



GATE - MATHEMATICS MOCK TEST PAPER

- There are total of 65 questions in this paper which are of multiple choice type or numerical answer type.
- Questions Q.1 - Q.25 carry 1 mark each. Questions Q.26 - Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.
- Questions Q. 56 - Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 - Q.60 carry 1 mark each, and questions Q. 61 - Q.65 carry 2 marks each.
- There will be negative marking of 1/3 marks for each wrong answer for 1 mark questions. For all 2 marks questions 2/3 marks will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.

TIME : 3 HOURS

MAX. MARKS : 100

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1. Let $f(x, y) = \begin{cases} \frac{x^2 y^2}{(x+y)^2} \{1 + \log(1 - (x+y))\}, & (x, y) \neq (0, 0) \\ k, & (x, y) = (0, 0) \end{cases}$

Then the value of k for which $f(x, y)$ is continuous at $(0, 0)$ is _____.

2. For what value of λ the following matrix have nullity 1 ?

(A) $\lambda \neq 2$

(B) $\lambda \neq 3$

(C) $\lambda = 4$

(D) for all λ

3. If 3×3 is skew - Hermitian matrix and if have an eigen value $-2i$ then one of the remaining eigen values is _____.

4. Consider the l.p.p. Max $z = 2x_1 - 4x_2$

$$\text{s.t. } x_1 + 2x_2 \leq 3$$

$$3x_1 + 4x_2 \leq 5$$

$$\text{and } 0 \leq x_1 \leq 5$$

$$0 \leq x_2 \leq 5$$

the total no. of extreme points is _____.

5. Let x^4 be an optimal solution to the l.p.p.

$$\text{minimize } c^T x$$

$$\text{Subject to } Ax \geq b$$

$$\text{and } x \geq b$$

which one of the following is true ?

(A) The value of the objective function at a feasible solution to the dual L.P.P. is bounded above by $c^T x$.

(B) x is an extreme point of the feasible region

(C) The dual L.P.P. has an optimal solution with optimum value $c^T x$

(D) If a variable is zero in x then the corresponding constraint in the dual is satisfied as a strict inequality

6. The life time of two brands of bulbs X and Y are exponentially distributed with a mean life time of 100 hours. Bulb X is switched on 15 hours after bulb Y has been switched on. The probability that the bulb X fails before Y is

(A) $\frac{15}{100}$

(B) $\frac{1}{2}$

(C) $\frac{85}{100}$

(D) 0

7. Consider the algebraic extension $E = \mathbb{Q}(\sqrt[3]{2}, \sqrt[4]{3})$ of the field \mathbb{Q} of rational numbers. Then $[e : \mathbb{Q}]$ the degree of E over \mathbb{Q} is _____.

8. The general solution of the partial differential equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 12(x + y)$

(A) $(x + y)^2 + \phi_1(x + iy) + \phi_2(x - iy)$

(B) $(x + y)^3 + \phi_1(y + ix) + \phi_2(y - ix)$

(C) $(x + y)^3 + \phi_1(x + iy) + \phi_2(x - iy)$

(D) $(x + y)^2 + \phi_1(y + ix) + \phi_2(y - ix)$

9. The numerical value obtained by applying the two point trapezoidal rule to the integral

$$\int_0^{\frac{\pi}{2}} e^{\sin x} dx \text{ is}$$

(A) $\frac{1+e}{2}$

(B) $\frac{1-e}{2}$

(C) $\frac{1}{2}$

(D) $\frac{e-1}{2}$

10. Let $f(x)$ be continuous whose values are known at $-2, -1, 1, 2$ if the Lagrange's interpolation formula $f(x) = L_1f(-2) + L_2f(-1) + L_3f(1) + L_4f(2)$ is used to approximate $f(0)$ then L_3 is

(A) 0

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{4}{3}$

11. A steam boat is moving with velocity v_1 when steam is shut off. If the retardation at any subsequent time is equal to the magnitude of the velocity at that time, then the velocity fin time t after the steam is shut off is

(A) $v_1 e^t$

(B) $v_1 e^{-t}$

(C) $2v_1 e^t$

(D) $2v_1 e^{-t}$

12. Which one of the following statement is not correct?

(A) Every positive integer is either even or odd

(B) No integer is both even and odd

(C) For any integer, a , a^2 is even if and only if a is even

(D) No integer is both even and prime

13. Which one of the following is correct?

(A) Between any two rational numbers, there is an integer

(B) Between any two irrational numbers, there is a rational number

(C) Between any two irrational numbers, there is an integer

(D) Sum of two irrational numbers is always irrational

14. If $f(x)$ and $g(x)$ are differentiable function for $0 \leq x \leq 1$ such that $f(1) - f(0) = k[g(1) - g(0)]$. $k \neq 0$ then there exists c satisfying $0 < c < 1$.

What is equal to $\frac{f'(c)}{g'(c)}$?

