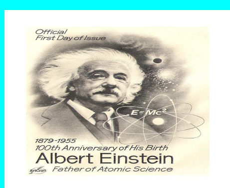

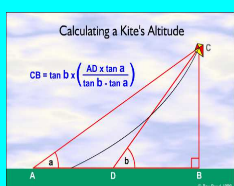

IIT-JAM - PHYSICS

MOCK TEST PAPER(According to new pattern)

- Attempt ALL the 60 questions.
- SECTION-A Consists of 30 questions. These questions are Multiple Choice Questions (MCQs),First 20 questions carries one marks for each,and remaining 10 questions carries two marks for each.
- Section-B Consists of 10 questions. These questions are Multiple Select Questions (MSQs), each question carries two marks.
- Section-C Consists of 20 Numerical Answer Type (NAT) questions each question carries two marks. For each NAT type question, the answer is a signed real number.
- In Section A, for all 1 mark questions,1/3 marks will be deducted for each wrong answer and for all 2 marks questions, 2/3 marks will be deducted for each wrong answer.There is no negative marking in Section B and Section C.

- ***Total marks : 100***
- ***Duration of test : 3 Hours***

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SECTION-A

MULTIPLE CHOICE QUESTIONS (MCQs) (Q. 1-30)

1. A particle is moving in space with 0 as the origin. Some possible expression for it's position velocity is given in polar coordinates (r, θ, ϕ,) are given below. Which of the following options is correct ?

(A) $\vec{r} = r\hat{e}_r + r\hat{e}_\theta + \hat{e}_\phi$, $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta + r\sin\theta\,\dot{\phi}\,\hat{e}_\phi$

(B) $\vec{r} = r\hat{e}_r$ $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r\sin\theta\,\dot{\phi}\,\hat{e}_\phi$

(C) $\vec{r} = r\hat{e}_r + r\theta\hat{e}_\phi$ $\vec{v} = \dot{r}\,\hat{e}_r + r^2\,\dot{\theta} + r^2\sin\theta\,\dot{\phi}\,\hat{e}_\phi$

(D) $\vec{r} = r\hat{e}_r + r\theta\,\hat{e}_\theta$ $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

2. A satellite moves around a planet in a circular orbit at a distance R from its centre . The time period of revolution of the satellite is T. If the time period of satellite is become 8T then what will be the new radius of orbit .

(A) $\frac{R}{2}$

(B) 4R

(C) $\frac{R}{4}$

(D) 2R

3. A solid sphere of mass 2m and radius a/2 is rolling with a linear v speed on a flat surface without slipping. The magnitude of the angular momentum of the sphere w.r.t a point along the path of the sphere on the surface is,

(A) $\frac{2}{5}mav$

(B) $\frac{m}{2}av$

(C) mav

(D) 2mav

4. An object of mass m moving with a velocity v is approaching a second object of the same mass but at rest. The total kinetic energy of the two objects as viewed from the centre of mass is,

(A) mv^2

(B) $\frac{1}{2}mv^2$

(C) $\frac{1}{4}mv^2$

(D) $\frac{1}{8}mv^2$

5. Moment of inertia of a uniform solid cylinder about its axis having length l and radius R is

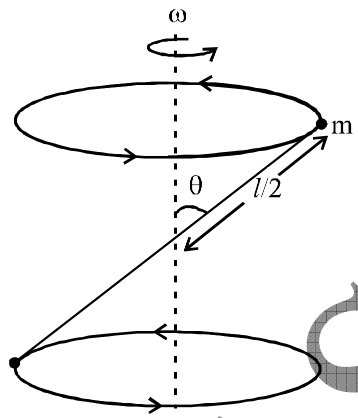
(A) $\frac{1M}{2}\ell^2$

(B) $\frac{1}{2}MR^2$

(C) $\frac{1}{2}M\left(l^2+R^2\right)$

(D) $\frac{1}{4}M\left(l^2+2R^2\right)$

6. A thin massless rod of length l has equal point masses m attached at its length (see figure). The rod is rotating about an axis passing through its centre and making angle θ with it. The magnitude of the rate of change of its angular momentum



(A) $2ml^2 \omega^2 \sin\theta \cos\theta$

(B) $ml^2 \omega \sin 2\theta$

(C) $\frac{ml^2}{4}\omega^2 \sin 2\theta$

(D) $\frac{ml^2}{2}\omega \sin 2\theta$

7. The weight of a body at a height is equal to the radius of the earth. W is its weight at a height which is equal to three times the radius of the earth which will be

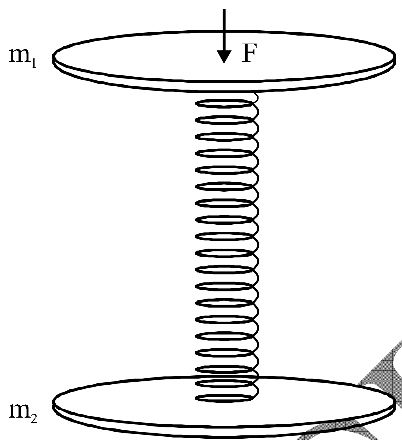
(A) $\frac{W}{2}$

(B) $\frac{W}{3}$

(C) $\frac{4}{3}W$

(D) $\frac{W}{4}$

8. Disc of mass m_2 is placed on a table. A stiff spring is attached to it and is vertical. The other end of the spring is attached to a disc of mass m_1.(Fig.). What minimum force should be applied to the upper disc to press the spring such that the lower disc is lifted off the table when the external force is suddenly removed ?



(A)

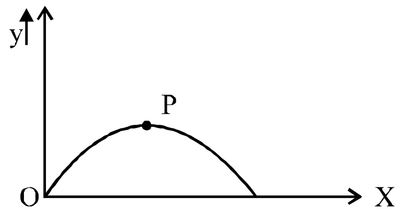
(A) $F_{min} = m_1 g$

(B) $F_{min} = m_2\ g$

(C) $F_{min} = (m_1 + m_2)\ g$

(D) $F_{min} = \frac{m_2 + m_1}{m_1 m_2} g$

9. A projectile is fixed from the origin O at an angle of 30° from the horizontal. At the highest point P of its trajectory the radial and transverse components of its acceleration in terms of the gravitational acceleration g are



(A) $a_r = -\frac{g}{\sqrt{13}}$, $a_\phi = -\frac{g}{\sqrt{12}}$

(B) $a_r = -\frac{g}{\sqrt{12}}$, $a_\phi = g\sqrt{\frac{12}{13}}$

(C) $a_r = -\frac{g}{\sqrt{12}}$ $a_\phi = -g\sqrt{\frac{13}{12}}$

(D) $a_r = -\frac{g}{\sqrt{13}}$, $a_\phi = -g\sqrt{\frac{12}{13}}$

10. A particle of mass 3 kg moves under a force of $\left[4\vec{i} + 8\vec{j} + 10\vec{k}\right]$ newton. If the particle starts from rest and was at the origin initially, what are its new co-ordinates after 3 s?

(A) $(6,12,15)$

(B) $(6,15,13)$

(C) $(6,12,6)$

(D) $(6,15,12)$

11. In the formula $x = 3yz^2$ x and z have dimensions of capacitance and magnetic induction what are the dimensions of y -

(A) $M^{-3} L^{-2} T^4 A^2$

(B) $M^{-3} L^{-2} T^8 A^4$

(C) $M^{-1} L^{-2} T^8 A^2$

(D) $M^{-1} L^{-2} T^4 A^6$

12. A body of mass m_1 collides elastically and head on with another of mass m_2 at rest . If initial velocity of m_1 is u, and final velocity after the collision is $U_1/1.5$ then what is the ratio of $\frac{m_1}{m_2}$?

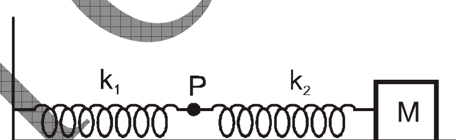
(A) 1 : 25

(B) 1 : 5

(C) 5 : 1

(D) 25 : 1

13. The mass M shown in figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



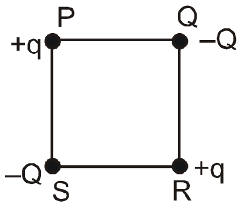
(A) $\frac{k_1 A}{k_2}$

(B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$

(D) $\frac{k_2 A}{k_1 + k_2}$

14. Four charges are placed at the corners of a square of side ℓ as shown. The charge at P enjoys equilibrium. Then q/Q is equal to



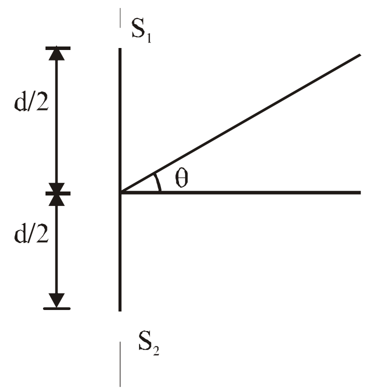
(A) $2\sqrt{2}$

(B) 2

(C) $\sqrt{2}$

(D) $2\sqrt{3}$

15. In an interference arrangement similar to Young's double-slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by a distance d = 150.0 m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is the maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by



(A) $I(\theta) = I_0 / 2$ for $\theta = 30°$

(B) $I(\theta) = I_0 / 4$ for $\theta = 90°$

(C) $I(\theta) = I_0 / 3$ for $\theta = 0°$

(D) $I(\theta)$ is constant for all values of (θ)

16. Two point charges +q and –q are held fixed at (–d, 0) and ℓ, 0) respectively of a x-y coordinate system. Then

(A) The electric field E at all point on the x-axis has the same direction

(B) Work has to be done in bringing a test charge from ∞ to the origin.

(C) Electric field at all point on y-axis is along x-axis

(D) The dipole moment is 2qd along the x-axis.

17. A particle with specific charge $\frac{q}{m}$ moves rectilinearly due to an electric field $\vec{E} = E_0 - ax$, where a is positive constant, x is the distance from the point where the particle was initially at rest then the distance covered by the particle till the moment it came to a stand still.

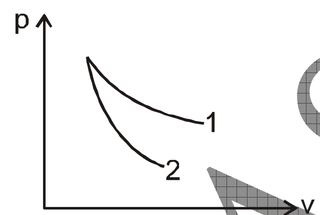
(A) $\frac{E_0}{a}$

(B) $\frac{E_0}{2a}$

(C) $E_0 \frac{a}{2}$

(D) $\frac{2E_0}{a}$

18. p-v plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to.



