

GATE - ENGINEERING SCIENCE

SAMPLE THEORY

VPM CLASSES

For IIT-JAM, JNU, GATE, NET, NIMCET and Other Entrance Exams

1-C-8, Sheela Chowdhary Road, Talwandi, Kota (Raj.) Tel No. 0744-2429714

Web Site www.vpmclasses.com E-mail-vpmdclasses@yahoo.com

Toll Free: 1800-2000-092

Mobile: 9001297111, 9829619614, 9001894073, 9829567114

Website: www.vpmdasses.com

FREE Online Student Portal: examprep.vpmdasses.com

E-Mail: vpmdclasses@yahoo.com / info@vpmdclasses.com

SECTION A: ENGINEERING MATHEMATICS

CAUCHY'S THEOREM

If $f(z)$ is an analytic function of z and if $f'(z)$ is continuous at each point within and on a closed contour C , then $\int_C f(z)dz = 0$.

Proof. Let D be the region which consists of all points within and on the contour C . If $P(x, y)$, $Q(x, y)$, $\frac{\partial Q}{\partial x}$, $\frac{\partial P}{\partial y}$ are all continuous functions of x and y in the region D , then Green's

theorem states that
$$\int_C (Pdx + Qdy) = \int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy.$$

Since $f(z) = u + iv$ is continuous on the simple curve C and $f'(z)$ exists and is continuous in D , therefore u, v, u_x, u_y, v_x, v_y are all continuous in D . The conditions of Green's theorem are thus satisfied. Hence

$$\begin{aligned} \int_C f(z)dz &= \int_C (u + iv)(dx + idy) \\ &= \int_C (udx - vdy) + i \int_C (vdx + udy) \\ &= - \iint_D \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx dy + i \iint_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) dx dy = 0 \quad [\text{By Green's theorem}] \\ &= - \iint_D \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx dy + i \iint_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) dx dy \quad [\text{By Cauchy-Riemann equations}] \end{aligned}$$

Hence $\int_C f(z)dz = 0$.

SECTION B: FLUID MECHANICS

The Navier-Stokes equations

The flow of an incompressible homogeneous fluid with density ρ_0 and viscosity μ is described by the incompressible Navier-Stokes equations,

$$\rho_0 (\vec{u}_t + \vec{u} \cdot \nabla \vec{u}) + \nabla p = \mu \Delta \vec{u},$$

$$\nabla \cdot \vec{u} = 0.$$

Here, $\vec{u}(\vec{x}, t)$ is the velocity of the fluid, and $p(\vec{x}, t)$ is the pressure. The first equation is conservation of momentum, and the second equation is conservation of volume.

Remark. It remains an open question whether or not the three-dimensional Navier-Stokes equations, with arbitrary smooth initial data and appropriate boundary conditions, have a unique, smooth solution that is defined for all positive times.

This is one of the Clay Institute Millennium Prize Problems.

Let U, L be a typical velocity scale and length scale of a fluid flow, and define dimensionless variables by

$$\vec{u}^* = \frac{\vec{u}}{U}, \quad p^* = \frac{p}{\rho U^2}, \quad \vec{x}^* = \frac{\vec{x}}{L}, \quad t^* = \frac{Ut}{L}.$$

Using these expressions in, and dropping the stars on the dimensionless variables, we get

$$\vec{u}_t + \vec{u} \cdot \nabla \vec{u} + \nabla p = \frac{1}{R} \Delta \vec{u},$$

$$\nabla \cdot \vec{u} = 0,$$

where R is the Reynolds number defined in Euler equations.

