

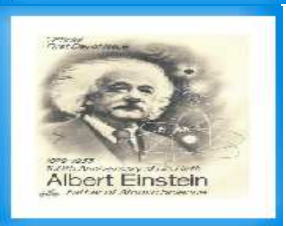


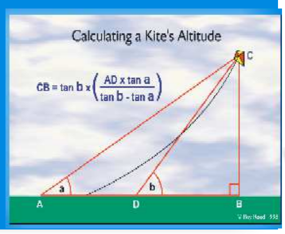

INDRAPRASTHA UNIVERSITY - MCA

SAMPLE THEORY

- HIGHER ORDER DIFFERENTIAL EQUATION
- COMPLEMENTS

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HIGHER ORDER DIFFERENTIAL EQUATION

(i) Definition

The general form of linear equation is

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + P_ny = Q, \qquad \text{...(i)}$$

where Q and $P_1, P_2, P_3, \dots\dots, P_n$ are all constants or functions of x.

If $P_1, P_2, P_3, \dots\dots\dots, P_n$ are all constants (Q may not be constant), then the equation is said to be a linear differential equation with constant coefficients.

Note. The general solution of (i) is y = C.F. + P.I. where C.F. is complementary function and P.I. is particular Integral.

Let us consider the equation $\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + P_ny = 0,$ ...(ii)

where $P_1, P_2, P_3, \dots, P_n$ are all constants. or $[D^n + P_1D^{n-1} + \dots + P_n]y = 0$

Let $y = e^{mx}$ be a solution of (ii). Then substituting e^{mx} for y in (ii) we have

$(m^n + P_1m^{n-1} + P_2m^{n-2} + \dots + P_n)e^{mx} = 0$, which is true if

$$m^m + P_1m^{n-1} + \dots + P_n = 0 \qquad \text{...(iii)}$$

The equation (iii) is called the auxiliary equation. If the roots of (iii) are $m_1, m_2, \dots, m_n$ and are all different, then $e^{m_1x}.e^{m_2x}, e^{m_3x}, \dots\dots\dots\dots$ are all different and linearly independent.

Hence the general solution of (ii) is $y = C_1e^{m_1x} + C_2e^{m_2x} + \dots + C_ne^{m_nx}$

Note. Symbol D. The symbols D and D^n are generally used for the operators $\frac{d}{dx}$ and $\frac{d^n}{dx^n}$ respectively.

Thus the equation $4\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 5y = 0$ can be written in symbolic form as $(4D^2 + 3D - 5)y = 0$.

Solved Examples

Ex.1 Solve $(D^3 + 6D^2 + 11D + 6)y = 0$.

Sol. Here D stands for $\frac{d}{dx}$ and so the given equation is $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$

Its auxiliary equation is $m^3 + 6m^2 + 11m + 6 = 0$ or $(m + 1)(m^2 + 5m + 6) = 0$

or $(m + 1)(m + 2)(m + 3) = 0$ or $m = -1, -2, -3$

$\therefore$ The required solution is $y = C_1e^{-x} + C_2e^{-2x} + C_3e^{-3x}$, where C_1, C_2, C_3 are arbitrary constants.

(ii). Auxiliary equation having equal roots

If the auxiliary equation has two equal roots say $m_1 = m_2$, then general solution

$y = (C_1 + C_2x)\, e^{m_1x} + C_3e^{m_3x} + \ldots + C_ne^{m_nx}$

Ex.2 Solve $(D^3 - D^2 - D + 1)y = 0$.

Sol. The auxiliary equation is $m^3 - m^2 - m + 1 = 0$

or $m^2(m - 1) - (m - 1) = 0$ or $(m - 1)(m^2 - 1) = 0$ or $(m - 1)(m - 1)(m + 1) = 0$

or $m = 1, 1, -1$

$\therefore$ The required solution is $y = (c_1x + c_2)e^x + c_3e^{-x}$, where c_1, c_2, c_3 are arbitrary constants.

(iii). Auxiliary equation having imaginary roots

Let $\alpha + i\beta$ and $\alpha - i\beta$ be a pair of imaginary roots, then the solution corresponding to these roots will be $C_1e^{(\alpha + i\beta)x} + C_2e^{(\alpha - i\beta)x} = e^{ax}[C_1e^{i\beta x} + C_2e^{-i\beta x}] =$

$e^{ax}[C_1'\cos\beta x + C'_2\sin\beta x]$, ...(i)

where C'_1 and C'_2 are arbitrary constants $= Ae^{ax}\cos(\beta x + B)$. Similarly $Ae^{ax}\sin(\beta x + B)$

Ex.3 Solve $\left(\frac{d^2y}{dx^2}\right) + 4y = 0$, given that y = 2 and $\left(\frac{dy}{dx}\right) = 0$ when x = 0.

