d\tau + K_D \frac{de}{dt} \qquad ...(20)$$

In the s-domain, the output signal from the controller is

$$E_a(s) = \left(K_P + \frac{K_I}{s} + K_D s\right)E(s) \qquad ...(21)$$

Figure shows a PID controller for second-order system. The closed-loop transfer function of a PLD controller for a second-order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2(K_D s^2 + K_P s + K_I)}{[s^3 + (2\zeta\omega_n + K_D\omega_n^2)s^2 + K_P\omega_n^2 s + K_I\omega_n^2]} \qquad ...(22)$$

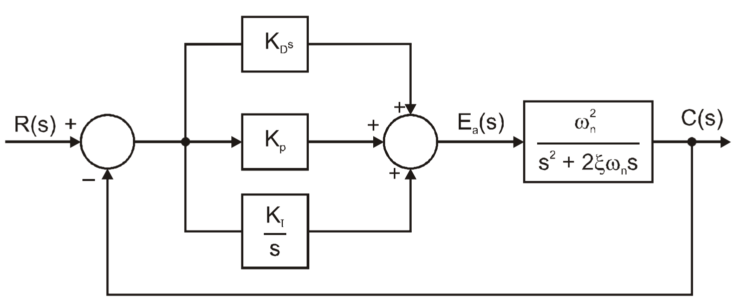


**Fig.7** Block diagram of a PID controller for second-order system.

The mode of control is combination of the previous modes and is given by the expression

$$p = K_c \in + K_c \tau_D \frac{d\in}{dt} + \frac{K_c}{\tau_1}\int_0^1 \in dt + p_s \qquad ...(23)$$

In this case, the controller contains three knobs for adjusting $K_c$, $\tau_D$, and $\tau_1$.

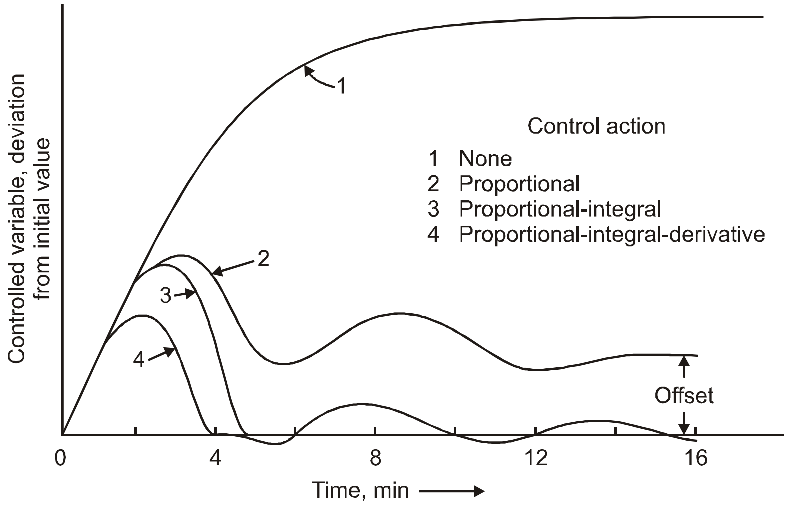


**Fig.8** Response fo a typical control system showing the effects of various modes of control.