(A) $2k$

(B) k

(C) $-k$

(D) $\frac{1}{k}$

15. Which one of the following functions is continuous at origin?

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- (A) $f(x) = \cos\left(\frac{1}{x}\right)$; when $x \neq 0$, $f(0) = 0$
- (B) $f(x) = \sin x \sin\left(\frac{1}{x}\right)$; when $x \neq 0$, $f(0) = 0$
- (C) $f(x) = x + \sin\left(\frac{1}{x}\right)$; when $x \neq 0$, $f(0) = 1$
- (D) $f(x) = \sin x \sin\left(\frac{1}{x}\right)$, when $x \neq 0$, $f(0) = 1$

16. If $u = \frac{xy}{x+y}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to:

- (A) 1
 (B) u
 (C) -u
 (D) 0

17. For the function $f(x,y) = x^5 F\left(\frac{y}{x}\right)$, the value of the differential $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is equal to:

- (A) $5x^5 F\left(\frac{y}{x}\right)$
 (B) $5x^4 F\left(\frac{y}{x}\right)$
 (C) $4x^5 F\left(\frac{y}{x}\right)$
 (D) $4x^4 F\left(\frac{y}{x}\right)$

18. The set $S_1 = \left\{ \alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = \begin{bmatrix} 2 & 4 & 8 \\ 6 & 0 & -2 \end{bmatrix} \right\}$ and $S_2 = (f = u^3 + 3u + 4, g = u^3 + 4u + 3)$ are :

- (A) Both linearly dependent
- (B) Both linearly independent
- (C) S_1 is linearly dependent but S_2 is not
- (D) S_2 is linearly dependent but S_1 is not

19. If the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is such that $T(1, 0) = (2, 3, 1)$ and $T(1, 1) = (3, 0, 2)$, then:

- (A) $T(x, y) = (x + y, 2x + y, 3x - 3y)$
- (B) $T(x, y) = (2x + y, 3x - 3y, x + y)$
- (C) $T(x, y) = (2x - y, 3x + 3y, x - y)$
- (D) $T(x, y) = (x - y, 2x - y, 3x + 3y)$

20. Multiplication of a complex number z by $(1 + i)$ rotates the radius vector to z in the complex plane by an angle:

- (A) 90° clockwise
- (B) 45° clockwise
- (C) 90° anticlockwise
- (D) 45° anticlockwise

21. If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$, then $a^2 + b^2$ is equal to _____.

22. In a city, three daily newspapers A, B, C are published, 42% of the people in that city read A, 51% read B and 68% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers. The percentage of persons who read all the three papers is _____.

23. If the set Z of integers is a group under the binary operation $*$ defined as $m * n = m + n + 1$, $n \in Z$, then the inverse of the element 5 is _____.

24. In an abelian group, the order of an element a is 4 and the order of an element b is 3, then $(ab)^{14}$ is equal to :

(A) a^2b^{-1}

(B) $(ab)^{-2}$

(C) a^2

(D) b

25. Which of the following rings are integral domains?

1. Z_{60} 2. Z_{71} 3. Z_{82} 4. Z_{97}

Select the correct answer using the codes given below:

(A) 1 and 2

(B) 2 and 3

(C) 2 and 4

(D) 3 and 4

26. A ring $(R, +, \cdot)$ whose all elements are idempotent is:

(A) Always abelian

(B) An integral domain

(C) A division ring

(D) A field

27. If $f(x) = \begin{cases} x^{(\frac{1}{k})-1} \cos\left(\frac{1}{x}\right); & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ then under what condition is $f(x)$ differentiable at $x = 0$?

(A) $k > \frac{1}{2}$

(B) $k \geq \frac{1}{2}$

(C) $k \leq \frac{1}{2}$

(D) $k < \frac{1}{2}$

28. Which one of the following differential equations is exact?

(A) $(3x^3 + 2y \sin 2x) dx + (2 \sin 2x + 3y^3) dy = 0$

(B) $ye^{xy} dx + (e^{xy} + 2y) dy = 0$

(C) $(2xy \cos x^3 + 2xy + 1) dx + (\sin - x^2) dy = 0$

(D) $y \left(1 + \frac{1}{x}\right) + \cos y + (x + \log x - x \sin y) \frac{dy}{dx} = 0$

29. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\tan y}{x}$ is :

(A) $y = c x \sin x$

(B) $y/x = \sin x$

(C) $\sin(y/x) = cx$

(D) $\sin(y/x) = c \sin x$

where c is an arbitrary constant.

30. If integers $a, b > 1$; then the set of all integers of the form $ma + nb$ (m, n integers) includes:

(A) Both their GCD and LCM

(B) Their GCD but not LCM

(C) Their LCM but not GCD

(D) Neither their LCM nor GCD

31. Which one of the following statements for sets A, B, C is correct?

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(A) $A - (B \cup C) = (A - B) \cup (A - C)$

(B) $A \cup (B - C) = (A \cup B) - (A \cup C)$

(C) $A - (B \cap C) = (A - B) \cap (A - C)$

(D) $A - (B \cup C) = (A - B) \cap (A - C)$

32. If $u = x^2 + y^2 + z^2$, then

$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ is equal to :

(A) $4u$

(B) u^2

(C) u

(D) $2u$

33. If $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(x, y, z) = (x - y, y + 3z, x + 2y)$, then T^{-1} is

(A) $\left(2x + z, -x + z, \frac{x}{3} + y - \frac{z}{3} \right)$

(B) $\frac{1}{3} \left(2x + y, -x + y, \frac{1}{3}x - \frac{1}{3}y + z \right)$

(C) $\frac{1}{3} \left(x + 2y, x - y, -\frac{1}{3}x + \frac{1}{3}y - z \right)$

(D) $\frac{1}{3} \left(x - 2y, x + y, \frac{x}{3} - \frac{y}{3} - z \right)$

34. Let V be a vector space of 2×2 matrices over \mathbb{R} . Let T be the linear mapping $T : V \rightarrow V$, such that $T(A) = AB - BA$,

$B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, then the nullity of T is _____.

35. The value of $i^{1/3}$ are :

(A) $-i, \frac{i \pm \sqrt{3}}{2}$

(B) $i, \frac{i \pm \sqrt{3}}{2}$

(C) $-i, \frac{\sqrt{3}i \pm 1}{2}$

(D) $i, \frac{\sqrt{3}i \pm 1}{2}$

36. If α is a complex number such that $\alpha^2 + \alpha + 1 = 0$, then α^{31} is equal to _____.

37. If the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} -i & 0 \\ 0 & 0 \end{pmatrix}$ from a group with respect to matrix multiplication, then which one of the following statements about the group, thus formed, is correct?