The non-dimensionalized equation suggests that for flows with high Reynolds number, we may neglect the viscous term on the right hand side of the momentum equation, and approximate the Navier-Stokes equation by the incompressible Euler equations

$$\vec{u}_t + \vec{u} \cdot \nabla \vec{u} + \nabla p = 0,$$

$$\nabla \cdot \vec{u} = 0.$$

The Euler equations are difficult to analyze because, like the Navier-Stokes equations, they are nonlinear. Moreover, the approximation of the Navier-Stokes equation by the Euler equations is problematic. High-Reynolds number flows develop complicated small-scale structures (for instance, boundary layers and turbulence) and, as a result, it is not always possible to neglect the second-order spatial derivatives $\Delta \vec{u}$ in the viscous term in

comparison with the first-order spatial derivatives $\vec{u} \cdot \nabla \vec{u}$ in the inertial term, even though the viscous term is multiplied by a small coefficient.

SECTION C: MATERIAL SCIENCE

Corrosion

Corrosion, oxidation and degradation commonly spoil the aesthetics, leads to component weakening, the requirement for remedial actions and, in the worse case, failure of materials in products. Often this is directly due to the aggressive environment that a product is exposed to over its operational life. In the case of plastics or paint coatings it can be embrittlement or discolouration, paint delamination or solvent swelling. Metals can be attacked by aqueous based corrosion or oxidised at elevated temperature.

Aqueous Corrosion

In order for aqueous corrosion to occur the following have to be present:-

- Anodic area where components degradation occurs
- Cathodic area
- Electrolyte to transport corrosive species and various ions
- Electron conduction path connecting anode and cathode

The corrosion rate is dependent on many factors, some of which are listed below:-

- Relative areas of the anode and cathode
- Electrolyte conductivity and whether the metal is immersed in water, underground or just corroding as a result of surface moisture related to the humidity
- Availability of fresh reactants; corrosive species such as oxygen and chloride ions

Mechanisms to minimise the impact of the environmental attack are often in place, but have failed to be effective for a range of possible reasons:

- Unusual or unexpected exposure conditions (chemical, humidity, pH, turbulence from flowing water etc, differential aeration etc.)
- Galvanic cell set up with another nearby metal component electrically in contact with the corroding part, with the circuit completed by an electrolyte, which can be as little as surface adsorbed water

- Ineffective or insufficient biocides or inhibitors in a closed system like a heating system as microbiological attack can be very aggressive
The protection system designed to slow down or prevent corrosion can be
- Passivation having or deliberately growing a protective oxide. Stainless steel has a protective surface oxide and aluminium and titanium are often anodised to grow thicker oxides
- Choice of material / alloy to insure the material is passive or immune in the environmental conditions the component is likely to see over life
- Avoid two phase alloys which can provide the anode and cathode locally close together and give phenomena such as dezincification of brasses where one phase can be preferentially eaten away
- Painting, spraying or dip coating to provide a diffusion barrier to slow down or stop the ingress of corrosive species such as oxygen or chloride ions
- Avoid rough surfaces as points protruding into the electrolyte will preferentially corrode
- Cracks or small pits should be avoided as, from the catchment area principle, the anode at the crack tip (because of a lower availability of oxygen) is small with respect to the cathode and attack can be relatively fast giving pits. Aluminium and stainless steels are susceptible to pitting corrosion for example.

Stress Corrosion Cracking and Fatigue

Corrosion, with the exception of pitting, is generally a slow process. However, it can combine with mechanical loading to produce a particularly aggressive form. In stress corrosion cracking (SCC) a tensile stress or residual stress from component manufacture forces the crack tip open to allow the attacking elements in and concentrated chemical attack. The corrosion component of SCC allows areas crack growth to continue even when the stress is lowered.

Combining fatigue with corrosion is probably the most catastrophic form of attack affecting aluminium alloys in aircraft for example. SCC is distinct in SEM/EDX and AES (Auger).