(A) He and O_2

(B) O_2 and He

(C) He and Ar

(D) O_2 and N_2

19. In the vicinity of the triple point the saturated vapour pressure of carbon dioxide depends on temperature T as $\log P = a - \frac{b}{T}$, where a and b are constants. If p is expressed in atmospheres, then for the sublimation process a = 9.05 and b = 1.80 kK, and for vaporization process a = 6.78 and b = 1.31kK. then the temperature of triplet point.

(A) 216 K

(B) 275 K

(C) 196.46 K

(D) 242.0K

20. What is the value of line integral of $\vec{A}\int_C A\cdot dr$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ where $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$

(A) 2π

(B) $\frac{1}{2}\pi$

(C) 0

(D) π

21. A particle of rest mass m_0 moving with 0.6 speed makes a completely inelastic collision with a particle of rest mass $2m_0$ that is initially at rest, what is the rest mass of the resulting single body.

(A) $3m_0$

(B) $4.5\ m_0$

(C) $3.4\ m_0$

(D) $3.6\ m_0$

22. A particle of mass m energy E is moving in one dimensional potential box of width a then

(A) The eigen function are orthonormalised.

(B) The expectation value of position is a/2.

(C) In n = 3 level the degeneracy is 9

(D) The energy difference between consecutive energy levels are same.

23. Which of the structure having largest density

(A) Simple cubic structure

(B) Face-centered cubic structure

(C) Body-centered structure

(D) Diamond

24. Which of the following statement is correct

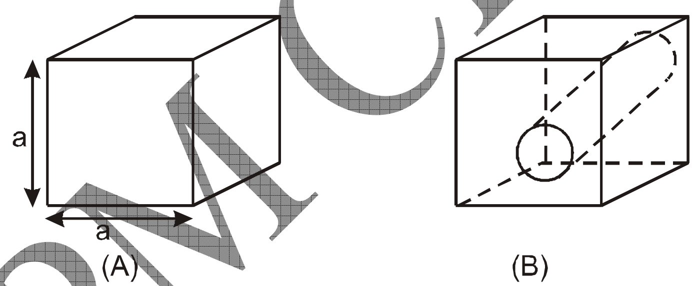
(A) Paramagnetic substance have large value of susceptibility of magnetic permeability.

(B) Ferromagnetic substance large susceptibility but less negative magnetic permeability

(C) Diamagnetic substance have temperature independent susceptibility.

(D) Susceptibility of diamagnetic and paramagnetic substance is depend on temperature.

25. Body A and body B are made up an isotropic material. Body B has a cavity inside it



(A) Expansion in volume of body A is more than that of body B.

(B) Expansion in volume of body B is more than that of A.

(C) Expansion in volume of two bodies is same

(D) Data is incomplete

26. Two cars start of to race with velocity V_1 and V_2 travel in a straight line with uniform acceleration a_1 and a_2. If the race end in a dead heat, then the length of the course is

(A) $\frac{2(V_1a_1 - V_2a_2)}{(a_1 - a_2)^2}$

(B) $\frac{2(V_1 - V_2)(V_1a_2 - V_2a_1)}{(a_1 - a_2)^2}$

(C) $\frac{2(V_1a_1 - V_2a_2)}{(a_1 - a_2)^2}$

(D) $\frac{2(V_1 - V_2)(V_1A_1 + V_2a_2)}{(a_1 + a_2)^2}$

27. A solid sphere of mass 2m and radius 71/2 is rolling with a linear ν speed on a flat surface with out slipping. The magnitude of the angular momentum of the sphere w. r. to a point along a path of sphere on surface is

(A) $\frac{7}{2}ma\nu$

(B) 7 maν

(C) $\frac{7}{4}ma\nu$

(D) 14 maν

28. The temperature of a diatomic gas is 300 K. Find the angular root mean square velocity of a rotating molecule of its m.o. 1 is = 2.1×10^{-39} gm. cm^2. (Take K = 1.z8 × 10^{-23})

(A) 2.21×10^{12} rad / s

(B) 4.43×10^{12} rad / s

(C) 44.3×10^{12} rad / s

(D) 22.1×10^{12} rad / s

29. Two large tanks a and b, open at the top, contain different liquids. A small hole is made in the side of each tank at the same depth h below the liquid surface, but the hole in a has twice the area of the hole in b. the ratio of the densities of the liquids in a and b so that the mass flux is the same for each hole should be

(A) 2

(B) 0.5

(C) 4

(D) 0.25

30. A mass m is suspended to a spring of length L and force constant k. The frequency of vibration is f_1. The spring is cut into two equal parts and each sorubg us cut half is loaded with same mass m. The new frequency f_2 is given by

(A) $f_2 = \sqrt{2}f_1$

(B) $f_2 = \frac{f_1}{\sqrt{2}}$

(C) $f_2 = f_1/2$

(D) $f_2 = \frac{\sqrt{2}}{f_1}$

SECTION-B

MULTIPLE SELECT QUESTIONS (MSQs) (Q. 31-40)

31. A particle of mass 2m with a velocity $\vec{v} = v_0(2\hat{\ell} + \hat{J}),$, collides elastically with another particle of mass 3 m which is at rest initially. Which of the following statement/s is/are not correct-

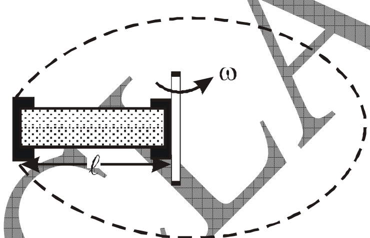
(A) The velocity of centre of mass frame is $\frac{4V_0}{\sqrt{5}}$

(B) Before collision velocity of particle of mass 2m is $\frac{2V_0}{\sqrt{5}}$

(C) Before collision velocity of particle of mass 2m is $\frac{3V_0}{\sqrt{5}}$

(D) Before collision velocity of particle of mass 3m is $\frac{2V_0}{\sqrt{5}}$

32. A straight tube of length L contains incompressible liquid of mass M and the closed tube is whirled in horizontal plane about one of the ends. If ω is the uniform angular velocity, the force not exerted by the liquid on the other is/are



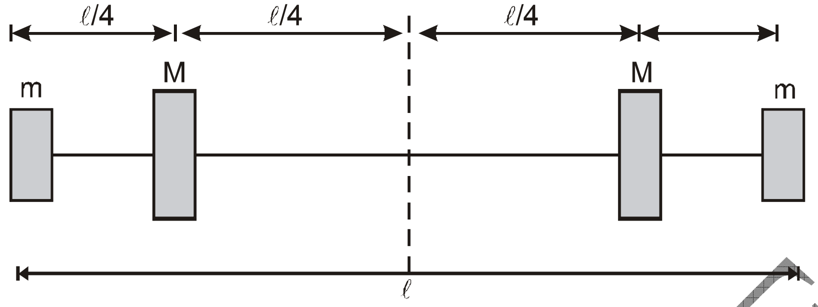
(A) $\frac{ML\,\omega^2}{4}$

(B) 2.5 ML ω^2

(C) $\frac{ML\,\omega^2}{2}$

(D) ML ω^2

33. A rod which is massless has four masses fixed on it, as shown in the figure. The moment of inertia about an axis passing through the centre of rod is/are



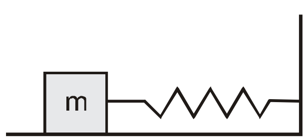
(A) $\ell^2\left(M + \frac{m}{4}\right)$

(B) $\frac{\ell^2}{2}\left(m + \frac{M}{4}\right)$

(C) $\frac{\ell^2}{2}\left(\frac{m}{4} + M\right)$

(D) $0.125\,\ell^2\ (4m + M)$

34. The block shown in the figure is acted upon by a spring having spring constant k and a weak friction force of constant magnitude f. The block is pulled a distance X_0 from equilibrium and released. It then oscillates many times and finally comes to rest. Friction is causing the damping of oscillations.



Make the incorrect statement(s)

(A) The decrease in amplitude is same for each cycle of oscillation is equal to $\frac{2f}{k}$.

(B) The decrease in amplitude is same for each cycle of oscillation and is equal to $\frac{4f}{k}$.

(C) The decrease in amplitude is different for each cycle of oscillation.

(D) The decrease in amplitude is different for each cycle of oscillation and for every subsequent cycle it decreases by a factor of 2.

35. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work not done in stretching is/are

(A) $\frac{F}{2\ell}$

(B) $0.5F\ell$

(C) $2F\ell$

(D) $\frac{F\ell}{2}$

36. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3, What relationship between t and t_0 is/are true?

(A) $t_0 = 0.5t$

(B) $t = t_0/2$

(C) $t = 2t_0$

(D) $t = 4t_0$

37. A wire of cross-sectional area 4×10^{-4} m^2, modulus of elasticity 2×10^{11} Nm^{-2} and length 1 m is stretched between two vertical rigid poles. A mass of 1kg is suspended at its middle. Calculate the angle it makes with horizontal.

(A) $\frac{1}{2}\times 10^{-2}$rad.

(B) 5×10^{-3}rad.

(C) $\frac{1}{4}\times 10^{-2}$rad.

(D) 2×10^{-2}rad.

38. Power transferred from the driving force to the oscillator is/are not maximum at the frequency of

(A) Quality resonance

(B) amplitude resonance

(C) Velocity resonance

(D) All of these

39. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_1 - \omega_2 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can only detect signals of intensity $\geq 2\ A^2$.

(a) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(b) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

(A) 6.28×10^{-3} s ; 1.57×10^{-3} s

(B) 6.28×10^{-4} s ; 157×10^{-3} s

(C) 6.28×10^{-2} s ; 157×10^{-4} s

(D) 628×10^{-5} s ; 15.7×10^{-4} s

40. A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite to the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20° C; Density of river water = 10^3 kg/m^3;

Bulk modulus of the water =9 2.088×10^9 Pa;

Gas constant R = 8.31 J/mol-K;

Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4)

(A) 1.007×10^5 Hz ; 1.0304×10^5 Hz

(B) 1.117×10^5 Hz ; 1.57×10^5 Hz

(C) 10.07×10^4 Hz ; 103.04×10^3 Hz

(D) 10.07×10^5 Hz ; 103.04×10^7 Hz

SECTION-C

NUMERICAL ANSWER TYPE (NAT) (Q. 41-60)

41. A copper strip 2.0 cm wide and 1.0 mm thick is placed in a magnetic field with B = 1.5Wb/m^2 (The number of free electrons per unit volume of copper is 8.4×10^{28} m^{-3} and charge on electron is 1.6×10^{-19}C). If a current of 200 A is set up in the strip, then calculate the Hall potential difference appears across the strip.

42. The electric potential V as a function of distance x (in metre) is given by :

$$V = (5x^2 + 10x - 9)V$$

What would be the value of electric field of x = 1m?