(A) The group has no element of order 4α

(B) The group has an element of order 3α

(C) The group is non-abelian

(D) $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ is its own inverse

38. A random sample of size n is chosen from a population with probability density function $f(x,$

$$\theta) = \begin{cases} \frac{1}{2} e^{-(x-\theta)}, & x \geq \theta \\ \frac{1}{2} e^{(x-\theta)}, & x < \theta \end{cases}$$

Then, the maximum likelihood estimator of θ is the

(A) Mean of the sample

(B) Standard deviation of the sample

- (C) Median of the sample
 (D) Maximum of the sample
39. The ring of integers (mod 6) is
 (A) A finite integral domain
 (B) An infinite integral domain
 (C) A field
 (D) Not an integral domain
40. If n denotes the number of elements in a field, then n must be :
 (A) A prime
 (B) A prime of the form $4k + 1$
 (C) A product of distinct primes
 (D) A power of a prime
41. The singular solution of the differential equation
 $(xp - y)^2 = p^2 - 1$, where $p = dy/dx$, is :
 (A) $x^2 + y^2 = 1$
 (B) $x^2 - y^2 = 1$
 (C) $x^2 + 2y^2 = 1$
 (D) $2x^2 + y^2 = 1$
42. The singularity of the equation

$$= \frac{2}{3}x \frac{dy}{dx} - \frac{2}{3x} \left(\frac{dy}{dx}\right)^2, x > 0 \text{ is :}$$

- (A) $y = x^3$

- (B) $y = x$
 (C) $y = x^3/6$
 (D) $y = x^3/2$

43. The orthogonal trajectories of the system of parabolas $y^2 = 4a(x + a)$, a being the parameter, is given by the system of the curves:

- (A) $y^2 = 4a(x + a)$
 (B) $y^2 = 4a(x - a)$
 (C) $y^2 = 4ax$
 (D) $x^2 = 4ay$

44. Consider a mapping f from the set of natural numbers to the set of integers Z defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -n/2, & \text{when } n \text{ is even} \end{cases}$$

Then $f : N \rightarrow Z$ is :

- (A) one-one but not onto
 (B) Onto but not one-one
 (C) Both one-one and onto
 (D) Neither one-one nor onto

45. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then

$$\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$$
 is equal to :

- (A) 0

(B) $\frac{a}{2}(\alpha - \beta)^2$

(C) $-\frac{a}{2}(\alpha - \beta)^2$

(D) $\frac{a^2}{2}(\alpha - \beta)^2$

46. If $f(x) = \begin{cases} x^\alpha \cos 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$, then:

(A) $\alpha < 0$

(B) $\alpha > 0$

(C) $\alpha = 0$

(D) α may be positive, negative or zero

47. The value of $\lim_{n \rightarrow \infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$ is _____.

Statement for COMMON DATA Questions 48 and 49

Let a random variable X follow the exponential distribution with mean 2. Define $Y = [X - 2 | X > 2]$

48. The value of $P(Y > t)$ is

(A) $e^{-t/2}$

(B) e^{-2t}

(C) $\frac{1}{2} e^{-t/2}$

(D) $\frac{1}{2} e^{-t}$

49. The value of $E(Y)$ is equal to _____.

Statement for COMMON DATA Questions 50 and 51

Let the random variables X and Y be independent Poisson variates with parameters λ_1 and λ_2 respectively.

50. The conditional distribution of X given $X+Y$ is

- (A) Poisson
- (B) Hypergeometric
- (C) Geometric
- (D) Binomial

51. The regression equation of X on $X+Y$ is given by

- (A) $E(X | X+Y) = XY \frac{\lambda_1}{\lambda_1 + \lambda_2}$
- (B) $E(X | X+Y) = (X+Y) \frac{\lambda_2}{\lambda_1 + \lambda_2}$
- (C) $E(X | X+Y) = (X+Y) \frac{\lambda_1}{\lambda_1 + \lambda_2}$
- (D) $E(X | X+Y) = XY \frac{\lambda_2}{\lambda_1 + \lambda_2}$

Statement for Linked Answer Questions 52 and 53:

Let $f(z) = \cos z - \frac{\sin z}{z}$ for non-zero $z \in \mathbb{C}$ and $f(0) = 0$. Also, let $g(z) = \sinh z$ for $z \in \mathbb{C}$

52. Then $f(z)$ has a zero at $z = 0$ of order _____.

53. Then $\frac{g(z)}{zf(z)}$ has a pole at $z = 0$ of order _____.

Statement for linked answer Questions 54 and 55

Consider the boundary value problem

$$u_{xx} = u_{yy} = 0, \quad x \in (0, \pi), \quad y \in (0, \pi),$$

$$u(x, 0) = u(x, \pi) = u(0, y) = 0.$$

54. Any solution of this boundary value problem is of the form

(A) $\sum_{n=1}^{\infty} a_n \sinh nx \sin ny$

(B) $\sum_{n=1}^{\infty} a_n \cosh nx \sin ny$

(C) $\sum_{n=1}^{\infty} a_n \sinh nx \cos ny$

(D) $\sum_{n=1}^{\infty} a_n \cosh nx \cos ny$

55. If an additional boundary condition $u_x(\pi, y) = \sin y$ is satisfied, the $u(x, \pi/2)$ is equal to

(A) $\frac{\pi}{2}(e^{\pi} - e^{-\pi})(e^{\pi} - e^{-\pi})$

(B) $\frac{\pi(e^x + e^{-x})}{(e^{\pi} - e^{-\pi})}$

(C) $\frac{\pi(e^x + e^{-x})}{(e^{\pi} + e^{-\pi})}$

(D) $\frac{\pi}{2}(e^{\pi} + e^{-\pi})(e^x + e^{-x})$

General Aptitude (GA) Questions (Q. 56-65)

56. A butler stole wine from a butt of sherry, which contained 30% spirit and he replaced what he had stolen by wine containing 12% spirit. The butt was then of 18% strength. How much of the butt did he steal ?

