

## JEST - PHYSICS

### SAMPLE THEORY

- BLACKBODY RADIATION
- QUANTUM MECHANICS : DE-BROGLIE WAVES

# VPM CLASSES

For IIT-JAM, JNU, GATE, NET, NIMCET and Other Entrance Exams

1-C-8, Sheela Chowdhary Road, Talwandi, Kota (Raj.) Tel No. 0744-2429714

Web Site [www.vpmclasses.com](http://www.vpmclasses.com) E-mail-[vpmclasses@yahoo.com](mailto:vpmclasses@yahoo.com)

## BLACKBODY RADIATION

The photons in a given particle-state interact only with the walls, and the remaining states do not act as a particle and energy reservoir for them. As the photons do not 'diffuse' from one particle-state to another (our subsystems previously), their chemical potentials have no significance in relation to one another. The photons in a given particle-state chosen for the application of grand canonical distribution interact with the walls of the cavity which in turn is kept at a constant temperature. When the photons go from this state to the walls, and vice versa, only the energy of the walls (now acting as a reservoir) changes, all the constituents making up the walls remain unchanged.

Therefore, while applying grand canonical distribution to photons, chemical potential of the reservoir has to be taken as zero. Equation  $\mu = \mu$  then suggests that the chemical potential of the photon gas shall also be zero at equilibrium.

The foregoing conclusion which states that chemical potential of a photon gas is zero can also be arrived at in another way. The enclosure absorbs and re-emits photons of various energies. The walls can convert a high energy photon into a number of low energy photons, and vice versa. This means that the total energy of the (photon) system and its other thermodynamic parameters have no direct relationship to the number of particles (photons) in it. Therefore from eq,  $\mu = \left( \frac{\partial G}{\partial N} \right)_{T,P}$  chemical potential of the photon gas is zero.

As  $\mu = 0$ , the distribution of photons in various quantum states of energies  $\epsilon_s = h\nu_s = \hbar\omega_s$  is

$$\bar{n}_s = \frac{1}{e^{h\nu_s/kT} - 1} \quad \text{..(1)}$$

which is called *Planck's distribution*.

The number of vibration modes of electromagnetic waves between propagation vector  $k$  and  $k + dk$  is

$$\frac{V k^2 dk}{2\pi^2} \quad \text{..(2a)}$$

and between frequency  $\nu$  and  $\nu + d\nu$  is

$$\frac{4\pi V \nu^2 d\nu}{c^3} \quad \text{..(2b)}$$

the radiation energy in this segment of the spectrum is

$$U(\nu)d\nu = \frac{8\pi h V}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/KT} - 1} \quad \text{..(3)}$$

and the energy per unit volume is

$$u(\nu)d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/KT} - 1}$$

Equation (3) is the *Planck formula* for spectral energy distribution of black-body radiation.

In terms of wavelength, the Planck formula becomes

$$U(\lambda)d\lambda = \frac{8\pi hc V}{\lambda^5} \frac{d\lambda}{e^{hc/(\lambda KT)} - 1} \quad \text{..(4)}$$

On longer wavelength side ( $hc/\lambda \ll KT$ ), this relation reduces to

$$U(\lambda)d\lambda = \frac{8\pi k T V d\lambda}{\lambda^4} \quad \text{..(5)}$$

This is the *Rayleigh –Jeans formula*.

On shorter wavelength side where  $hc/\lambda \gg kT$ . Eq. (4) becomes

$$U(\lambda) d\lambda = \frac{8\pi hc V}{\lambda^5} e^{-hc/(\lambda KT)} d\lambda \quad \text{..(6)}$$

This is the *Wien radiation formula*.

Spectral distribution of energy density at 1646 K according to relations (4)– (6) is shown in Fig.(1). Planck's formula is in very good agreement with the experiment. Planck's curves, which pass through experiment points, are also shown at two lower temperatures.