High Temperature Oxidation

High temperature oxidation of metals occurs when the temperature is sufficient to allow interdiffusion of metal and oxygen. It can be deliberate:

- Furnace produced coloured metal for controlled heat transfer characteristics (emissivity)
 - Growing a certain layer thicknesses of silicon oxide insulation or silicon oxynitride as a dielectric on semiconductor wafers
 - Furnace oxidation method of cleaning contaminants off metals prior to thermal reduction
- In other cases oxidation is to be avoided or only permitted until a protective oxide is produced. Areas of interest may be:-
- Inside a jet engine
 - Automotive engine components
 - Air leaks in a reducing or other gas furnace process such as hardening or annealing
 - Welding under a protective inert gas shield

Corrosion Protection

1. Cathodic protection

The principle of this method is to alter the electrode potential of the metallic structure so that they can lie in the immunity region. This is the region where the metal is in the stable state of the element and corrosion reactions are not possible. It is mostly used in steel structures in marine and underground regions.

Two methods are used to apply the cathodic protection to a metal structure.

- **Impressed Current** - This method is used for the protection of pipelines and the hulls of ships in sea water. In this method, an electric current is applied to the metal surface by use of DC electrical circuit. The negative and positive terminal of the current source is connected to the metal requiring protection and an auxiliary anode respectively. The flow of electric current charges the structure with electrons and changes the electrode potential in the negative direction. This process continues till it reaches the immunity region. The current flows from anode to cathode. Thus it protects the metal surface from corrosion.
- **Sacrificial Anode** - This is especially used for ships, offshore oil and gas production platform etc. In this technique, the more reactive metal is used to alter the electrode potential and get the immunity region. Zinc is generally used as sacrificial anode. It

generates the anodic dissolution current with more negative potential. The cathodic curve intersection is now at a more negative potential which is the immunity region. At this region, the corrosion rate of steel is negligible.

SECTION D: SOLID MECHANICS

STRESS AND STRAIN

A member develops resisting force when it is subjected to loads. These resisting forces may be conveniently split into normal and parallel to the sectional planes. The resistance force parallel to the plane is called *shearing resistance*. The intensity of resisting force normal to the sectional plane is called *Normal stress* (σ) and parallel to the sectional plane is called *shearing stress*.

$$\text{Normal stress, } \sigma = \frac{dR}{dA}, \text{ and Shear stress, } \tau = \frac{dQ}{dA}$$

where R = normal resistance

Q = the shearing resistance.

Unit of stress : N/m^2 .

Simple stresses

These are of two types

- **Normal stress** : When a bar is subjected to force P as shown in the figure in such a manner that the applied forces must be same, the resistance R acting on a section is given by

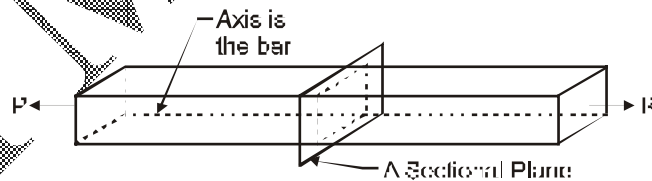


Fig. Bar subjected to tensile force

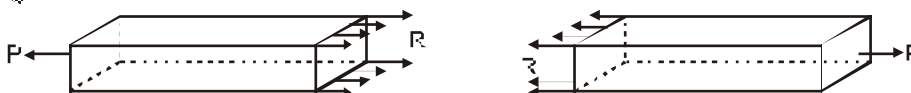


Fig. Free body diagram

$$R = \int \sigma dA = \sigma \int dA = \sigma A$$

where, A = cross sectional area.

From figure, $P = R$

$$P = \sigma A$$

or $\sigma = \frac{P}{A}$

- **Shear stress** : The force parallel to the cross section of bar is shear force. Let Q be the shear stress acting on the section. Then with the usual assumption that stresses are uniform, we get

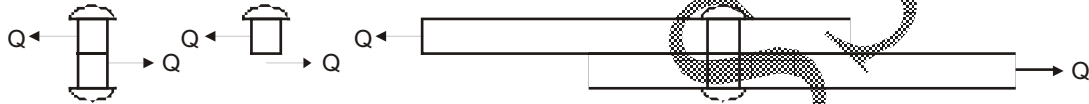


Fig. Rivet in direct shear

$$R = \int \tau dA = \tau \int dA = \tau A$$

For equilibrium, $Q = R = \tau A$

Shear stress, $\tau = \frac{Q}{A}$

Hence direct shear force is equal to the shearing force per unit area.