43. The interference pattern is obtained using a yellow light of wavelength in which 20 equally spaced fringes occupy 2.0 cm on the screen. On replacing the yellow source by another monochromatic source but making no other changes, it is noticed that 30 fringes occupy 2.4 cm on the screen. What is the wavelength of the second source ?

44. Two strong lines in the spectrum of sodium have wavelengths of 5890 Å and 5896 Å. In order to resolve these lines in the second order spectrum how many lines of the grating must be illuminated ?

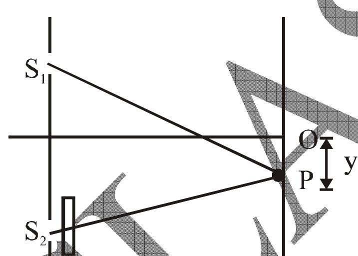
45. A vessel of volume V = 30 litre contains an ideal gas at temperature, T = 0°C. Keeping temperature constant, a part of the gas is allowed to escape from the vessel causing the pressure to fall down by $\Delta P = 0.78$ atm. Find the mass of the gas released. Its density under normal conditions is $\rho = 1.3$ g/lt.

46. A cart is moving along x direction with a velocity of 4 ms^{-1}. A person on the cart throws a stone with velocity of 6 ms^{-1} relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z axis. At the highest point of its trajectory the stone hits an object of equal mass hung vertically from branch of tree by means of a string of length L. The stone gets embedded in the object. Calculate the speed of combined mass immediately after the embedding w.r.t. an observer on the ground.

47. A man of weight W is standing on a spring balance in a satellite revolving about the earth. Then what would be his weight read by spring balance?

48. A particle is projected with a velocity of 19.6 ms^{-1} at an angle of 60° to the horizontal. What is the value of greatest distance of the projectile from a plane inclined at 30° to the horizontal?

49. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in the figure. Find the location of central maximum (bright fringe with zero path difference) on the y-axis.



50. The ionization energy of a hydrogen like Bohr atom is 8 rydberg. What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state?

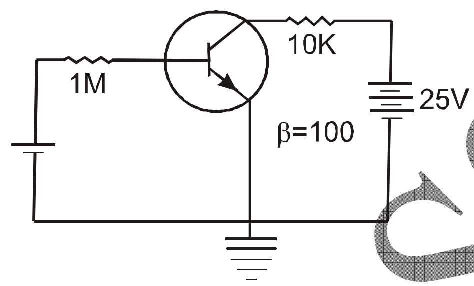
51. The ionization energy of a hydrogen like Bohr atom is 8 rydberg. What is the radius of the first orbit for this atom?

52. In an experiment of Compton effect the wavelength of incident radiation is 1.81. Calculate the wavelength of scattered radiation at , $\theta = 30°$. Also calculate the velocity of corresponding recoiled electron.

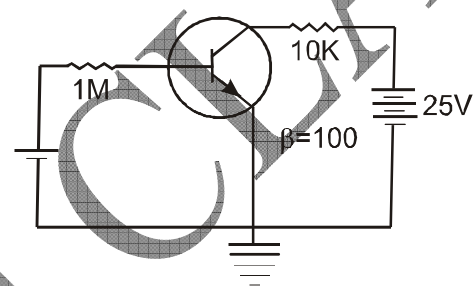
53. Find the Coulomb barriers of $_{8}O^{16}$, $_{41}Nb^{93}$ and $_{83}Bi^{209}$ as seen by a proton.

54. NaCl crystallizes as cubic structure. Taking the molecular weight of NaCl as 58.46 and the density at room temperature as 2.167 gm/ cm^3, calculate the lattice constant. (Avogadro number = 6.02×10^{23} per gmole)

55. For the common emitter configuration of transistor, as shown in the figure find emitter current.



56. For the common emitter configuration of transistor, as shown in the figure find voltage across emitter.



57. An electrically operated lift weighs 3 ton. It is raised 7 m in 5 second by a constant lifting force which is removed by switching off the current at such an instant that the weight of the lift bring it to rest in completing 7m lift. Neglecting the frictional and other resistance. Find the maximum velocity which the lift attains.

58. An electrically operated lift weighs 3 ton. It is raised 7 m in 5 second by a constant lifting force which is removed by switching off the current at such an instant that the weight of the lift bring it to rest in completing 7m lift. Neglecting the frictional and other resistance. Find the acceleration produced by the constant lifting force.

59. An electrically operated lift weighs 3 ton. It is raised 7 m in 5 second by a constant lifting force which is removed by switching off the current at such an instant that the weight of the lift bring it to rest in completing 7m lift. Neglecting the frictional and other resistance. Find the height to which the lift has risen when the current is switched off and

60. An electrically operated lift weighs 3 ton. It is raised 7 m in 5 second by a constant lifting force which is removed by switching off the current at such an instant that the weight of the lift bring it to rest in completing 7m lift. Neglecting the frictional and other resistance. Find the accelerating force.

Answer Key

[redacted]

HINTS AND SOLUTIONS

SECTION-A

MULTIPLE CHOICE QUESTIONS (MCQs) (Q. 1-10)

1.(B) We know position vector in Cartesian coordinate system.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

In polar spherical coordinate

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

So, $\vec{r} = r\,\hat{e}_r$

and $v = \dot{r} = \dot{r}\,\hat{e}_r + r\,\dot{\hat{e}}_r$

$$v = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta + r\sin\theta\,\dot{\phi}\,\hat{e}_\phi$$

2.(B) By kepler's law

$$T^2 \propto a^3$$

$T^2 = k\,a^3$ $\quad a = R$

$T^2 = K\ R^3$ ...(1)

In new case $T_2 = \frac{8Y}{T}$

$(8T)^2 = K(R)^3$ ...(2)

eq. (2) / eq. (1)

$$\frac{R_2^3}{R^3} = \frac{(8)^2}{1}$$

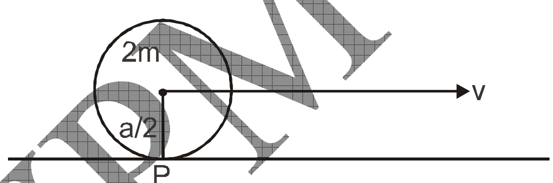
$$R_2^3 = (8)^2\ R^3$$

$$R_2 = (8)^{2/3}\ R = \frac{8}{(8)^{1/3}} R = \frac{8}{2} R$$

$$\boxed{R_2 = 4R}$$

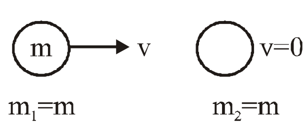
3.(C) A solid sphere is rolling along the flat surface without slipping and p is the point on the surface of sphere.

So, angular momentum of sphere about P is $L = 2mv \times \frac{a}{2}$



$$\boxed{L = mva}$$

4.(C)



Let v_1 is velocity of first mass after collision and v_2 is velocity of second mass after collision.

By the conservation law of momentum,

$$mv = m_1v_1 + m_2v_2 \quad ...(i)$$

But $m_1 = m_2 = m$

$\Rightarrow$ $mv = mv_1 + mv_2$

$\Rightarrow$ $v = v_1 + v_2 \quad ...(ii)$

By conservation law of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$\Rightarrow$ $v^2 = v_1^2 + v_2^2 \quad ...(iii)$

Solving Eqs. (ii) and (iii) we get

$$v_1 = 0, v_2 = v$$

Velocity of first mass is zero whereas velocity of second mass attain the velocity of first .

Now, the velocity of centre of centre of mass is given

$$(m + m)\, v_{CM} = mv$$

$\Rightarrow$ $v_{CM} = \frac{1}{2}v \quad ...(iv)$

Hence, the velocity of first mass w.r.t centre of mass = $0 - v_{CM}$

$= -\frac{1}{2}v \quad ...(v)$

That of second = $v - \frac{1}{2}v = \frac{1}{2}v \quad ...(vi)$

Total kinetic energy as seen from centre of mass

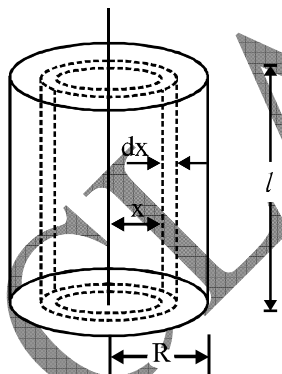
$$= \frac{1}{2} m \left(\frac{1}{2}v\right)^2 + \frac{m}{2}\left(\frac{1}{2}v\right)^2 = \frac{m}{2}\left[\frac{v^2}{4}+\frac{v^2}{4}\right] = \frac{mv^2}{4}$$

5.(B) Let M be the mass, l be the length and R be the radius of a solid cylinder as shown in fig. . We have to calculate moment of inertia of this solid cylinder about its axis.

Volume of the cylinder = $\pi R^2 l$ ...(i)

mass per unit volume of the cylinder ,

$$\rho = \frac{M}{\pi R^2 l}$$



Draw two cylindrical surface of radii x and (x + dx) coaxial with the given cylinder as shown in Fig. This part of the cylinder may be considered as a hollow cylinder of radius x and thickness dx

Area of cross section of the wall of this hollow cylinder = $2\pi x \cdot dx$.

Volume of material in this elementary hollow cylinder = $(2\pi x\, dx)\, l$

Mass of the elementary hollow cylinder

$$m = (2\pi x\, dx)\, l \times \rho = (2\pi x dx)\, l \times \frac{M}{\pi R^2 l}$$

$m = \frac{2M}{R^2} x\,dx.$

As radius of this cylinder is x, moment of inertia of the elementary cylinder about the given axis is

$$dI = mx^2 = \left(\frac{2M}{R^2} x\,dx\right) x^2 = \frac{2M}{R^2} x^3\,dx$$

∴ Moment of inertia of the solid cylinder about the given axis is

$$I = \int_{x=0}^{x=R} \frac{2M}{R^2} x^3\,dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_{x=0}^{x=R} = \frac{2M}{4R^2}\left(R^4 - 0\right)$$

$$\boxed{I = \frac{1}{2} MR^2}$$

Note that this formula for I does not depend upon length of the cylinder.