- (A) $\frac{2}{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{3}{4}$
57. Find the average of all prime numbers between 30 and 50.
- (A) 39.8
- (B) 37.3
- (C) 35
- (D) 41
58. 64, 144, 256, 400,.....
- (A) 529
- (B) 484
- (C) 676
- (D) 576
59. What is the Antonym of Aggressive?
- (A) Not getting justice
- (B) Militant
- (C) Retiring
- (D) Noisy
60. What is the synonym of Voracious?
- (A) Tenacious
- (B) Truthful

- (C) Spacious
(D) Ravenous
61. What is the synonym of Abortive
- (A) Fruitful
(B) Familiar
(C) Unsuccessful
(D) Consuming
62. Fragile : Hardy
- (A) Awkward: clumsy
(B) Orthodox: traditional
(C) Amateur: professional
(D) Cautious: flippant
63. Chapter: Book
- (A) alcove: nook
(B) paragraph: sentence
(C) Page: rip
(D) room : house
64. What is the synonym of Bias
- (A) Prejudice
(B) Tendency
(C) Resent
(D) Inclination

65. What is the Antonym of Coincidence?

- (A) incidence
- (B) Accident
- (C) Chance
- (D) Adaptation

ANSWER KEY

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer.	0	B	0	3	C	C	12	B	D	B	B	D	B	B	B	B	A	C	B	D
Question	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer.	4	25	-7	A	C	C	D	D	C	B	D	D	A	0	A	1	D	D	D	A
Question	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer.	B	C	A	A	D	B	3	A	2	D	C	2	3	A	C	A	A	D	C	D
Question	61	62	63	64	65															
Answer.	C	C	D	A	A															

HINTS AND SOLUTIONS

1. 0

Given that $f(x, y)$ is continuous at $(0, 0)$ for that $(x, y) \rightarrow (0, 0)$ $f(x, y) = f(0, 0)$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \left[\frac{x^2 y^2}{(x+y)^2} [1 + \log(1 - (x+y))] \right] = k$$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \left[\frac{x^2 y^2}{(x+y)^2} \left[1 - (x+y) - \frac{(x+y)^2}{2!} \dots \right] \right] = k$$

$$\Rightarrow 0 = k$$

2. (B) Since Let $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$

by elementary matrix transformations changing A in echelon form.

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

Rank (A) = 3 for $\lambda \neq 3$

$$\begin{aligned} \Rightarrow \text{nullity } A &= \text{no. of columns} - \text{Rank } A \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

3. 0

A skew Hermitian matrix have all its eigenvalues either pure imaginary or zero and we know that imaginary values comes in pair so another eigenvalues of that matrix can be either 0 or 2i

4. 3

Given l.p.p. Max $z = 2x_1 - 4x_2$

$$\text{s.t. } x_1 + 2x_2 \leq 3$$

$$3x_1 + 4x_2 \leq 5$$

$$x_1 \leq 5$$

$$x_2 \leq 5$$

here three extreme points but no. of constraint \geq no. of variables so we get no. of basic feasible solution is more than no. of extreme points.

5.(C) If primal problem have an optimal solution then it's dual must have optimal solution and optimum values of primal and dual are same.

6. (C) Probability that the bulb X fails before Y = $\frac{100-15}{100} = \frac{85}{100}$

7. 12

Clearly $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q] = Q[(\sqrt[3]{2}, \sqrt[4]{3}) : Q^{\sqrt[3]{2}}] \cdot [Q(\sqrt[3]{2}) : Q]$

By $[K : F] = [K : E] [E : F]$

and $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q] = [Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q^{\sqrt[4]{3}}] [Q(\sqrt[4]{3}) : Q]$

Since $[Q^{\sqrt[3]{2}} : Q] = 3$

and $[Q^{\sqrt[4]{3}} : Q] = 4$

divide $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q]$ in both

Thus $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q] \geq 0$

on the other hand $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q^{\sqrt[3]{2}}]$ is at most 4 since $\sqrt[4]{3}$ is a zero for $x^4 - 3 \in Q^{\sqrt[3]{2}} x$.

Therefore $[Q(\sqrt[3]{2}, \sqrt[4]{3}) : Q] = [Q^{\sqrt[3]{2}, \sqrt[4]{3}} : Q^{\sqrt[3]{2}}] [Q^{\sqrt[3]{2}} : Q]$

$$\leq 4.3$$

$$= 12$$

8. (B) The given equation can be written as

$$(D^2 + D'^2) = 12(x + y) \quad \dots(1)$$

It's auxiliary equation is $m^2 + 1 = 0$

so that $m = \pm i$

$$\text{C.F.} = \phi_1(y + ix) + \phi_2(y - ix)$$

$\phi_1 \phi_2$ being arbitrary functions ‘

$$\begin{aligned} \text{Now PI} &= \frac{1}{D^2 + D'^2} 12(x + y) \\ &= 12 \frac{1}{D^2 + D'^2} (x + y) \\ &= \frac{12}{1^2 + 1^2} \int \int v dv \\ &= 6 = v^3 = (x + y)^3 \end{aligned}$$

Required GS $V = \phi_1(y + ix) + \phi_2(y - ix) + (x + y)^3$

9.(D) Two point trapezoidal rule is

$$\int_a^b f(x) dx = \frac{f(b) - f(a)}{2}$$

$$\begin{aligned} \text{Hence } \int_0^{\frac{\pi}{2}} e^{\sin x} dx &= \frac{f\left(\frac{\pi}{2}\right) - f(0)}{2} \\ &= \frac{e - 1}{2} \end{aligned}$$

10.(B) L_3 is coefficient of term $f(1)$ is given by

$$\frac{1}{(1+2)(1+1)(1-2)} = \frac{1}{3}$$

11.(B) A steam boat is moving with a velocity v_1 when steam is shut off. If the retardation at any subsequent time is equal to the magnitude of the velocity at the time, the velocity in time t after the steam is shut off will be equal to $v_1 e^{-t}$.

