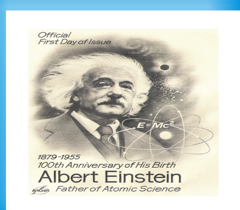

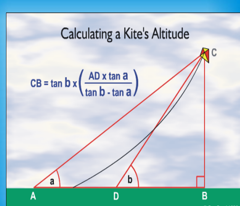

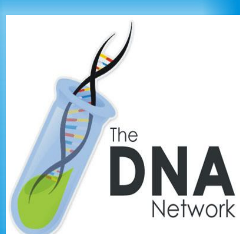

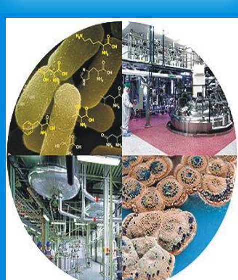

GATE - MECHANICAL ENGINEERING

SAMPLE THEORY

- **BOUNDARY LAYER THEORY**
- **RADIATIVE HEAT TRANSFER**

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BOUNDARY LAYER THEORY

- **The Boundary Layer Equations**

Newton's law applied to a fluid element in 2-D is:

$$\rho\, \Delta x\, \Delta y \frac{DV}{Dt} = F$$

and, in the x direction:

$$F = \Delta x\, \Delta y \left(-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y}\right)$$

This leads, directly to:

$$\rho\,(u_t + u\,u_x + v\,u_y) = -p_x + (\mu\, u_y)_y$$

This boundary layer equation is combined with the continuity equation and dp/dy = 0 to obtain the 3 equations in the unknowns: p, u, and v.

One of the basic results in this development of boundary layer theory is that the static pressure is constant through the boundary layer (dp/dy = 0). This is a rather important result, but it is not exactly true.

We start with the 2-D boundary layer equation shown above. For steady flow, this reduces to:

$$u\,u_x + v\,u_y = -\frac{p_x}{\rho} + \nu\, u_{yy}$$

The approach to the solution of this equation is to assume that the pressure does not vary with y, so it is specified by the external velocity distribution. v is computed from the continuity equation, leaving a PDE in u to be integrated. However, this holds only for laminar flow since turbulent boundary layers are inherently unsteady.

The balance of normal forces on a fluid element we can see that $dp/dn = \rho V^2/R$

where R is the radius of curvature. This holds even for viscous fluids since we expect viscosity to produce shear stresses and not normal stresses.

Now, if $C_p = (p-p_0)/(.5\,\rho\, U_0^2)$

then, $dCp/dn = 2\,(V/U_0)^2 / R$

Thus, the change in Cp through the boundary layer is: Cp ~ $\delta/R * (V/U_0)^2$

So, as long as the radius of curvature is much larger than the boundary layer thickness, and the local velocities are not too large, this is true.

Laminar Boundary Layer Theory

Flat Plate Flow

The boundary layer equations reduce to the following when the boundary layer is assumed to be steady:

$$u u_x + v u_y = -\frac{p_x}{\rho} + \nu u_{yy}$$

This is especially simple in the case of incompressible flow with no pressure gradient. This laminar, flat plate, boundary layer flow satisfies:

$$u u_x + v u_y = \nu u_{yy} \qquad u_x + v_y = 0$$

These equations are easily solved by introducing the variable η: $\eta \equiv \frac{y}{2}\left(\frac{u_e}{\nu x}\right)^{1/2}$

also introducing the function: $f(\eta)$ so that $u = \frac{1}{2} u_e f'$ where $f' = df / d\eta$

From continuity, then:

$$v = \frac{1}{2}\left(\frac{u_e \nu}{x}\right)^{1/2} (\eta f' - f)$$

and the boundary layer equation becomes:

f''' + ff'' = 0

This ordinary differential equation is accompanied by the boundary conditions which state that the velocity right at the surface is 0 and that far away the velocity approaches the specified U_e. These B.C.'s are written:

f = f' = 0 at h = 0

f' = 2 at h = infinity

The equation seems simple, but it is nonlinear and no closed form solution is known. The problem can be solved by assuming a series approximation for f.

When all is said and done, the following relations are found:

$$\delta = \frac{5.2\,x}{\sqrt{Re_x}} \qquad C_{f_1} = \frac{0.664}{\sqrt{Re_x}} \qquad C_f = \frac{1.328}{\sqrt{Re}} \qquad \theta = \frac{0.671\,x}{\sqrt{Re_x}}$$

$$\delta^* = \frac{1.721\,x}{\sqrt{Re_x}}$$

C_f is about twice the momentum thickness for a flat plate.