6.(C) $\vec{L} = I\,\omega = mr^2\,\omega = m\left[l/2 \sin\theta\right]^2 \omega = \frac{ml^2}{4}\,\omega \sin^2\theta$ $\qquad \because \theta = \omega t$

$$\vec{L} = \frac{ml^2}{4}\omega \sin^2 \omega t$$

$$\frac{d\vec{L}}{dt} = \frac{ml^2}{4}\,\omega \frac{d}{dt}\left(\sin^2 \omega t\right) = \frac{ml^2}{4}\omega\,[2 \sin \omega t \cos \omega t \times \omega] = \frac{ml^2}{2}\,\omega^2 \sin \omega t \cos \omega t$$

$$= \frac{ml^2}{2}\,\omega^2 \sin\theta \cos\theta$$

$$\frac{d\vec{L}}{dt} = \frac{ml^2}{4}\,\omega^2 \sin 2\theta$$

7.(D) Here $\frac{mg'}{mg} = \left(\frac{R}{R+h}\right)^2$ Now

Now $\frac{W_2}{W_0} = \left(\frac{R}{R+3R}\right)^2$ and $\frac{W_1}{W_0} = \left(\frac{R}{R+R}\right)^2$

$\therefore$ $\frac{W_2}{W_1} = \left(\frac{R+R}{R+3R}\right)^2 = \frac{1}{4}$.

But W_1 is given to be W $\therefore$ $W_2 = \frac{W_1}{4} = \frac{W}{4}$.

8.(C) In Fig. , position A of the upper disc shows underformed spring . Let the external force applied be F. If on applying the force F, the upper disc of mass m_1 is pressed downward downwards by x_1 (as at B)

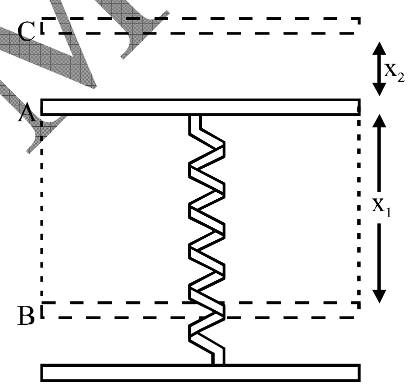
$$F + m_1 g = kx_1 \qquad ..(1)$$

Now if on releasing the upper disc the extension of the spring is x_2 (as at C), then by conservation of mechanical energy

$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}kx_2^2 + m_1 g(x_1 + x_2)$$

i.e., $$\tfrac{1}{2}k\left(x_1^2 - x_2^2\right) = m_1 g(x_1 + x_2)$$

i.e., $$x_1 = \frac{2m_{1g}}{k} x_2 \qquad ..(2)$$



(B)

Now the lower disc will leave the table if and only if

$$kx_2 > m_2g \qquad \text{i.e.,} \qquad x_2 > m_2g/k$$

Substituting the values of x_1 and x_2 from Eqns. (1) and (3) in (2)

$$\frac{F+m_1 g}{k} = \frac{2m_1g}{k} + \left(> \frac{m_2g}{k}\right)$$

or $\qquad F > (m_1 + m_2)g$ i.e., $\quad F_{min} = (m_1 + m_2)\, g$

So the lower disc will spring back and rise off the table if the spring is pressed by a face greater than the weight of the system.

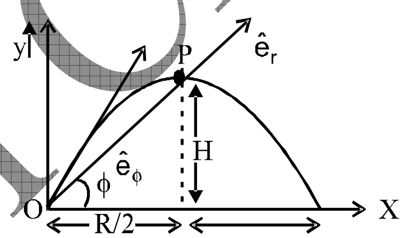
9.(D) For projectile motion in polar coordinates

$$\hat{y} = \hat{e}_r \sin\theta + \hat{e}_\phi \cos\theta$$

so, $$\vec{a} = -g\left[\hat{e}_r \sin\phi + \hat{e}_\phi \cos\phi\right]$$

$a_r = -g \sin\phi$

$a_\phi = -g\cos\phi$



At highest point

$$\sin\phi = \frac{H}{\sqrt{\left(\frac{R}{2}\right)^2 + H^2}} \qquad ..(1)$$

$\theta = 30^o$

So, $H = \frac{U^2 \sin^2 \theta}{2g}$ ...(2)

$H = \frac{U^2}{8g}$

and $R = \frac{u^2 \sin 2\theta}{g}$

$\frac{R}{2} = \frac{u^2 \times \sin 60^o}{2g}$

$\frac{R}{2} = \frac{\sqrt{3}u^2}{4g}$ ...(3)

so, $\sin \phi = \frac{\frac{u^2}{8g}}{\sqrt{\frac{3u^4}{16g^2} + \frac{u^4}{64\,g^2}}} = \frac{1/8}{\sqrt{\frac{12+1}{64}}}$

$\sin \phi = \frac{1}{\sqrt{13}}$

$\because \cos \phi = \sqrt{1 - \sin^2 \phi} = \sqrt{1 - \frac{1}{13}}$

$\cos \phi = \sqrt{\frac{12}{13}}$

So, $a_r = -g \sin \phi = -\frac{g}{\sqrt{13}}$ Ans.

$a_\phi = -g \cos \phi = -g\sqrt{\frac{12}{13}}$

10.(A) We know that according to Newton's law of motion

$\vec{F} = m\vec{a}$ i.e., $\vec{a} = \vec{F}/m$

But here $\vec{F} = 4\vec{i} + 8\vec{j} + 10\vec{k}$ and m = 3 kg

$\vec{a} = (1/3)\left[4\vec{i} + 8\vec{j} + 10\vec{k}\right]$ m/s^2

Further according to the equation of motion

$\vec{s} = \vec{u}t + (1/2)\,\vec{a}t^2$

$\vec{s} = 0 \times + (1/2)(1/3)\left[4\vec{i} + 8\vec{j} + 10\vec{k}\right] \times 3^2$

i.e., $\vec{s} = \left[6\vec{i} + 12\vec{j} + 15\vec{k}\right]$

so the co-ordinates of the particle after 3 sec. are [6, 12, 15].

11.(B) Given $x = 3yz^2$

$x = c = M^{-1} L^{-2} T^4 A^2$

and $z = B\ MT^{-2} A^{-1}$

So, $y = \dfrac{x}{3z^2} = \dfrac{\left[M^{-1}L^{-2}T^4A^2\right]}{\left[MT^{-2}A^{-1}\right]^2} = M^{-3} L^{-2} T^{4+4} A^{2+2}$

$y = M^{-3} L^{-2} T^8 A^4$

12.(C) Let v_2 be velocity of m_2 after collision.

By law of conservation of momentum

$m_1 u_1 = \dfrac{m_1 \times u_1}{1.5} + m_2 v_2$

$m_2 v_2 = m_1 u_1 - \dfrac{10 m_1 u_1}{15} = \dfrac{1}{3} m_1 u_1$

By conservation of energy ..(1)

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1\left(\frac{u_1}{1.5}\right)^2 + \frac{1}{2}m_2 v_2^2$$

$$m_2 v_2^2 = \frac{5}{2}m_1u_1^2 \qquad ..(2)$$

So, from eq. (1) and (2) $\frac{m_1}{m_2}=\frac{5}{1}$

13.(D) $x_1 + x_2 = A$ .....(1)

and $k_1x_1 = k_2x_2$

or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$

$$x_2 = \frac{k_1}{k_2}x_1 \qquad ....(2)$$

By equation (1) and (2)

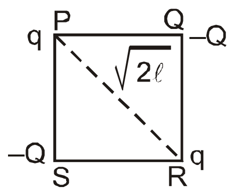
$$x_1 = A - x_2$$

$$x_1 = A - \frac{k_1}{k_2}x_1$$

$$x_1\left[1+\frac{k_1}{k_2}\right] = A$$

$$\boxed{x_1 = \frac{Ak_2}{k_1+k_2}}$$

14.(A) Here $F_{PQ} = \frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2}\ell\right)^2}$



For equilibrium

$$\frac{q^2}{4\pi\varepsilon_0\left(\sqrt{2}\ell\right)^2} = \frac{qQ\cos 45°}{4\pi\varepsilon_0\ell^2} + \frac{qQ\cos 45°}{4\pi\varepsilon_0\ell^2} = \frac{2qQ}{4\pi\varepsilon_0\left(\sqrt{2}\ell\right)^2}$$

i.e. $\frac{q}{Q} = \frac{2\sqrt{2}}{1}$

15.(C) The intensity of light is $I(\theta) = I_0 \cos^2\left(\frac{\delta}{2}\right)$

where $\delta = \frac{2\pi}{\lambda}(\Delta x) = \left(\frac{2\pi}{\lambda}\right)(d \sin\theta)$

(a) For θ = 30°

$\lambda = \frac{c}{v} = \frac{3\times10^8}{10^6} = 300\text{m}$ and d = 150 m

$$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2}$$

$\therefore \frac{\delta}{2} = \frac{\pi}{4}$

$\therefore I(\theta) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$ [Option (a)]

(b) For θ = 90°

$$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi$$

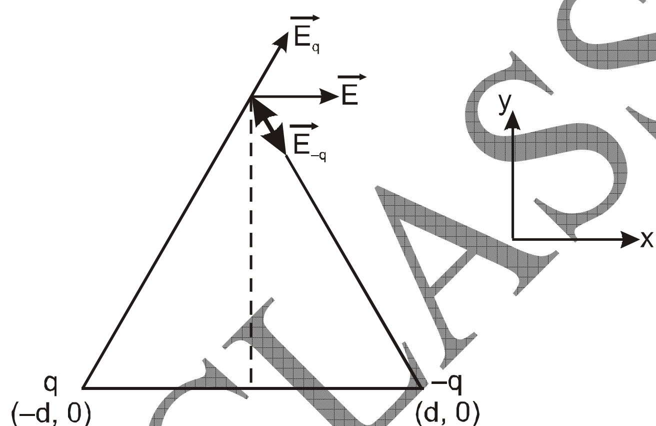
or $\frac{\delta}{2} = \frac{\pi}{2}$

and $I(\theta) = 0$

(c) For $\theta = 0°$, $\delta = 0$ or $\frac{\delta}{2} = 0$

$\therefore$ $I(\theta) = I_0$ [option (c)]

16.(C) The diagrammatic representation of the given question is shown in figure.



The electrical field $\vec{E}$ at all points on the x-axis will not have the same direction.

For $-d \le x \le d$, electric field is along positive x-axis while for all other points it is along negative x-axis.

The electric field $\vec{E}$ at all points on the y-axis will be parallel to the x-axis (ie, $\hat{i}$)

The electrical potential at the origin due to both the charges is zero, hence, no work is done in bringing a test charge from infinity to the origin.