Strain

It is defined as the change in length per unit length. The strain may be tensile or compressive depending upon whether the length increases (under tensile load) or decreases under (compressive load).

$$\text{Linear strain, } e = \frac{\text{Change in length}}{\text{Original length}}$$

$$\text{Lateral strain} = \frac{\text{Change in the lateral dimension}}{\text{Original lateral dimension}}$$

Poisson's ratio

The ratio of lateral strain to the longitudinal strain in a constant quantity and is called the

Poisson's ratio and is denoted by ν or $\frac{1}{m}$

$$\nu = \frac{\text{Lateral strain}}{\text{Logitudinal strain}}$$

For the most metals its value is between 0.25 to 0.33. For cement concrete 0.15, for rubber 0.5, for steel 0.3. Its value is the same in tension as well as in compression.

Types of strain.

- **Normal strain** : It produces under the action of normal stress and it is measured by change in length.
- **Shear strain** : It produces under the action of shear stress and it is measured by change in angle.

Thus, shear strain, $\gamma = \tan \phi$

For small straining, then $\phi = \gamma$

$$\phi = \frac{d}{l}$$

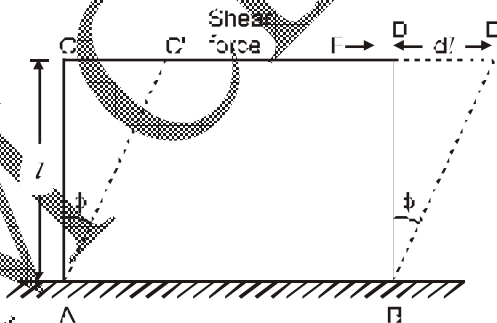


Fig.

- **Superficial strain** : It is the ratio of change in area of cross-section per unit original area.
- **Volumetric strain** : When a uniform stress is applied on all the three faces of the body, then all the three dimensions will change resulting in change in volume. Thus

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

STRESS AND STRAIN RELATIONSHIP (Hooke's Law)

Hooke's law states that stress is directly proportional to strain upto the elastic limit.

Let σ be the stress and e be the strain, then

$$\sigma \propto e$$

or $\sigma = E e$

where, E = constant of proportionality of material and is known as modulus of elasticity.

Stress-Strain Relation in aluminium and high strength Steel

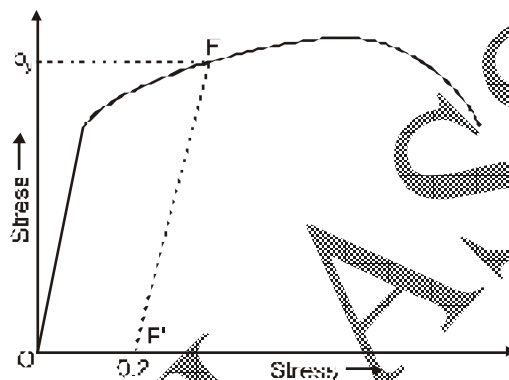


Fig. Stress strain relationship in Aluminium and high strength steel

Stress strain diagram is shown in the figure. This stress p at which if unloading is made there will be 0.2 percent permanent set is known as 0.2 percent proof stress and this point is treated as yield point for all practical purposes.

Stress-Strain Diagram (brittle material)



Fig. Stress Strain relations for brittle material.

In brittle materials there is no appreciable change in rate of strain. There is no yield point and no necking takes place. Ultimate point and breaking point are one and the same. The strain at failure is very small.

SECTION E: THERMODYNAMICS

The Clausius Inequality

The first law is simply an energy balance. However, the second law leads to an inequality; an irreversible process is less efficient than a reversible process. Another important inequality in thermodynamics is the Clausius inequality.

$$\oint \frac{\delta Q}{T} \leq 0$$

That is, the cyclic integral of $\delta Q/T$ is always less than or equal to zero. This is valid for all cycles, reversible or irreversible.