12.(D) No integer is both even and prime

Prime number-an integer $P \geq 2$ is said to be a prime number if its divisors are ± 1 and $\pm P$.

∴ Integer 2 is prime and even both.

13.(B) Between any two irrational numbers, there is a rational number is a correct statement.

14.(B) By the cauchy second mean value theorem we know that "if two function f, g defined on [a, b] are

1. continuous on [a, b]
2. derivable on (a, b) and
3. $g'(x) \neq 0$ for any $x \in (a, b)$

then there exists at let one real no. c between a and b such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

Now Here use this theorem we get

$$\frac{f'(c)}{g'(c)} = k$$

15.(B) option-a $f(x) = \cos$ at $x \neq 0$ $f(0) = 0$

$$\text{then } f(0+h) = \lim_{h \rightarrow 0} \cos \frac{1}{h} = [\text{Value b/w } + \text{ to } 1]$$

$$f(0-h) = \lim_{h \rightarrow 0} \cos \left(\frac{1}{-h} \right) = [\text{Value b/w } -1 \text{ to } 1]$$

$$f(0) = 0$$

Since $f(0+h) \neq f(0) \neq f(0-h)$

Thus, $f(x)$ is not continuous at $x = 0$

$$\text{option-c } f(x) = x + \sin \frac{1}{x} \text{ at } x \neq 0, f(0) = 1$$

$$\begin{aligned} \text{then } f(0 + h) &= \lim_{h \rightarrow 0} h + \sin \frac{1}{h} \\ &= 0 + [\text{Value b/w } -1 \text{ to } 1] \end{aligned}$$

$$\begin{aligned} f(0 - h) &= \lim_{h \rightarrow 0} -h - \sin \left(\frac{1}{h} \right) \\ &= 0 - [\text{Value b/w } -1 \text{ to } 1] \end{aligned}$$

Since $f(0^+) \neq f(0) \neq f(0^-)$

Thus $f(x)$ is not continuous at $x = 0$

option -b

$$f(x) = \sin x \frac{1}{x} \text{ sin at } x = 0, f(0) = 0$$

$$\begin{aligned} \text{then } f(0 + h) &= \lim_{h \rightarrow 0} \sin h \sin \frac{1}{h} \\ &= \sin 0 \sin \left(\frac{1}{0} \right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(0 - h) &= \lim_{h \rightarrow 0} \sin(-h) \sin \left(\frac{-1}{h} \right) \\ &= \lim_{h \rightarrow 0} \sin h \frac{1}{h} \\ &= 0 \end{aligned}$$

Since $f(0 + h) = f(0) = f(-h)$

Thus, the function is continuous at origin

16.(B) Here, it is given that

$$u = \frac{xy}{x+y}$$

∴ u is a homogeneous equation of degree one

∴ By Euler's theorem

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1 \cdot u = u$$

17.(A) Given function $f(x, y) = x^5 F\left(\frac{y}{x}\right)$

∴ We know Euler's theorem is

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n f(x, y)$$

where $n \rightarrow$ degree of the function

∴ Using this theorem,

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 5x^5 F\left(\frac{y}{x}\right)$$

18.(C) Here, the sets

$$S_1 = \left\{ \alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = \begin{bmatrix} 2 & 4 & 8 \\ 6 & 0 & -2 \end{bmatrix} \right\}$$

$$\text{or, } S_1 = \left\{ \alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = 2 \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix} \right\}$$

So, we can see that in set $S_1, \beta = 2\alpha$

∴ S_1 is linearly dependent.

and set

$$S_2 = \{f = u^3 + 3u + 4, g = u^3 + 4u + 3\}$$

then, $f = u^3 + 3u + 4$

and $g = u^3 + 4u + 3$

Now, $af + bg = 0$

$\therefore a(u^3 + 3u + 4) + b(u^3 + 4u + 3)$

or $au^3 + 3au + 4a + bu^3 + 4bu + 3b = 0$

or $(a + b)u^3 + (3a + 4b)u + (4a + 3b) = 0$

$\Rightarrow a + b = 0$

$3a + 4b = 0$

and $4a + 3b = 0$

\therefore From Eq. (i) $a = -b$

Putting this value in Eq. (ii), we get

$3(-b) + 4b = 0$

or $-3b + 4b = 0$

$\therefore b = 0$

then $a = 0$

and $af + bg = 0 \Rightarrow a = 0$ and $b = 0$

Hence, f and g are linearly independent.

19.(B) First we will show that the set $\{(1, 0), (1, 1)\}$ is a basis of \mathbb{R}^2 .

For linear independent of this set.

Let $a(1, 0) + b(1, 1) = (0, 0)$ where $a, b \in \mathbb{R}$

Then $(a + b, b) = (0, 0)$

$\Rightarrow a + b = 0, b = 0$

$$\Rightarrow a = 0, b = 0$$

Hence, the set $\{(1, 0), (1, 1)\}$ is linearly independent.