Sol. The auxiliary equation is $m^2 + 4 = 0$ or $m = \pm 2i$.

$\therefore$ The solution of the given equation is $y = c_1 \cos 2x + c_2 \sin 2x$, ...(i)

where c_1, c_2 are arbitrary constants

Also from (i) we get $\frac{dy}{dx} = -2c_1 \sin 2x + 2c_2 \cos 2x$ ...(ii)

Now given that y = 2, $\frac{dy}{dx} = 0$ when x = 0. From (i) and (ii) we get

$2 = c_1 \cos 0 + c_2 \sin 0$ and $0 = -2c_1 \sin 0 + 2c_2 \cos 0$ or $2 = c_1$ and $0 = 2c_2$

or $c_1 = 2, c_2 = 0$.

From (i) the required solution is y = 2 cos2x.

COMPLEMENTS

Complements are used in digital computers for simplifying the subtraction operation and for logical manipulation. There are two kinds of complements for any base-r system.

(a) The r's complement

(b) The (r–1)'s complement

When the value of the base is substituted, the two receive the name 2's and 1's complement for binary system and 10's & 9's complement for decimal system.

R's COMPLEMENT

Given a positive number N, in base r, with an integer part of n digits, the r's complement of N is defined as $r^n - N$ for $N \neq 0$ and 0 for N = 0

The following example will explain it more.

The 10's complement of $(52520)_{10}$ is $10^5 - 52520 = 47480$

The number of digits in the number is n = 5

The 10's complement of $(0.3267)_{10}$ is $1 - 0.3267 = 0.6733$

No integer part, so $10^n = 10^0 = 1$

The 10's complement of $(25.639)_{10}$ is $10^2 - 25.639 = 74.361$

The 2's complement of $(101100)_2$ is $(2^6)_{10} - (101100)_2 = (1000000 - 101100)_2 = 010100$

The 2's complement of $(0.0110)_2$ is $(1 - 0.0110)_2 = 0.1010$

The 10's complement of a decimal number can be formed by leaving all least significant zeros unchanged, subtracting the first nonzero least significant digit from 10 and then subtracting all other higher significant digits from 9. The 2's complement can be formed by leaving all least significant zeros and the first nonzero digit unchanged, and then replacing 1's by 0's and 0's by 1's in all other higher significant digits.

The r's complement of a number exists for any base r (r greater than 0 but not equal to 1) and may be obtained from the definition given above.

THE (r – 1)'s COMPLEMENT

Given a positive number N in base r with an integer part of n digits and a fraction part of m digits, the (r – 1)'s complement of N is defined as $r^n - r^{-m} - N$. Some numerical examples are as follow:

The 9's complement of $(52520)_{10}$ is $(10^5 - 1 - 52520) = 99999 - 52520 = 47479$

No fraction part, so $10^{-m} = 10^0 = 1$

The 9's complement of $(0.3267)_{10}$ is $(1 - 10^{-4} - 0.3267) = 0.9999 - 0.3267 = 0.6732$

No integer part, so $10^n = 10^0 = 1$

The 9's complement of $(25.639)_{10}$ is $(10^2 - 10^{-3} - 25.639) = 99.999 - 25.639 = 74.360$

The 1's complement of $(101100)_2$ is $(2^6 - 1) - (101100) = (111111 - 101100)_2 = 010011$

The 1's complement of $(0.0110)_2$ is $(1 - 2^{-4})_{10} - (0.0110)_2 = (0.1111 - 0.0110)_2 = 0.1001$

The 9's complement of a decimal number is formed simply by subtracting every digit from 9. The 1's complement of a binary number is even simpler to form- the 1's are changed to 0's and the 0's to 1's. Since the (r – 1)'s complement is very easily obtained, it is sometimes convenient to use it when the r's complement is desired. From the definitions and from a comparison of the results obtained in the examples, it follows that the r's complement can be obtained from the (r – 1)'s complement after the addition of r^{-m} to the least significant digit. The 2's complement of 10110100 is obtained from the 1's complement 01001011 by adding 1 to give 01001100.

It is worth mentioning that the complement of the complement restores the number to its original value. The r's complement of N is $r^n - N$ and the complement of $(r^n - N)$ is $r^n - (r^n - N) = N$, and similarly for the 1's complement.