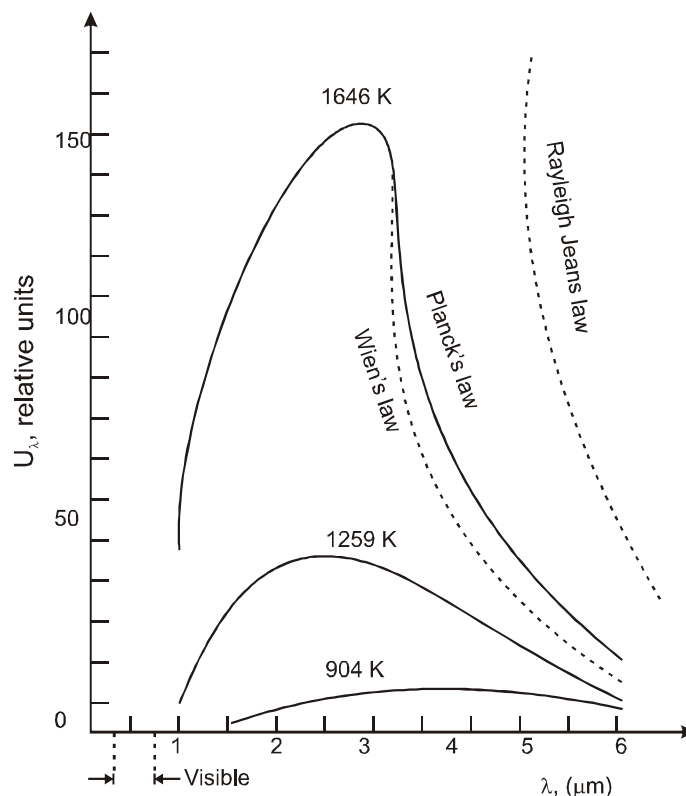
The maximum energy density  $u(\lambda) = U(\lambda)/V$ , then it can be seen that  $\lambda_m T$  is very nearly equal to  $hc/5k$ . In fact,

$$\lambda_m T \equiv b = \frac{hc}{4.9651k} \quad \text{..(7)}$$

Substitution the values of constants in Eq. (7), we obtain

$$\lambda_m T = b = 2.898 \times 10^{-3} \text{ m K}$$

when temperature rises, the position of the distribution maximum shifts towards lower wavelength is inverse proportion to  $T$ . This is *Wien's displacement law*.



**Fig. (1) Spectral distribution of energy in black-body radiation.**

At equilibrium, the total energy of the black-body radiation inside an enclosure of volume  $V$  and at temperature  $T$ , from Eq. (3), is

$$U(T) = \frac{8\pi h V}{c^3} \int_0^\infty \frac{v^3 dv}{e^{hv/kT} - 1}$$

Put  $hv/kT = x$ . Then

$$U(T) = \frac{8\pi^5 V (kT)^4}{15(hc)^3} \quad \text{..(8)}$$

The total energy per unit volume, or the total energy density, at temperature  $T$  is

$$u(T) = \frac{8\pi^5 (kT)^4}{15(hc)^3} = aT^4 \quad \text{..(9a)}$$

Where

$$a = \frac{8\pi^5 k^4}{15(hc)^3} \quad \text{..(9b)}$$

Thus the energy density or the total energy of the blackbody radiation is proportional to the fourth power of absolute temperature. This is the **Stefan-Boltzmann law**.

## THERMODYNAMICS OF BLACK-BODY RADIATION

The grand partition function of a system

$$Z(\mu, T) = \sum_{U_r, N_r} e^{-\beta U_r} \times e^{\mu \beta N_r} = \sum_r [\exp(-\beta \sum_s n_{rs} \epsilon_s)] [\exp(\mu \beta \sum_s n_{rs})]$$

$$= \sum_r \exp \left[ \sum_s \beta (\mu - \epsilon_s) n_{rs} \right] = \sum_r \prod_s \exp [\beta (\mu - \epsilon_s) n_{rs}]$$

or  $Z(\mu, T) = \prod_s Z_s(\mu, T)$  ..(10)

As the photons do not interact and  $\mu = 0$  for a photon system, we get from eq. (10), the relation

$$Z(T) = \prod_s Z_s(T) \quad \text{..(11)}$$

where, from eq.  $Z_s(\mu, T) = \frac{1}{1 - e^{(\mu - \epsilon_s)/kT}}$  reduce to

$$Z_s(T) = \frac{1}{1 - \exp(-\epsilon_s/kT)} \quad \text{..(12)}$$

The grand potential of the system becomes

$$\Omega = -kT \ln Z(T) = kT \sum_s \ln [1 - \exp(-\epsilon_s/kT)]$$

we get

$$\Omega = - \frac{8\pi^5 V (kT)^4}{45(hc)^3} \quad \text{..(13)}$$

From Eqs. (12) And (13), we get

$$\Omega = -u(T)/3 \quad \text{..(14)}$$

As  $\Omega = -pV$ , the pressure of radiation is given as

$$p = U(T)/3V = u(T)/3$$

## QUANTUM MECAHNICS :DE-BROGLIE WAVES

### VELOCITY OF DE-BROGLIE WAVES ; QUANTUM PICTURE OF MATERIAL PARTICLE

According to de Broglie hypothesis a material particle in motion has a wave of wavelength  $\lambda = h/mv$  ; associated with it, (where  $m$  is the mass of the particle and  $v$  its velocity.) If  $E$  is the energy of the particle, then the frequency  $\nu$  of the wave can be specified by the quantum condition  $E = h\nu$ , thereby giving

$$\nu = E/h \quad \text{..(1)}$$

But according to Einstein's mass-energy relation  $E = mc^2$ , so that from (1),  $\nu = mc^2/h$