Now, we shall show that the set $\{(1, 0), (1, 1)\}$ spans \mathbb{R}^2 . Let

$$(x, y) \in \mathbb{R}^2 \text{ and Let } (x, y) = a(1, 0) + b(1, 1)$$

$$(x, y) = (a + b, b)$$

$$\Rightarrow a + b = x, b = y$$

$$\Rightarrow a = x - b$$

$$a = x - b$$

$$(\because b = y)$$

$$\text{or here } T(e_1) = (2, 3, 1)$$

$$\text{and } T(e_2) = (3, 0, 2)$$

$$\text{or set } (x, y) = ae_1 + be_2$$

$$T(x, y) = aT(e_1) + bT(e_2)$$

$$= x - y(2, 3, 1) + y(3, 0, 2) = (2x - 2y + 3y), 3(x - y), (x - y + 2y)$$

$$T(x, y) = (2x + y, 3x - 3y, x + y)$$

20.(D) $\text{Arg } z(1 + i) = \text{Arg } z + \text{Arg } (1 + i)$

$$= \text{Arg } z + \tan^{-1} 1$$

$$= \text{Arg } z + 45^\circ$$

So, the multiplication of complex number z by $(1 + i)$ rotates vector to z in the complex plane by an angle 45° anticlockwise.

21. 4

Here, we will write

$$(\sqrt{3} + i)^{100} = 2^{100} \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^{100}$$

When we change $\frac{\sqrt{3}}{2} + \frac{i}{2}$ into polar form, we get

$$\text{Let } z = \frac{\sqrt{3}}{2} + \frac{i}{2} = r(\cos\theta + i\sin\theta)$$

$$\text{Then } r\cos\theta = \frac{\sqrt{3}}{2} \text{ and } r\sin\theta = \frac{1}{2}$$

Now, squaring and adding these, we get

$$r^2(\cos^2\theta + \sin^2\theta) = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$r^2 = \frac{3}{4} + \frac{1}{4} = 1$$

$$\text{or } r = 1$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$

\therefore The value of θ , such that $-\pi < \theta < \pi$ and satisfying both the above equations is given by

$$\theta = \frac{\pi}{6}$$

$$\therefore \text{Required polar form} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$$

$$\begin{aligned} \therefore (\sqrt{3} + i)^{100} &= 2^{100} \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \right)^{100} \\ &= 2^{100} \left(\cos\frac{100\pi}{6} + i\sin\frac{100\pi}{6} \right) \end{aligned}$$

$$\begin{aligned}
 &= 2^{100} \left(\cos \frac{50\pi}{3} + i \sin \frac{2\pi}{3} \right) \\
 &= 2^{100} \left\{ \cos \left(8 \times 2\pi + \frac{2\pi}{3} \right) + i \sin \left(8 \times 2\pi + \frac{2\pi}{3} \right) \right\} \\
 &= 2^{100} \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\} \\
 (\sqrt{3} + i)^{100} &= 2^{100} \left(-\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 2^{100} \left(-\frac{1+i\sqrt{3}}{2} \right) \\
 &= 2^{99} (-1+i\sqrt{3})
 \end{aligned}$$

Comparing with $2^{99}(a+ib)$, we get

$$a = -1, b = \sqrt{3}$$

$$\begin{aligned}
 \therefore a^2 + b^2 &= (-1)^2 + (\sqrt{3})^2 \\
 &= 1 + 3 = 4
 \end{aligned}$$

Short method

$$\frac{1}{2^{99}}(\sqrt{3} + i)^{100} = (a + ib)$$

$$2 \left[\frac{\sqrt{3}}{2} + i \frac{1}{2} \right]^{100} = (a + ib)$$

$$\Rightarrow 2 \left[\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right]^{100} = (a + ib)$$

$$\Rightarrow 2 \left[\cos \frac{100\pi}{6} + i \sin \left(\frac{100\pi}{6} \right) \right] = a + ib$$

$$\Rightarrow a = 2 \cos \frac{50\pi}{3}, b = 2 \sin \left(\frac{50\pi}{3} \right)$$

Now $a^2 + b^2 = 4$ **Ans.(D)**

22. 25

Here, A, B, C are three published newspapers.

\therefore Public read A newspaper $n(A) = 42\%$

Public read B newspaper $n(B) = 51\%$

Public read C newspaper $n(C) = 68\%$

Public read newspaper A and B both $n(A \cap B) = 30\%$

Public read newspaper B and C both $n(B \cap C) = 28\%$

Public read newspaper A and C both $n(A \cap C) = 36\%$

Public who do not read any paper = 8%

Let the persons who read all the three papers = x

$$\begin{aligned} \therefore n(A \cup B \cup C) &= n(A) + n(B) + n(A \cap B) \\ &\quad - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C) \end{aligned}$$

$$92 = 41 + 51 + 68 - 30 - 28 - 36 + x$$

or, $92 = 161 - 94 - 161 + 94$

$$\therefore x = 92 - 161 + 94$$

$$x = 25$$

23. -7

Here, It is given that the set Z of integers is group under the binary operation * defined as $m * n = m + n + 1$, $m, n \in Z$.

\therefore Identity element : let e be the identity element in Z for binary operation *, then

$$m * e = m = e * m \quad \forall m \in N$$

$$\Rightarrow m * e = m$$

$$\Rightarrow m + e + 1 = m$$

$$\Rightarrow e = -1$$

Inverse element : Let m be the inverse of the elements, then

$$m * 5 = -1 = 5 * m$$

$$\Rightarrow m + 5 + 1 = -1$$

$$\Rightarrow m = -1 - 6 = -7$$

24.(A) Here, it is given that in an abelian group, the order of an element a is 4 and the order of an element b is 3.

$$\therefore \text{Order of } a \text{ is } 4 \therefore a^4 = e$$

Where e is the identity of the group and order of b is 3

$$\therefore b^3 = e$$

\therefore Group is abelian, then

$$(ab)^{14} = a^{14} \cdot b^{14}$$

$$= a^{12} \cdot a^2 \cdot b^{12} \cdot b^2$$

$$= (a^4)^3 \cdot a^2 \cdot (b^3)^4 \cdot b^2$$

$$= e^3 \cdot a^2 \cdot e^4 \cdot b^2$$

$$= a^2 b^2 \cdot b b^{-1}$$

$$= a^2 b^3 b^{-1}$$

$$= a^2 e b^{-1}$$

$$= a^2 b^{-1}$$

25.(C) We know that the number of elements in a integral domain is either p or p^n .