Dipole moment is directed from the –q charge to the +q charge (ie, $-\hat{i}$ direction)

17.D) The equation of motion is

$$F = \frac{m\,dV(x,t)}{dt} = qE$$

$$\frac{dv}{dt} - \frac{vdv}{dx} = \frac{q}{x}(E_0 - ax)$$

By integrating

$$\frac{1}{2}v^2 - \frac{q}{m}\left[E_0 x - \frac{1}{2}ax^2\right] = \text{const.}(c)$$

at x = 0, v = 0, so, c = 0

$$v^2 = \frac{2q}{m}\left[E.x - \frac{1}{2}ax^2\right]$$

$\because$ v = 0 $|x = x_m$

So, $x = \boxed{x_m = \frac{2E_0}{a}}$

18.(B) In adiabatic process slop of p-v graph

$$\frac{dp}{dv} = -\gamma\frac{p}{v}$$

slope $\propto \gamma$ (–ve sign)

from given graph

$(\text{slope})_2 > (\text{slope})_1$

$\gamma_2 > \gamma_1$

Therefore 1 should correspond to O_2 gas whose $\gamma = 1.4$ and 2 should correspond to He gas ($\gamma = 1.67$)

19.(A) The equations of the transition lines are

$$\log P = 9.05 - \frac{1800}{T} \quad \text{(solid gas)}$$

$$\log P = 6.78 - \frac{1310}{T} \quad \text{(Liquid gas)}$$

At triplet point ($T = T_r$)

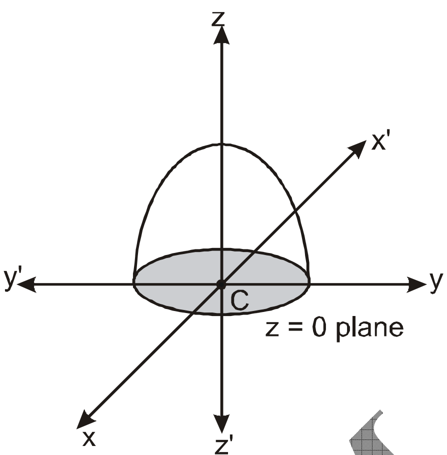
$$(\log P)_{s.g.} = (\log P)_{L..g.}$$

$$9.05 - \frac{1800}{T_r} = 6.78 - \frac{1310}{T_r}$$

$$T_r = \frac{490}{2.27} = 216° \text{ K}$$

20.(D) ($\because$ $z = 0, x^2 + y^2 = 1$)

from stokes-curl theorem.



$$\int_C \vec{A}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{A})\cdot dS$$

$$\therefore \quad \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x - y & -yz^2 & -y^2 z \end{vmatrix} = \hat{i}(-2yz + 2yz) + 0\hat{j} + \hat{k}[0 + 1]$$

$$\vec{\nabla}\times\vec{A} = \hat{k}$$

So, $\int_C \vec{A}\cdot d\vec{r} = \iint_S \hat{k}\cdot\hat{k}\,dx\,dy = \int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx\,dy$

$\int_C \vec{A}\cdot d\vec{r} = 2\int_{-1}^{+1}\sqrt{1-x^2}\,dx = 4\int_0^1\sqrt{1-x^2}\,dx$

$\begin{Bmatrix} \text{Let} & x = \cos\theta \\ & dx = -\sin\theta\, d\theta \end{Bmatrix}$

So, $\int_C \vec{A}\cdot d\vec{r} = 4\int_{\frac{\pi}{2}}^{0}\sin\theta(-\sin\theta\, d\theta) = 2\int_0^{\pi/2} 2\sin^2\theta d\theta = 2\int_0^{\pi/2}(1-\cos 2\theta)d\theta$

$= 2\cdot\frac{\pi}{2} = \pi$

$$\boxed{\int_C \vec{A}\cdot d\vec{r} = \pi}$$

21.(D) $P_{final} = P_{initial}$

$$\frac{M_0' v_f}{\sqrt{1-\frac{v_f^2}{c^2}}} = \frac{M_0 v_i}{\sqrt{1-\frac{v_i^2}{c^2}}} = \frac{M_0(0.6c)}{\sqrt{1-(0.6)^2}} = \frac{m_0(0.6c)}{(0.8)}$$

$$\frac{M_0' v_f}{\sqrt{1-\frac{v_f^2}{c^2}}} = \frac{3}{4}m_0 c \qquad ....(1)$$

$\because \quad E_{final} = E_{initial}$

$$\frac{M_0' c^2}{\sqrt{1-\frac{v_f^2}{c^2}}} = \frac{m_0 c^2}{\sqrt{1-\frac{v_i^2}{c^2}}} + 2m_0c^2 = \frac{m_0c^2}{(0.6)} + 2m_0c^2$$

$$\frac{M_0' c^2}{\sqrt{1-\frac{v_f^2}{c^2}}} = \frac{5}{3} m_0 c^2 + 2 m_0 c^2$$

$$\frac{M_0' c^2}{\sqrt{1-\frac{v_f^2}{c^2}}} = \frac{11}{3} m_0 c^2 \qquad .....(2)$$

equation (1), equation (2)

$$\frac{v_f}{c^2} = \frac{3}{4} \times \frac{3}{11c} \Rightarrow \quad v_f = \frac{9}{44} c$$

$$\boxed{V_f = 0.2045c = 0.20c}$$

so from equation (2)

$$M_0' = \frac{11}{3} m_0 \sqrt{1-(0.20)^2} = \frac{11}{3} M_0' (0.979)$$

$$M_0' = 3.59 m_0$$

$$\boxed{M_0' = 3.6 m_0}$$

22.(B) For a particle moving in one-dimensional box energy eigen value is

$$E = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \qquad ....(1)$$

and eigen function is

$$\psi = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$\because \int_{-\infty}^{+\infty} \psi_n^*(x) \psi_m(x) dx = \frac{2}{a} \int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = 0$$

so wave function are orthogonal not normalised. One-dimensional box is not degenerate. Degeneracy is appear in 3-D box.

Energy difference $\Delta E = E_2 - E_1 = \dfrac{3\pi^2\hbar^2}{2ma^2}$

$\Delta E = E_3 - E_2 = \dfrac{5\pi^2\hbar^2}{2ma^2}$

So, $\Delta E = 3 : 5 : 7......$

So, energy difference ΔE is of increasing order not constant.

$\because \quad \langle x \rangle = \int_0^a \psi_x^* \psi dx = \frac{2}{a}\int_0^a x \sin^2 \frac{n\pi x}{a} dx = \frac{2}{a} \times \frac{a^2}{4} = \frac{a}{2}$

$\langle x \rangle = \dfrac{a}{2}$

23.(B) F.CC have highest packing factor 74% So, having highest density.

24.(C) **Paramagnetic Substance.** X = +positive (less) magnetic permeability μ > 1 (positive)

$X \propto \frac{1}{T} \Rightarrow \boxed{X = \frac{C}{T}}$

Ferromagnetic substance. X → positive (large)

μ >> 1 (positive)

$\boxed{X = \frac{C}{T - T_C}}$

Diamagnetic substance. X of diamagnetic substance is negative and does not depend on temperature.

$\boxed{X \not\propto \frac{1}{T}}$

25.(C) Thermal expansion of isotropic object does not depends on shape. Size and any hole or cavity. So, Expansion in volume of both A, B bodies are same.

26.(B) Suppose the length of the course be S and let car takes t seconds. As is clear from the statement both the cars reach their destination at the same time since the race ends in a dead heat.

∴ For the first car

$$S = V_1 t + \frac{1}{2} a_1 t^2$$

For the 2nd Car

$$S = V_2 t + \frac{1}{2} a_2 t^2.$$

on subtracting, we get

$$0 = (V_1 - V_2)t + \frac{1}{2}(a_1 - a_2)t^2.$$

or $$t\left[\frac{1}{2}(a_1 - a_2)t + (V_1 - V_2)\right] = 0$$

∴ $$t = 0 \text{ and } t = -\frac{2(V_1 - V_2)}{(a_1 - a_2)} \text{ in eq. (1)}$$

Here t = 0 indicates the starting position

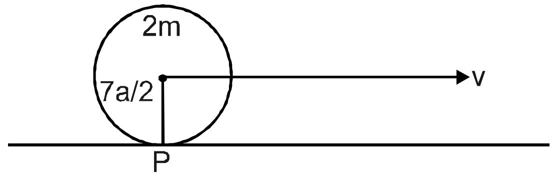
∴ Put $t = \frac{2(V_1 - RV_2)}{a_1 - a_2}$ in eq (1)

we get, $$S = \frac{-2V_1(V_1 - V_2)}{(a_1 - a_2)} + \frac{1}{2} a_i \frac{4(V_1 - V_2)^2}{(a_1 - a_2)}$$

on simplifying we get the desired expression

$$S = \frac{2(V_1 - V_2)(V_1 a_2 - V_2 a_1)}{(a_1 - a_2)^2}.$$

27.(B) Angular momentum about point P is



$L = mvr$

$m \to 2M$

$r \to \frac{7a}{2}$

So, $L = 2m \times \frac{7a}{2} \times v$

$\boxed{L = 7\,mav}$ Ans. (B)

28.(B) We know

$$\frac{1}{2}I\omega^2 = \frac{1}{2}kT \text{ or } \omega^2 = \frac{kT}{I} \text{ or } \omega = \frac{\sqrt{KT}}{I} = \sqrt{\frac{1.38\times10^{-23}\times300}{2.1\times10^{-39}\times10^{-3}\times10^{-4}}}$$

on simplifying we get = $\omega = 4.43 \times 10^{12}$ red/s

29.(B) Mass flux = $v_a A_a \rho_a = v_b A_b \rho_b$

Here $A_a = 2A_b$ and $v_a = v_b = \sqrt{2gh}$

$\therefore \frac{\rho_a}{\rho_b} = \frac{1}{2} = 0.5$

30.(B) Here $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}}$

and $k_1 = \frac{mg}{\Delta L}$

Now $k_2 = \frac{mg}{\Delta L/2} = 2k_1$

$\therefore \; f_2 = \frac{1}{2\pi}\sqrt{\frac{2k_1}{m}}$

and $\quad f_2 = \sqrt{2} f_1$

SECTION-B

MULTIPLE SELECT QUESTIONS (MSQs) (Q. 11-35)

31.(A,B) $\quad \vec{V}_1 = V_0(2\hat{\ell} + \hat{J}) \qquad m_1 = 2m$

$\vec{V}_2 = 0 \qquad m_2 = 3m$

Velocity of center of mass $\quad \vec{V}_{cm} = \frac{V_1 m_1 + V_2 m_2}{m_1 + m_2}$

$$\vec{V}_{cm} = \frac{2m(V_0(2\hat{\ell} + \hat{J}))}{2m + 3m}$$

$$\vec{V}_{cm} = \frac{2}{5} V_0 [2\hat{\ell} + \hat{J}] \qquad \text{...(1)}$$

$$|\vec{V}_{cm}| = \frac{2V_0}{\sqrt{5}} \text{ m/sec.}$$

So, velocity of particle of mass '2m' before collision

$$\vec{V}_1 = \vec{V}_1 - V_{cm} = V_0(2\hat{\ell} + \hat{J}) - \frac{2}{5} V_0 [2\hat{\ell} + \hat{J}]$$

$$\vec{V}_1 = \frac{3}{5} V_0 (2\hat{\ell} + \hat{J})$$

$$\boxed{V_1' = \frac{3V_0}{\sqrt{5}} \text{ m/sec.}}$$

Similarly velocity of particle of mass '3m' before collision $\vec{V}_2 = \vec{V}_2 - V_{cm}$

$= 0 - \frac{2}{5}V_0[2\hat{\ell} + \hat{J}] = -\frac{2}{5}V_0[2\hat{\ell} + \hat{J}]$

$$\vec{V}_2{}' = \frac{2V_0}{\sqrt{5}} m/sec$$

32.(A,B,D) Let there be a small element of length $d\ell$ at distance L from the end of rotation axis.