For internally reversible cycles, it can be shown that:

$$\oint \left. \frac{\delta Q}{T} \right|_{\text{int, rev}} = 0$$

The Clausius inequality forms the basis for the definition of a new property called entropy.

As can be seen in the equation above, for an internally reversible process the cyclic integral of $\delta Q/T$ is zero. A quantity whose cyclic integral is zero depends on the state only and not the process path, and thus it is a property.

Clausius in 1865 realized that he discovered a new property and he called it entropy:

$$ds = \left(\frac{\delta Q}{T} \right)_{\text{int, rev}} \quad (\text{kJ/K})$$

Entropy per unit mass is designated by s (kJ/kg.K).

The entropy change of a system during a process can be calculated:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int, rev}} = (\text{kJ/K})$$

To perform this integral, one needs to know the relation between Q and T during the process.

Note that the cyclic integral of $\delta Q/T$ will give us the entropy change only if the integration carried out along an internally reversible path between two states.

For irreversible processes, we may imagine a reversible process between the two states (initial and final) and calculate the entropy change (since entropy is a property).

The Increase of Entropy Principle

Entropy change of a closed system during an irreversible process is greater than the integral of $\delta Q/T$ evaluated for the process. In the limiting case of a reversible process, they become equal.

$$dS \geq \frac{\delta Q}{T}$$

The entropy generated during a process is called entropy generation, and is denoted by S_{gen} .

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

Note that the entropy generation S_{gen} is always a positive quantity or zero (reversible process). Its value depends on the process, thus it is not a property of a system.

The entropy of an isolated system during a process always increases, or in the limiting case of a reversible process remains constant (it never decreases). This is known as the increase of entropy principle.

The entropy change of a system or its surroundings can be negative; but entropy generation cannot.

$$S_{\text{gen}} = \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

1. A process must proceed in the direction that complies with the increase of entropy principle, $S_{gen} > 0$. A process that violates this principle is impossible.
2. Entropy is a non-conserved property, and there is no such thing as the conservation of entropy. Therefore, the entropy of universe is continuously increasing.
3. The performance of engineering systems is degraded by the presence of irreversibility. The entropy generation is a measure of the magnitudes of the irreversibilities present during the process.

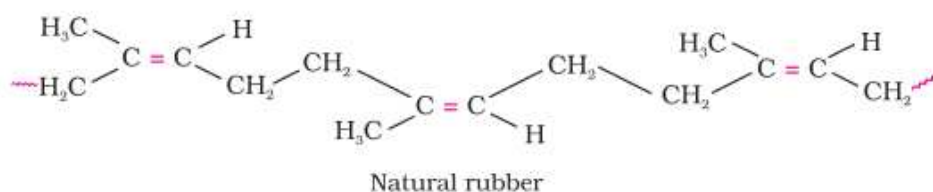
SECTION F: POLYMER SCIENCE AND ENGINEERING

Rubber

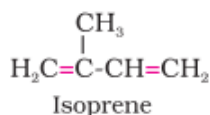
1. Natural rubber

Rubber is a natural polymer and possesses elastic properties. It is also termed as elastomer and has a variety of uses. It is manufactured from rubber latex which is a colloidal dispersion of rubber in water. This latex is obtained from the bark of rubber tree and is found in India, Sri Lanka, Indonesia, Malaysia and South America.

Natural rubber may be considered as a linear polymer of isoprene (2-methyl-1, 3-butadiene) and is also called as cis - 1, 4 - polyisoprene.