∴ Only possible options are Z_{71} and Z_{97} containing prime number.

26.(C) A ring $(R, +, \cdot)$ whose all elements are idempotent is a division ring.

27.(D) Given that the function $f(9x)$ is differentiable at $x = 0$ i.e.

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(9h)^{\frac{1}{k}-1} \cos\left(\frac{1}{9h}\right)}{h} \\ &= \lim_{h \rightarrow 0} \cos\left(\frac{1}{9h}\right) \lim_{h \rightarrow 0} 9^{\left(\frac{1}{k}-1\right)} h^{\frac{1}{k}-2} \\ &= [\text{value b/w } -1 \text{ to } 1] 9^{\frac{1}{k}-1} \lim_{h \rightarrow 0} h^{\left(\frac{1}{k}-2\right)} \\ &= 0 \text{ possible only when } \frac{1}{k} - 2 > 0 \text{ i.e. } k < \frac{1}{2} \end{aligned}$$

28.(D) We can check for exact differential equation

$$y\left(1 + \frac{1}{x}\right) + \cos y + (x + \log x - x \sin y) \frac{dy}{dx} = 0 \quad \dots (i)$$

It can be written as

$$\left\{ y\left(1 + \frac{1}{x}\right) + \cos y \right\} dx + \{x + \log x - x \sin y\} dy = 0$$

Here $M = y\left(1 + \frac{1}{x}\right) + \cos y$

$$\therefore \frac{\partial M}{\partial y} = \left(1 + \frac{1}{x}\right) - \sin y$$

and $N = x + \log x - x \sin y$

$$\therefore \frac{\partial N}{\partial x} = \left(1 + \frac{1}{x}\right) - \sin y$$

So, we can see that

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$$

\(\therefore\) Differential equation is exact.

29.(C) Here, given differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x} \quad \dots (i)$$

which is a homogeneous differential equation.

\(\therefore\) Put $y = vx$

or,
$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{vx}{x} + \tan \frac{vx}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \tan v$$

$$\Rightarrow x \frac{dv}{dx} = \tan v$$

or,
$$\frac{dv}{\tan v} = \frac{dx}{x} \Rightarrow \cot v \, dv = \frac{dx}{x}$$

Now, integrating it, we get

$$\log \sin v = \log x + \log c$$

$$\Rightarrow \sin v = x c$$

$$\Rightarrow \sin \frac{y}{x} = cx$$

30.(B) Here, it is given that integers $a, b > 1$

Let c be the GCD of a and b , then

$$c = (a, b)$$

If there exist integers m, n such that

$$c = ma + nb$$

then the set of all integers of the form $ma + nb$ (m, n integers) includes their GCD but not LCM.

31.(D) Let $x \in A - (B \cup C)$

$$\Rightarrow x \in A \text{ and } x \notin (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \notin B, x \notin C)$$

$$\Rightarrow (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)$$

and $x \in (A - B)$ and $x \in (A - C)$

$$\Rightarrow x \in (A - B) \cap (A - C)$$

$$\therefore A - (B \cup C) \subseteq (A - B) \cap (A - C)$$

Similarly,

$$(A - B) \cap (A - C) \subseteq A - (B \cup C)$$

Hence, $A - (B \cup C) = (A - B) \cap (A - C)$

32.(D) Here $u = x^2 + y^2 + z^2$.

It is a homogeneous function of degree 2.

$$\therefore \text{By Euler's theorem } x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = nu = 2u$$

33.(A) Here, $T(x, y, z) = (x - y, y + 3z, x + 2y)$

Now, let $T(x, y, z) = (uvw)$

then $T^{-1}(uvw) = (x, y, z)$

So, $T(x, y, z) = (x - y, y + 3z, x + 2y) = (u, v, w)$

$$\Rightarrow x - y = u \quad \dots(i)$$

$$y + 3z = v \quad \dots(ii)$$

$$x + 2y = w \quad \dots(iii)$$

Taking equation (i) and (ii)

$$\dots(i)$$

$$\dots(ii)$$

$$\text{By subtracting } \begin{array}{r} x - y = u \\ -x + 2y = w \\ \hline 0 - 3y = u - w \end{array}$$

$$\text{or } y = -\frac{u+w}{3}$$

Putting this value in equation (i), we get

$$x - \left(-\frac{u+w}{3}\right) = u$$

$$\text{or } x + \frac{u+w}{3} = u$$

$$\text{or } x = u - \frac{(u+w)}{3}$$

$$x = \frac{3u - u - w}{3} = \frac{2u - w}{3}$$

and from equation (ii) $\left(\frac{-u+w}{3}\right) + 3z = v$

or $3z = v - \left(\frac{-u+w}{3}\right)$

$$3z = \frac{3v+u-w}{3}$$

$\therefore z = \frac{3v+u-w}{3 \times 3}$

$$= \frac{3v}{9} + \frac{u}{9} - \frac{w}{9}$$

$$z = \frac{1}{3} \left(v + \frac{u}{3} - \frac{w}{3} \right)$$

$\therefore T^{-1}(u \ v \ w) = (x, y, z)$

or $T^{-1}(u \ v \ w) = \left(\frac{1}{3}(2u+w), -\frac{u}{3} + v, \frac{u}{3} + v - \frac{w}{3} \right)$

or $T^{-1}(x, y, z) = \left(\frac{1}{3}(2x+z), -x+z, \frac{x}{3} + y - \frac{z}{3} \right)$