The de Broglie wave velocity  $v$  is given by

$$v = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v} \quad \text{..(2)}$$

According to Einstein's theory of relativity, the speed of light is maximum speed that can be attained by a particle in nature i.e. the speed of material particle ( $v$ ) is always less than the speed of light  $c$ . Accordingly equation (2) implies that the de Broglie wave velocity must be greater than  $c$ . This is an unexpected result. Furthermore, according to this result, the de Broglie wave associated with the particle would travel faster than the particle itself, thus leaving the particle far behind. Thus it is clear that material particle cannot be equivalent to a single wave train. The speed of a single wave-train is called the **phase-velocity**.

The difficulty was resolved by Schrodinger by postulating that a material particle in motion is equivalent to a wave packet rather than a single wave.

A wave packet comprises a group of wave ; each with slightly different velocity and wavelength, with phases and amplitudes so chosen that they interfere constructively over only a small region of space where the particle can be located, outside of which they produce destructive interference so that the packet, in general, resembles the curve of Fig.2 . Such a packet moves with its own velocity  $G$ , called the group velocity. The individual wave forming the packet possesses an average velocity,  $u$  called the *phase velocity*. It can be shown that the velocity of the material particle  $v$  is same as group velocity of the group of waves.

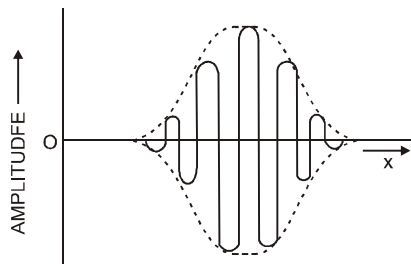


Fig.(2)

the phase velocity is given by

$$u = \frac{\omega}{k}.$$

the velocity

$$v_g = \frac{d\omega}{dk}$$

called the **group velocity**.

## Equation of Motion of Matter Waves

### (i) Time independent Schrodinger equation :

The Schrodinger time-independent wave equation,

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0. \quad \text{..(1)}$$

### (ii) Schrodinger equation for a free particle :

For a free particle  $V = 0$ ; therefore if we put  $V = 0$  in equation (1), it will become the Schrödinger equation for a free particle, i.e.,

$$\nabla^2 \psi + \frac{2mE}{\hbar^2} \psi = 0. \quad \text{..(2)}$$

### (iii) Time dependent Schrodinger Equation :

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = i\hbar \frac{\partial \psi}{\partial t}. \quad \text{..(3)}$$

This equation contains the time and hence is called the time-dependent Schrodinger equation. It be written as

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial \psi}{\partial t}. \quad \text{..(4)}$$

The operators  $\left( -\frac{\hbar^2}{2m} \nabla^2 + V \right)$  called Hamiltonian and is represented by H.

## PHYSICAL SIGNIFICANCE OF WAVE FUNCTION

$\psi(\vec{r}, t)$  is a probability that a particle be at  $\vec{r}$  at time  $t$ .  $\psi(\vec{r}, t)$  is more where probability of finding a particles is more and conversely. Maxborn defines it (in 1962) as :

'A particle defined by wave function  $\psi(\vec{r}, t)$ , the probability of that particle at  $(x, y, z)$  at time  $t$  is proportional to  $|\psi(\vec{r}, t)|^2$  i.e. position probability density of particle having wave function  $\psi(\vec{r}, t)$  is given by

$$p(\vec{r}, t) = |\psi(\vec{r}, t)|^2$$

$$= \psi^*(\vec{r}, t) \psi(\vec{r}, t) \quad \text{..(1)}$$

Where  $\psi^*(\vec{r}, t)$  is complex conjugate function of wave function  $\psi(\vec{r}, t)$ .

According to this definition

$$p(\vec{r}, t) dv = |\psi(\vec{r}, t)|^2 dv \quad \text{..(2)}$$

gives probability of finding a particle in volume  $dv = dx dy dz = d^3 \vec{r}$

So  $\psi(\vec{r}, t)$  is said to be probability amplitude. By the explanation of Born wave function of quantum mechanics have two restriction

- (1) Wave function cannot be infinity.
- (2)  $\psi(\vec{r}, t)$  gives unique value for any  $\vec{r}$ .