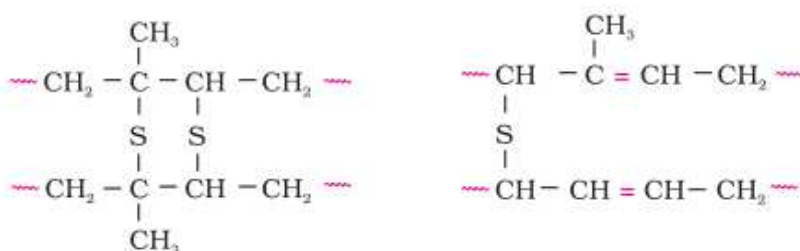


The cis-polyisoprene molecule consists of various chains held together by weak van der Waals interactions and has a coiled structure. Thus, it can be stretched like a spring and exhibits elastic properties.



Vulcanisation of rubber: Natural rubber becomes soft at high temperature (>335 K) and brittle at low temperatures (On vulcanisation, sulphur forms cross links at the reactive sites of double bonds and thus the rubber gets stiffened.

In the manufacture of tyre rubber, 5% of sulphur is used as a crosslinking agent. The probable structures of vulcanised rubber molecules are depicted below :



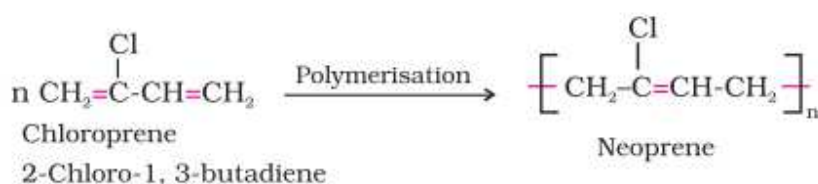
2. Synthetic rubbers

Synthetic rubber is any vulcanisable rubber like polymer, which is capable of getting stretched to twice its length. However, it returns to its original shape and size as soon as the external stretching force is released. Thus, synthetic rubbers are either homopolymers of 1, 3-butadiene derivatives or copolymers of 1, 3-butadiene or its derivatives with another unsaturated monomer.

Preparation of Synthetic Rubbers

1. Neoprene

Neoprene or polychloroprene is formed by the free radical polymerisation of chloroprene.

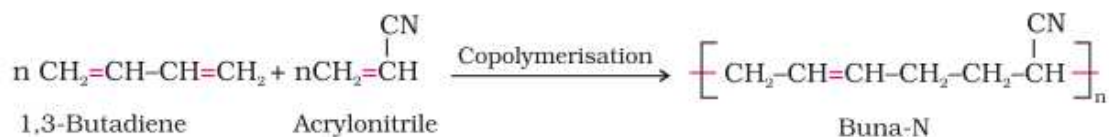


It has superior resistance to vegetable and mineral oils. It is used for manufacturing conveyor belts, gaskets and hoses.

2. Buna – N

Buna –N is obtained by the copolymerisation of 1, 3-butadiene and acrylonitrile in the

presence of a peroxide catalyst.



It is resistant to the action of petrol, lubricating oil and organic solvents. It is used in making oil seals, tank lining, etc.

SECTION G: FOOD TECHNOLOGY

Food preservation

Why are foods processed?

Food manufacturers process foods and raw ingredients to add value to their products in the form of longer shelf lives, added dietary nutrients, appealing textures and other features.

Preservation

- The most important reason to process or prepare foods has been to make them last longer before spoiling.
- Early civilizations used techniques like salting meats, fermenting dairy (into cheese or yogurt, for example) and pickling vegetables.
- More recently, in the 1790s, Napoleon Bonaparte offered a prize to the scientist who could best develop ways to preserve foods for the armies of France; the competition prompted the discovery of safe canning practices by Nicolas Appert.
- Louis Pasteur, working with beer and wine, would later discover pasteurization, a process that uses controlled amounts of heat to extend the shelf life of milk, juice and other products.
- Some argue that preservation is still the most important reason to process food because illness, even death, can result from eating spoiled food.

- When perishable products have longer shelf lives consumers can enjoy them for a greater part of the year, distributors can ship them over greater distances, and retailers can stock them on shelves for extended periods.