Thwaites Method

Thwaites method is used for computing the boundary layer characteristics in laminar flow with pressure gradients. It is based on the steady boundary layer equations with a specified external pressure gradient, and gives an approximate solution.

Starting with the Navier-Stokes Equations we make the usual boundary layer assumptions, that the flow is steady, incompressible, and we ignore higher order terms

In the x-direction:

$$\rho\,(u\,u_x + v\,u_y) = -p_x + (\mu\,u_y)_y$$

This may be rewritten in terms of the boundary layer variables, θ, H, U_e, and C_{fl}:

$$\theta_x + \frac{\theta}{u_e}(2 + H)\,u_{e_x} = \frac{C_{f_l}}{2}$$

where θ is the momentum thickness:

$$\theta = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy$$

H is the shape factor:

$$H = \delta^* / \theta$$

and C_{fl} is the local skin friction coefficient:

$$C_{f_l} = \frac{\tau}{q_\infty} = \mu \frac{du}{dy}$$

The basic idea behind many laminar boundary layer methods is to assume a particular form for the variation of u with y: i.e. $u/u_e = f(y)$. Pohlhausen's method is based on a quadratic polynomial for u(y). Thwaites method uses results from exact solutions to certain laminar boundary layers to obtain an approximate relationship between H, C_f, R_e, and θ.

$$\frac{\rho}{\mu}\frac{d}{dx}\left(\theta^2 U_e^6\right) = 0.45\, U_e^5$$

Substituting these results for θ and H into the expression above Thwaites obtained:

$$\theta^2 = \frac{0.45\, \nu}{U_e^6}\int_{x_0}^{x} U_e^5\, dx \;+\; \theta_0^2\left(\frac{U_{e_0}}{U_e}\right)^6$$

This is easily integrated for θ (x).

Turbulent Boundary Layer Theory

The equations for turbulent flow are the full Navier-Stokes equations. We can simplify the equations by using empirical models for the more complex aspects of the turbulence and making the assumption of a thin layer. The method described here is used to compute the characteristics of a "steady" turbulent boundary layer with a pressure gradient.

In this case two coupled first order ODE's are solved. The first is the von Karman integral equation:

$$\frac{d\theta}{dx} = C_f/2 - (H+2)\,\frac{\theta}{U_e}\frac{dU_e}{dx}$$

where θ and U_e are made dimensionless with the chord length and freestream velocity and H is the shape factor, δ^*/θ.

The second is an expression describing the entrainment of flow into the boundary layer:

$$\frac{dH}{dx} = \frac{1}{\theta}\,\frac{1}{dH_1/dH}\left(F - H_1\left(\frac{\theta}{U_e}\frac{dU_e}{dx} + \frac{d\theta}{dx}\right)\right)$$

Here, H_1 is the mass flow shape factor:

$$H_1 = \frac{\delta - \delta^*}{\theta}$$

and F is an entrainment parameter.

These variables are related to each other by the following expressions.

The skin friction coefficient is related to H and θ by the Ludwieg-Tillman skin friction law:

$$C_f = \frac{0.246 * 10^{-0.678\,H}}{Re_\theta^{0.268}}$$

The mass flow shape factor, H_1 can be related to H by the following fits:

$$H_1 = 0.8234\,(H-1.1)^{-1.287} + 3.3 \quad \text{for } H<=1.6$$

$$H_1 = 1.5501\,(H-0.6778)^{-3.064} + 3.3 \quad \text{for } H > 1.6$$

Finally, the entrainment parameter, F, is approximated by:

$$F = 0.0306\,(H_1 - 3.0)^{-0.6169}$$

Integrate this system of equation numerically to obtain the variation of θ and H with position along the airfoil.

These expressions must be evaluated numerically, but some useful results for flat plates are given below:

$$\delta = \frac{0.37x}{Re_x^{0.2}} \qquad C_f = \frac{0.455}{(\log Re)^{2.58}} \quad \text{or} \quad C_f = 0.074\,Re^{-0.2} \qquad \theta = \frac{0.036\,x}{Re_x^{0.2}}$$

$$\delta^* = \frac{.046\,x}{Re_x^{0.2}} \qquad C_{f_1} = \frac{.0592}{Re_x^{0.2}}$$

The formulas involving Re.2 are based on the assumption of a 1/7th power law shape for the boundary layer profile $u/U_e = (y/\delta)1/7$. Results agree with experiments up to Re = 20 million. Above this, the skin friction is underestimated. The results based on the logarithmic distribution agree well with limited test data out to Re = 500 million.