Mass of the element $d\ell = \frac{M}{L}d\ell$

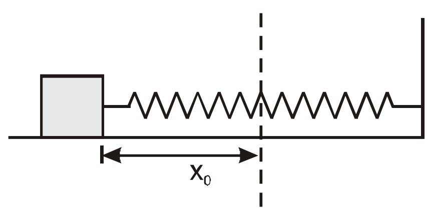
Small radial force on this element = $\left(\frac{M}{L}d\ell\right)\ell\omega^2$

$\therefore$ Total force $= \int_0^L \left(\frac{M}{L}d\ell\right)\ell\omega^2 = \frac{M}{L}\omega^2\int_0^L \ell\, d\ell = \frac{ML\omega^2}{2}$

33.(B,D) Here I = ΣMR² = $2\times m\times\left(\frac{\ell}{2}\right)^2 + 2\times M\times\left(\frac{\ell}{4}\right)^2 = \frac{m\ell^2}{2} + \frac{M\ell^2}{8} = \frac{\ell^2}{2}\left(m + \frac{M}{4}\right)$

$= 0.125\,\ell^2\,(4m + M)$

34.(A,C,D) Let x_1, x_2 represent the right extreme and left extreme position of block after completion of 1st half and 1st complete oscillations, respectively. These distances are measured from natural positions of the spring.



Similarly x_3, x_4 represent for IInd half and IInd complete oscillations and after completion of nth cycle of oscillation, the position is described as x_{2n}.

From work-energy theorem

$\frac{kx_0^2}{2} - \frac{kx_1^2}{2} = f(x_0 + x_1)$ [For left and right extreme positions of I[st] half of I[st] cycle].

$\Rightarrow \quad x_0 - x_1 = \frac{2f}{k}$

Now apply work-energy theorem for right and left extreme position of II[nd] half of I[st] cycle, we get

$$\frac{kx_1^2}{2} - \frac{kx_2^2}{2} = f(x_1 + x_2)$$

$\Rightarrow \quad x_1 - x_2 = 2f/k$

Similarly, for other subsequent cycles, we get

$x_2 - x_3 = 2f/k$

$x_3 - x_4 = 2f/k$

$\vdots$

$x_{2n-1} - x_{2n} = 2f/k$

where x_{2n} is the elongation in spring from natural length after n cycles.

Decrease in amplitude after I[st] cycle is ,$\Delta A_1 = x_0 - x_2 = 4f/k$

Decrease in amplitude after II[nd] cycle is, $\Delta A_2 = x_2 - x_4 = 4f/k$

and this continues, so decrease in amplitude is same after each cycle and is equal to 4f/k.

35.(A,B,C) Work done in stretching the wire = potential energy stored

$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2} \times \frac{F}{A} \times \frac{\ell}{L} \times AL = \frac{1}{2}F\ell$

36.(A,C) The time period of simple pendulum in air

$$T = t_0 = 2\pi\sqrt{\left(\frac{\ell}{g}\right)} \qquad ...(1)$$

ℓ, being the length of simple pendulum.

In water, effective weight of bob

w' = weight of bob in air – upthrust

$\Rightarrow \quad \rho V g_{eff} = mg - m'g = \rho V g - \rho' V g = (\rho - \rho')Vg$

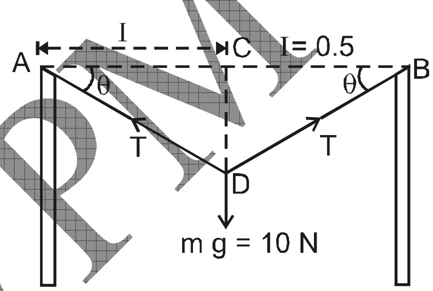
where ρ' = density of bob,

ρ = density of water

$$\therefore \quad g_{eff} = \left(\frac{\rho - \rho'}{\rho}\right)g = \left(1 - \frac{\rho'}{\rho}\right)g$$

$$\therefore \quad t = 2\pi\sqrt{\left[\frac{\ell}{(1-\rho'/\rho)g}\right]} \qquad ...(2)$$

Thus, $\frac{t}{t_0} = \sqrt{\left[\frac{1}{1-\rho'/\rho}\right]} = \sqrt{\left(\frac{1}{1-\frac{1000}{(4/3)\times 1000}}\right)} = \sqrt{\left(\frac{4}{4-3}\right)} = 2 \Rightarrow t = 2t_0 \Rightarrow t_0 = 0.5t$



37.(A,B)

Here AD = ℓ = $\delta\ell$

Stress = T/A and strain = $(\delta\ell/\ell)$

Now $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\delta\ell/\ell}$

or $T = YA\left(\frac{\delta\ell}{\ell}\right)$ ... (1)

From figure, $2\,T \sin\theta = m\,g$

or $T = \frac{mg}{2\sin\theta}$ ... (2)

From equation (1) and equation (2), we get

$$\frac{mg}{2\sin\theta} = YA\left(\frac{\delta\ell}{\ell}\right)$$

or $\sin\theta = \frac{mg}{2YA} \times \left(\frac{\ell}{\delta\ell}\right)$ ... (3)

From Δ ACD,

$$\frac{\ell}{\ell + \delta\ell} = \cos\theta \quad \text{or} \quad 1 + \frac{\delta\ell}{\ell} = \frac{1}{\cos\theta}$$

or $\frac{\delta\ell}{\ell} = \left(\frac{1}{\cos\theta} - 1\right) = \frac{1-\cos\theta}{\cos\theta}$ ... (4)

Substituting the value of ($\ell/\delta\ell$) from equation (4) in equation (3), we get

$$\sin\theta = \frac{mg}{2YA}\left(\frac{\cos\theta}{1-\cos\theta}\right)$$

$$\tan\theta\,(1-\cos\theta) = \frac{mg}{2YA}$$

or $\tan\theta\left(2\sin^2\frac{\theta}{2}\right) = \left(\frac{mg}{2YA}\right)$

when θ is small, then

$$\theta \times 2\left(\frac{\theta^2}{4}\right) = \left(\frac{mg}{2YA}\right)$$

$$\theta = \left(\frac{mg}{YA}\right)^{1/3} = \left(\frac{1\times 9.8}{2\times 10^{11}\times 4\times 10^{-4}}\right)^{1/3} = \frac{1}{2}\times 10^{-2}\text{rad.} = 5\times 10^{-3}\text{rad.}$$

38.(A,C,D) In amplitude resonance at resonance condition power transferred from driving force to oscillator is maximum.

39.(A,D) (a) If the detector is at x = 0, the two radiowaves can be represented as

$y_1 = A \sin \omega_1 t$ and $y_2 = A \sin \omega_2 t$

(Given : $A_1 = A_2 = A$)

By the principle of superposition

$$y = y_1 + y_2 = A \sin \omega_1 t + A \sin \omega_2 t$$

$$y = 2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)\sin\left(\frac{\omega_1+\omega_2}{2}t\right) = A_0 \sin\left(\frac{\omega_1+\omega_2}{2}t\right)$$

Here, $A_0 = 2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$

Since, $I \propto (A_0)^2 \propto 4A^2\cos^2\left(\frac{\omega_1-\omega_2}{2}t\right)$

So, intensity will be maximum when

$$\cos^2\left(\frac{\omega_1-\omega_2}{2}t\right) = \text{maximum} = 1$$

or $\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) = \pm 1$

or $\frac{\omega_1 - \omega_2}{2}$ t = 0, π, 2π...

ie, $t = 0, \frac{2\pi}{\omega_1-\omega_2}, \frac{4\pi}{\omega_1-\omega_2}, \ldots \frac{2n\pi}{\omega_1-\omega_2}$ n = 0,1,2 ..

Therefore, time interval between any two successive maxima is $\frac{2\pi}{\omega_1-\omega_2} = \frac{2\pi}{10^3}$ s or 6.28×10^{-3} s.

$= 628 \times 10^{-5}$ s

(b) The detector can detect if resultant intensity $\geq 2A^2$, or the resultant amplitude $\geq \sqrt{2}A$.

Hence, $2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) \geq \sqrt{2}A$

$\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) \geq \frac{1}{\sqrt{2}}$

Therefore, the detector lies idle. When value of $\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)$ is between 0 and $\frac{1}{\sqrt{2}}$

or when $\frac{\omega_1 - \omega_2}{2}$ t is between $\frac{\pi}{2}$ and $\frac{\pi}{4}$

or t lies between

$\frac{\pi}{\omega_1-\omega_2}$ and $\frac{\pi}{2(\omega_1-\omega_2)}$

$\therefore \quad t = \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2\times10^3}$

$t = 1.57 \times 10^{-3}$ s $= 15.7 \times 10^{-4}$ s

Hence, the detector lies idle for a time of 1.57×10^{-3} s in each cycle.