Food safety

- Processing and preparing foods can make them safer to eat by destroying toxins and eliminating or inhibiting pathogens.
- Techniques such as refrigerating, freezing, fermenting, drying and adding salt or sugar can slow or stop the growth of pathogens.
- Heat processes, such as pasteurization and cooking, can eliminate pathogens.
- Because these techniques help protect consumers, most cases of foodborne illness involve raw animal products, fruits and vegetables that have been contaminated by pathogens.

Variety

- Every year, food manufacturers introduce roughly 20,000 unique processed foods on retail shelves; of these, only the most highly successful remain.
 - o Most new products are variations on candy, gum, snacks and beverages.
 - o Some are fortified, low-fat, low-sugar, low-cholesterol or low-salt versions of existing products.
- By modifying the flavors, textures, aromas, colors and form of foods and raw ingredients, food processing can create greater variety in our food supply. Grains, for example, can be milled into flour, which is then used to make a wide variety of products.
- Grains are the core ingredients in most ready-to-eat breakfast cereals. In 1860, Dr. J. H. Kellogg invented a form of granola-like cereal designed to fit the austere diets of Seventh Day Adventists; he and his brother later founded the company that bears his name.
- Today, a trip down the breakfast aisle of a supermarket reveals a tremendous variety of products that are derived, in part, from wheat, rice, corn and several other grains.

- Breakfast cereal manufacturers achieve this by using techniques like adding flavors, cooking, drying, toasting and spraying on vitamins. Processes like shredding, flaking, puffing and extruding can alter the shapes of cereal grains.
- Changing the qualities of foods in these and other ways can make products more appealing to some consumers, helping manufacturers to increase sales in a highly competitive marketplace.

Convenience

- Food processing can create products that require little or no preparation on the part of consumers.
- Among the early examples of processed convenience food was pemmican, strips of dried buffalo meat mixed with fat and berries that could be eaten "on the go" by First Nations peoples of the Great Plains.
- Modern examples include baby food, canned foods, frozen pizzas, instant noodles, bottled juices and ready-to-serve cakes, cookies and pies.
- Fast food offers another form of processed, ready-to-eat calories for busy eaters.
- From 1966 to 1999, the amount of time U.S. adults ages 25 to 54 spent cooking meals decreased by 25% (43% among women). Americans, in general, are consuming fewer calories at home and more at full-service and fast-food restaurants. These trends may reflect a growing demand among busy consumers for convenient food.

Nutrition: Fortification

- Enrichment and fortification are two of the ways that processing can restore or raise nutrient levels in food.
- Enrichment means the restoration of some of the nutrients that are lost during certain forms of processing, such as milling grains.
- Fortification means adding certain nutrients above the levels that naturally occur in a food.
- The term fortification is also used more generally to refer to any addition of nutrients to a food.
- Fortification was designed to prevent nutrient deficiencies in the general population.

- In the 1830s, French chemist Jean Boussingault advocated adding iodine to table salt to prevent goiter (enlarged thyroid glands), a condition frequently caused by iodine deficiency.
- Today, many foods are typically fortified in the U.S., including salt (with iodine), milk (with vitamin D) and grain products (with thiamin, niacin, riboflavin, iron and folic acid).
- These additions have helped prevent nutrient deficiencies in the U.S. diet, but food manufacturers sometimes use fortification as a selling point for foods of questionable nutritional quality, such as candy, snacks and sweetened breakfast cereals.
- Some research also suggests that synthetic vitamins may not offer the same nutritional benefits as vitamins that occur naturally in food.

Nutrition: Preserving nutrients

- Some forms of preservation can also maintain nutrient levels in certain foods.
- After fruits and vegetables are harvested, they begin to lose their nutritional quality. Freezing or canning can preserve levels of certain nutrients.
- Despite a common belief, canned foods are not necessarily less nutritious than their fresh or frozen counterparts.
- Factors such as when the produce was harvested, how long it had been stored and how it will be prepared determine whether the fresh, frozen or canned version of a food is the most nutritious choice.