In many cases, the flow starts laminar, undergoes transition, and continues as a turbulent boundary layer.

RADIATIVE HEAT TRANSFER

Radiation Heat Transfer

Radiation differs from Conduction and Convection heat transfer mechanisms, in the sense that it does not require the presence of a material medium to occur. Energy transfer by radiation occurs at the speed of light and suffers no attenuation in vacuum.

Radiation can occur between two bodies separated by a medium colder than both bodies.

According to Maxwell theory, energy transfer takes place via electromagnetic waves in radiation. Electromagnetic waves transport energy like other waves and travel at the speed of light.

Electromagnetic waves are characterized by their frequency v (Hz) and wavelength λ (μm), where:

$$\lambda = c / í$$

where c is the speed of light in that medium; in a vacuum $c_0 = 2.99 \times 10^8$ m / s. Note that the frequency and wavelength are inversely proportional.

The speed of light in a medium is related to the speed of light in a vacuum,

$c = c_0 / n$

where n is the index of refraction of the medium, n = 1 for air and n = 1.5 for water.

Note that the frequency of an electromagnetic wave depends only on the source and is independent of the medium.

The frequency of an electromagnetic wave can range from a few cycles to millions of cycles and higher per second.

Einstein postulated another theory for electromagnetic radiation. Based on this theory, electromagnetic radiation is the propagation of a collection of discrete packets of energy called photons. In this view, each photon of frequency í is considered to have energy of

$$e = hí = hc / \lambda$$

where $h = 6.625 \times 10^{-34}$ J.s is the Planck's constant.

Note that in Einstein's theory h and c are constants, thus the energy of a photon is inversely proportional to its wavelength. Therefore, shorter wavelength radiation possesses more powerful photon energies (X-ray and gamma rays are highly destructive).

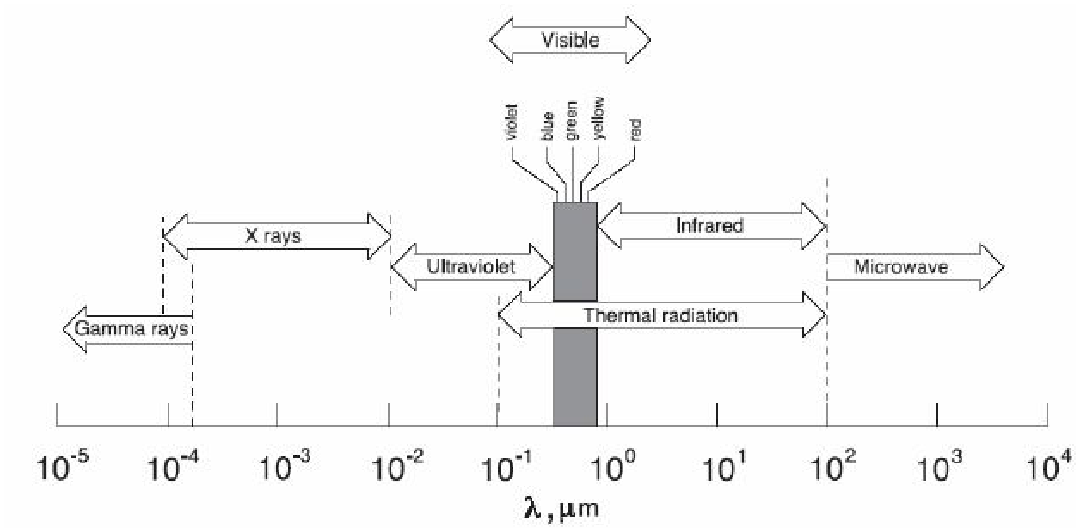

Fig.: Electromagnetic spectrum.

Electromagnetic radiation covers a wide range of wavelength, from 10^{-10} ìm for cosmic rays to 1010 ìm for electrical power waves.

As shown in Fig., thermal radiation wave is a narrow band on the electromagnetic wave spectrum.

Thermal radiation emission is a direct result of vibrational and rotational motions of molecules, atoms, and electrons of a substance. Temperature is a measure of these activities. Thus, the rate of thermal radiation emission increases with increasing temperature.

What we call light is the visible portion of the electromagnetic spectrum which lies within the thermal radiation band.

Thermal radiation is a volumetric phenomenon. However, for opaque solids such as metals, radiation is considered to be a surface phenomenon, since the radiation emitted by the interior region never reach the surface.

Note that the radiation characteristics of surfaces can be changed completely by applying thin layers of coatings on them.