40.(A,C) Velocity of sound in water is

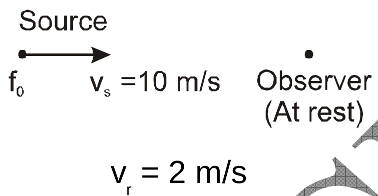
$$\nu_w = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445 \text{ m/s}$$

Frequency of sound in water will be

$$f_0 = \frac{\nu_w}{\lambda_w} = \frac{1445}{14.45 \times 10^{-3}} \text{Hz}$$

(a) Frequency of sound detected by receiver (observer) at rest would be

source



$$f_1 = f_0\left(\frac{\nu_w + \nu_r}{\nu_w + \nu_r - \nu_s}\right) = (10^5)\left(\frac{1445+2}{1445+2-10}\right)\text{Hz}$$

$= 1.007 \times 10^5$ Hz $= 10.07 \times 10^4$ Hz

(b) Velocity of sound in air is

$$\nu_a = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.31)(20+273)}{288 \times 10^{-3}}} = 344.\text{ m/s}$$

∴ Frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz

∴ Frequency of sound detected by receiver (observer) in air would be

$$f_2 = f_0\left(\frac{\nu_a - w}{\nu_a - w - \nu_s}\right) = 10^5 \left[\frac{344-5}{344-5-10}\right]\text{Hz.}$$

$f_2 = 1.0304 \times 10^5$ Hz = 103.04×10^3 Hz

SECTION-C

NUMERICAL ANSWER TYPE (NAT) (Q. 41-60)

41. **11**

Hall potential is given as

$$V_H = \frac{1}{ne}\left(\frac{IB}{t}\right)$$

Here, $n = 8.4 \times 10^{28}/m^3$

$I = 200$ A

$t = 1 \times 10^{-3}$m

$B = 1.5$ Wb/m^2

$$V_H = \frac{200 \times 1.5}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-3}}$$

So, $= 11\mu V$

42. **20**

$\vec{E} = -\vec{\Delta}V$

$\Rightarrow \vec{E} = (-10x - 10)\hat{i}$

At x = 1 m

$\vec{E} = -20\hat{i}$

$\Rightarrow |\vec{E}| = 20\,Vm^{-1}$

43. **4800**

If D is separation between source and screen, the fringe width is given as $\beta_1 = \frac{d\lambda}{D}$

where d = separation between virtual sources. If d and D are fixed, then $\frac{\beta_1}{\lambda_1} = \frac{\beta_2}{\lambda_2}$

In first case 20 fringes occupy 2 cm

$\Rightarrow \quad \beta_1 = \frac{2.0}{20}\text{cm} \qquad \beta_1 = 0.1\text{cm}$

In second case 30 fringes occupy 2.4 cm

$\Rightarrow \quad \beta_2 = \frac{2.4}{30}\text{cm} = 0.08\text{ cm}$

So, by Eq.(i) we get

$$\lambda_2 = \lambda_1 \frac{\beta_2}{\beta_1} = 6000 \times \frac{0.08}{0.10} = 4800\,\text{Å}$$

44. 492

In case of a grating ,

RP = $\frac{\lambda}{d\lambda} = nN$

So, As $d\lambda$ = 5896 – 5890 = 6 Å

and $\lambda = \frac{1}{2}$ [5896 + 5890] = 5893 Å

So, RP = $\frac{\lambda}{d\lambda} = \frac{5893}{6}$ = 982.17

(b) As RP = nN, i.e., N = (RP /n)

So, N = (982.17/2) ≃ 492

45. 30.4

Let m and P be the initial mass and pressure of the gas inside the vessel. Therefore ,

$$PV = (m/M)\, RT \qquad ...(1)$$

where M is the molecular weight of the gas in the vessel.

After a part of the gas is released , we have

$$(P - \Delta P)\, V = \frac{m'}{M}.RT. \qquad ...(2)$$

where , m´ is the mass of the remaining gas in the vessel.

Hence, mass of the gas released is equal to (subtracting (2) from (1)

$$\Delta m = m - m' = \frac{\Delta PVM}{RT}$$

Now, under normal conditions (P_0 = 1atm, T = 273K), density of the gas is given to be ρ . Therefore, we find,

$$P_0 (m/\rho) = (m/M).RT$$

or, $M / RT = \rho / P0$

Thus, $\Delta m = \frac{(\Delta P)V}{P_0}$

Now here $V = 30 \times 10^{-3}\ m^3$

$\rho = 1.3\ kg/m^3$

$\Delta P = 0.78$ atm

and , $P_0 = 1$ atm.

Therefore, $\Delta m = 1.3 \times 30 \times 10^{-3} \times 0.78$

$\cong$ 30.4 g.

46. 2.5

$$\vec{V}cart = 4\hat{i} \qquad ... (i)$$

$$\vec{V}_{stone+cart} = (6\sin 30)\hat{j} + (6\cos 30)\hat{k} \qquad ... (ii)$$

$= (3\hat{j} + 3\sqrt{3}\hat{k})$

V_{stone} = (ii) + (i) = $4\hat{i} + 3\hat{j} + 3\sqrt{3}\,\hat{k}$

Velocity of stone at higher point

$\vec{V}_{stone+height} = 4\hat{i} + 3\hat{j}$

[At highest point vertical component (i.e. z component) is zero]

or speed of stone at highest point

$V = \sqrt{4^2 + 3^2} = \sqrt{16+9}$ = 5 ms^{-1}

By the principle of law of conservation of momentum

$mV = 2mV_{combined}$

or $V_{combined} = \frac{V}{2} = \frac{5}{2} = 2.5 ms^{-1}$

47. **0**

We know in a satellite there is no derivational acceleration so, weight mg read by spring balance is zero

48. **16.9**

When the particle is at the greatest distance from the inclined plane, its velocity is parallel to the inclined plane.

i.e. V_y' (t) = 0,

V_y^2 (t) = $V_y^2(0) + 2a_y y'$

or 0 = [V sin (α – β)]2 – 2g cosβy

y' = $\frac{v^2.\sin^2(\alpha - \beta)}{2g\cos\beta}$

Here V = 19.6 mc^{-1}, α = 60°, β = 30°, g = 9.8 ms^{-2}

$$y' = \frac{(19.6)(19.6)}{2(9.8)\frac{\sqrt{3}}{2}}\frac{1}{4}m = 16.9m$$

49. 4.33

Given λ = 600 nm = 6×10^{-7} m,

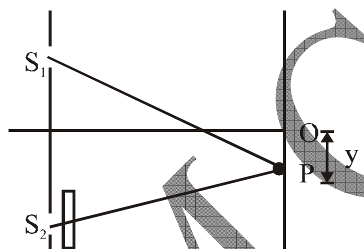
d = 0.45 mm = 0.45×10^{-3} m,

D = 1.5 m.

Thickness of glass sheet, t = 10.4 μm = 10.4×10^{-6} m Refractive index of medium, μ_m = 4/3

and refractive index of glass sheet μ_g = 1.5

Let central maximum is obtained at a distance y below point O. Then Δx_1 $= S_1P - S_2P = \frac{yd}{D}$



Path difference due to glass sheet

$$\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

Net path difference will be zero when

$$\Delta x_1 = \Delta x_2$$

or $$\frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t$$

$\therefore \quad y = \left(\frac{\mu_g}{\mu_m} - 1\right) t \frac{D}{d}$

Substituting the values, we have

$$y = \left(\frac{1.5}{4/3} - 1\right) \frac{10.4 \times 10^{-6}(1.5)}{0.45 \times 10^{-3}}$$

$y = 4.33 \times 10^{-3}$ m

or we can say y = 4.33 mm.

50. 150

$\because$ 1 rydberg = 2.2×10^{-18} J = Rhc

Ionisation energy is give as a 8 rydberg = $2.2 \times 10^{-18} \times 8 = \frac{17.6 \times 10^{-18}}{1.6 \times 10^{-19}}$ eV = 110 eV

So, energy in first orbit E_1 = –110 eV energy of radiation emitted when electron jumps from first excited state (n = 2) to ground state (n = 1)

$$E_{21} = \frac{E_1}{(2)^2} \frac{-E_1}{(1)^2} = \frac{-3E_1}{4} = 82.5 \text{ eV.}$$

so wavelength of photon emitted in this transition would be

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{12.37 \times 10^{-34} \times 10^{8} \times 10^{-19}}{82.5} = \frac{12.375 \times 10^{-7}}{82.5} = 0.14993 \times 10^{-7}$$

$\lambda = 149.93 \times 10^{-10}$m

$\boxed{\lambda \simeq 150 \text{ Å}}$

51. 0.176

$\because$ 1 rydberg = 2.2×10^{-18} J = Rhc

Ionisation energy is give as a 8 rydberg = $2.2 \times 10^{-18} \times 8$

$$= \frac{17.6 \times 10^{-18}}{1.6 \times 10^{-19}} \text{eV} = 110 \text{ eV}$$

Let Z be atomic number of given element

So, $E_1 = (-13.6)Z^2$

$-110 = -13.6\ Z^2$

$Z^2 = 8.08$

$\boxed{Z \sim 3}$

$\because \quad \text{radius } r \propto \frac{1}{Z}$

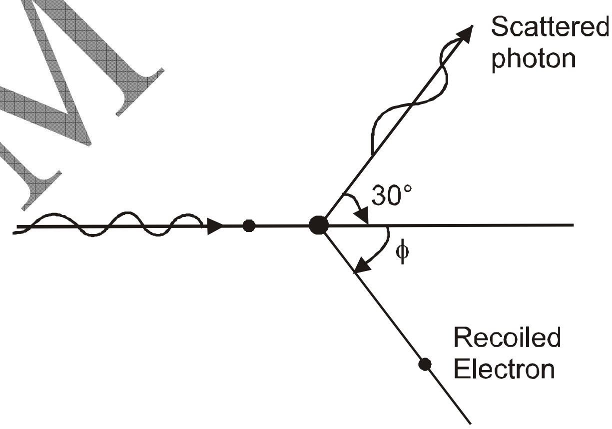
Radius of first orbit

$$r_1 = \frac{r_{H1}}{Z} = \frac{0.529}{3} \text{Å}$$

$\boxed{r_1 = 0.176\text{Å}}$

52. 75

The wavelength shift is given by



$\Delta\lambda = 0.0242\,(1 - \cos\theta) = 0.0242\,(1 - \cos 30°) = 0.0242\,(1 - 0.866) = 0.00324$ Å

The kinetic energy of ejected electrons.

$$\frac{1}{2}mv^2 = h(v - v') = \frac{hc}{\lambda^2}(\Delta\lambda)$$

or $$v = \left(\frac{2hc}{m\lambda^2}\Delta\lambda\right)^{1/2} = 2.1 \times 10^8 \text{ cm/ sec}$$

[by substituting the values given above]

Now from conservation of momentum

$$mv\cos\phi = \frac{hv}{c} - \frac{hv'}{c}\times 0.866 \simeq \frac{hv'}{c}\times 0.134$$

$$mv\sin\phi = \frac{hv'}{c}\times 0.5$$

or $$\tan\phi = \frac{0.5}{0.134}$$

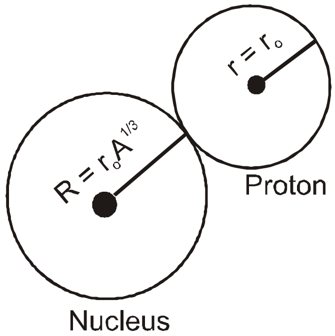
or $\phi = \tan^{-1}(3.62)$

$\therefore \quad \phi \cong 75°$

53. 12.33

The Coulomb barrier is the energy needed to bring the proton to the edge of the nucleus (see figure). If we define

$D = R + r = r_0\,(A^{1/3} + 1)$, then



$$E_r = k\frac{(Ze)e}{\Delta}$$

$$E_r = k\frac{Ze^2}{r_0\left(A^{1/3}+1\right)} = \left(\frac{1.44\text{ MeV .fm}}{1.4\text{ fm}}\right)\left(\frac{Z}{A^{1/3}+1}\right) = (1.03\text{ MeV})\left(\frac{Z}{A^{1/3}+1}\right)$$

For $_8O^{16}E_c = 1.03\text{ meV}\left(\frac{8}{(16)^{1/3}+1}\right)$ = 2.34 MeV

For $_{41}N^{93}E_c = 1.03\text{ meV}\left(\frac{41}{93^{1/3}+1}\right)$ = 7.64 MeV

and for $_{83}Bi^{209}E_c = 1.03\text{ meV}\left(\frac{80}{(209)^{1/3}+1}\right)$ = 12.33 MeV

54. 2.82

Mass of the NaCl molecule = $\frac{58.46}{6.02\times10^{23}} = 9.7\times10^{-23}\text{gm}$

Number of NaCl molecules per unit volume = $\frac{2.167}{9.7\times10^{-27}} = 2.23\times10^{22}\text{molecules/cm}^3$

Since NaCl is diatomic number of atoms per unit volume = $2 \times 2.23 \times 10^{22}$

= 4.46×10^{22} atoms/ cm^3.

$\simeq 4.5\times10^{22}$ atoms/ cm^3.

Let a be the lattice constant, i.e. the distance measured along the edge of the cube between adjacent atoms in the crystal and let n be the number of atoms along the edge of a 1-cm cube. . Then the length of an edge is na and the volume of the unit cube is n^3a^3. Since n^3 is the number of atoms in $1cm^2$, therefore

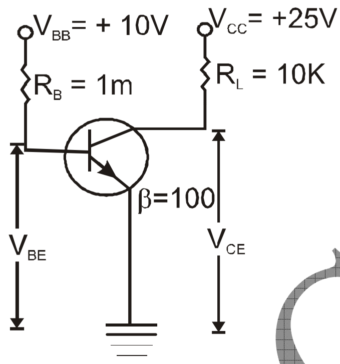
$$n^3a^3 = 4.5 \times 10^{22}\ a^3 = 1$$

or $$a^3 = 2.24 \times 10^{-23}\ cm^3$$

or $$a = 2.82 \times 10–8\ cm = 2.82\ Å$$

55. 1.01

The above figure can be simplified



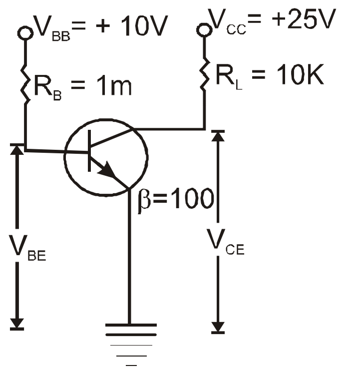
Base current = $I_B = \frac{V_{BB}}{R_B} = \frac{10V}{1M} = 10\mu A$

Collector current = βI_B = 100 × 10 μA = 1 mA

Emitter current = $I_E = I_c + I_n$ = 10 mA + 10μA = 1.01 mA

56. 15

The above figure can be simplified



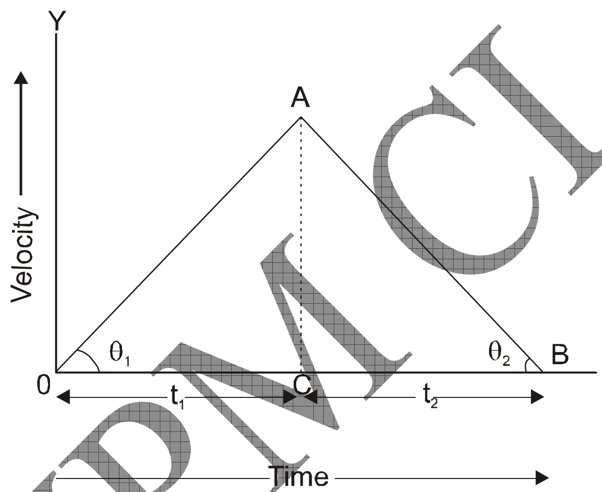
$V_{CE} = V_{CC} - I_C R_L = 25 - (1 \times 10)V = 15$ Volt

57. 2.8

Suppose

V = max velocity of the lift and

f = the acceleration as shown.



When the current is switched off retardation (negative acceleration) of the lift = 9.8 m/s^2.

From the figure we find

t_1 = time period for the acceleration

t_2 = time period for the retardation

Then from the figure we find area CAB show the distance travelled by the lift.

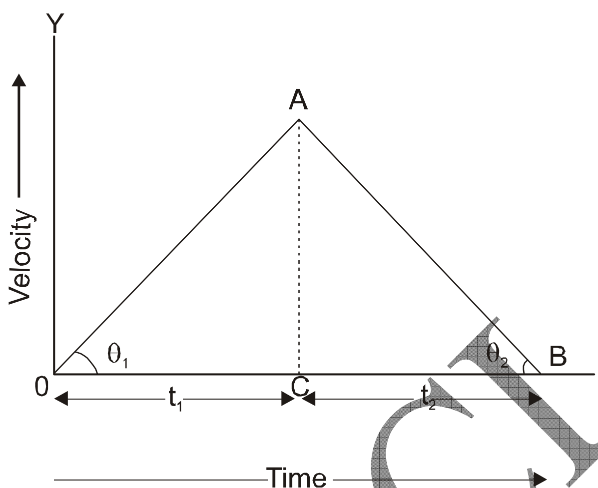
Area OAB = $\frac{1}{2}(t_1+t_2)V = \frac{1}{2}(5)V = 2.5V = 7$ or $V = \frac{7}{2.5} = 2.8 m/s.$

58. 0.6

Suppose

V = max velocity of the lift and

f = the acceleration as shown.



When the current is switched off retardation (negative acceleration) of the lift = 9.8 m/s^2.

From the figure we find

t_1 = time period for the acceleration

t_2 = time period for the retardation

Then from the figure we find area CAB show the distance travelled by the lift.

Area OAB = $\frac{1}{2}(t_1+t_2)V = \frac{1}{2}(5)V = 2.5V = 7$

or $V = \frac{7}{2.5} = 2.8 m/s.$

Retardation = $\tan\theta_2 = \frac{V}{t_2} = \frac{2.8}{t_2} = 9.8$

or $t_2 = \frac{2.8}{9.8} = 0.28$ seconds.

$\therefore$ $t_1 = 5 - 0.28 = 4.47$ seconds.

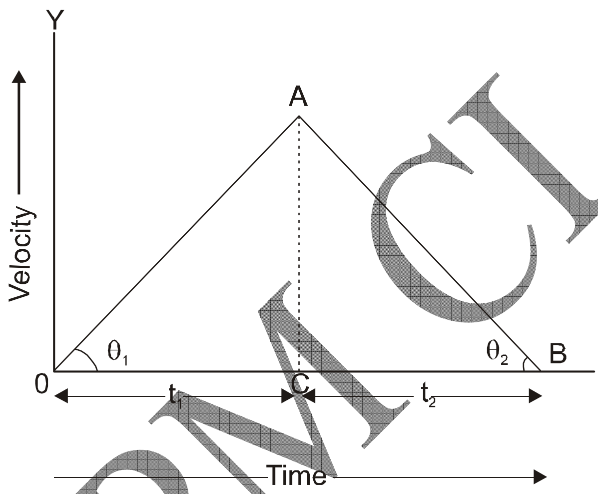
Next acceleration = $\tan\theta_1 = \frac{V}{t_1} = \frac{2.8}{4.72} = 0.6$ m/s^2.

59. 6.6

Suppose

V = max velocity of the lift and

f = the acceleration as shown.



When the current is switched off retardation (negative acceleration) of the lift = 9.8 m/s^2.

From the figure we find

t_1 = time period for the acceleration

t_2 = time period for the retardation

Then from the figure we find area CAB show the distance travelled by the lift.

Area OAB = $\frac{1}{2}(t_1 + t_2)V = \frac{1}{2}(5)V = 2.5V = 7$ or $V = \frac{7}{2.5} = 2.8 m/s.$

During acceleration, the lift rises to a height numerical equal to the area OCA.

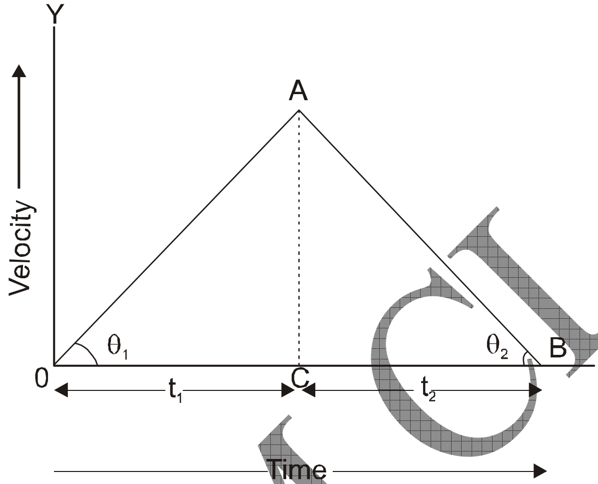
Area OCA = $\frac{1}{2}t_1 \times V = \frac{1}{2} \times 4.72 \times 2.8 = 6.6m$

60. 3.18

Suppose

V = max velocity of the lift and

f = the acceleration as shown.



When the current is switched off retardation (negative acceleration) of the lift = 9.8 m/s^2.

From the figure we find

t_1 = time period for the acceleration

t_2 = time period for the retardation

Then from the figure we find area CAB show the distance travelled by the lift.

Area OAB = $\frac{1}{2}(t_1 + t_2)V = \frac{1}{2}(5)V = 2.5V = 7$ or $V = \frac{7}{2.5} = 2.8 m/s.$

Retardation = $\tan\theta_2 = \frac{V}{t_2} = \frac{2.8}{t_2} = 9.8$ or $t_2 = \frac{2.8}{9.8} = 0.28$ seconds.

$\therefore$ $t_1 = 5 - 0.28 = 4.47$ seconds.

Next acceleration = $\tan\theta_1 = \frac{V}{t_1} = \frac{2.8}{4.72} = 0.6$ m/s^2.

Accelerating force = force to produce an acceleration of 0.6 m/s^2 + force to over come the weight of the lift. $= \frac{3\times 0.6}{9.81} + 3 = 3.18$ tonnes weight.