34. 0

Here linear mapping is $T : V \rightarrow V$, such that $T(A) = AB - BA$, where

$$B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2a & a+3b \\ 2c & c+3d \end{bmatrix}$$

and $BA = \begin{bmatrix} 2a+c & 2b+d \\ 3c & 3d \end{bmatrix}$

$$T(A) = AB - BA$$

$$= \begin{bmatrix} 2a & a+3b \\ 2c & c+3d \end{bmatrix} - \begin{bmatrix} 2a+c & 2b+d \\ 3c & 3d \end{bmatrix}$$

$$T(A) = \begin{bmatrix} -c & a+b-d \\ -c & c \end{bmatrix} = \begin{bmatrix} -c & a+b-d \\ 0 & c-a-b+d \end{bmatrix}$$

i.e. The matrix of T has two non zero rows

Hence rank (T) = 2

$$\Rightarrow \text{nullity of } T = 2 - 2 = 0$$

35.(A) Here, $i^{1/3}$

Let $z = i/3$

$$\Rightarrow z^3 = (i^{1/3})^3 = i$$

$$\Rightarrow z^3 - i = 0$$

$$\Rightarrow z^3 + i \cdot i^2 = 0$$

$$\Rightarrow z^3 + i^3 = 0$$

$$\Rightarrow (z+i)(z^2 + i^2 - iz) = 0 \quad [\because i^2 = -1]$$

$$\Rightarrow z^2 + i^2 - iz = 0$$

$$\Rightarrow z^2 - 1 - iz = 0$$

$$\Rightarrow z^2 - iz - 1 = 0$$

$$\therefore z = \frac{(-1) \pm \sqrt{(-i)^2 - 4 \cdot 1(-1)}}{2 \cdot 1}$$

$$= \frac{i \pm \sqrt{-1+4}}{2} = \frac{i \pm \sqrt{3}}{2}$$

$$\therefore z = -i, \frac{i \pm \sqrt{3}}{2}$$

36. 1

Here, given α is a complex number : $\alpha^2 + \alpha + 1 = 0$

$\therefore \alpha^2 + \alpha + 1 = 0$, we will consider $\alpha = \omega$

$\therefore \alpha^{31} = \omega^{31} = \omega^{31 \times 10 + 1} = \omega \quad [\because \omega^3 = 1]$

37.(D) Identity element of the group is $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Now $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Hence, $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ is its own inverse.

\therefore This statement is correct.

Ans. (D)

38.(D) Let x_1, x_2, \dots, x_n be a random sample of n observations from a population and probability density function is defined by

$$f(x, \theta) = \begin{cases} \frac{1}{2} e^{-(x-\theta)}, & x \geq \theta \\ \frac{1}{2} e^{(x-\theta)}, & x < \theta \end{cases}$$

it can be written as $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}$

The Likelihood function is defined by

$$L = \prod_{i=1}^n f(x_i, \theta) = \frac{1}{2^n} \exp\left[-\sum_{i=1}^n |x_i - \theta|\right]$$

$$L = \frac{1}{2^n} \exp[-n|\bar{x}_i - \theta|]$$

$$\log L = -2n \log 2 - n |\bar{x}_i - \theta|$$

The Likelihood equations for estimating θ . given

$$\frac{\partial}{\partial \theta} \log L = 0 = n$$

which is obviously inadmissible

Then we try to locate MLE' for θ by maximizing L directly

L is maximum $\Rightarrow \log L$ is maximum

$\log L$ is maximum if $(\bar{x} - \theta)$ is minimum which is so if θ maximum

if $x_{(1)} x_{(2)} \dots x_{(n)}$ is ordered sample from

This population then

39.(D) Since, we know that the ring of integers modulo P is an integral domain if P is prime and if P is not prime. So, here ring of integers (mod 6) is not an integral domain.

40.(A) Given that n is the number of elements in a field, and we know that the ring of modulo $(\mathbb{Z}_p, +, \cdot)$ is a field if and only if p a prime ($p = 2, 3, 5, 7, 11, \dots$)

i.e. order of the field is a prime no.

but, if $n = 4k + 1$ where $k \in \mathbb{N}$ then

$n = 5, 9, 13, \dots$ where 9 is not prime

i.e. n is not equal to a prime of the form $4k + 1$ and also we know that no. of elements in a field is either prime or some power of prime.

41.(B) Here, given differential equation is

$$(xp - y)^2 = p^2 - 1$$

$$\Rightarrow x^2p^2 + y^2 - 2xyp = p^2 - 1$$

$$\Rightarrow (x^2 - 1)p^2 - 2xyp + y^2 + 1 = 0$$

which is quadratic in p .

\therefore Singular solution will be

$$B^2 - 4A = 0$$

$$\Rightarrow (-2xy)^2 - 4(x^2 - 1)(y^2 + 1) = 0$$

$$\Rightarrow 4x^2y^2 - 4x^2y^2 + 4y^2 - 4x^2 + 4 = 0$$

$$\Rightarrow 4y^2 - 4x^2 = -4$$

$$\Rightarrow 4x^2 - 4y^2 = 4s$$

$$\Rightarrow x^2 - y^2 = 1$$

42.(C) Here, given differentiation equation is

$$y = \frac{2}{3}x \frac{dy}{dx} - \frac{2}{3x} \left(\frac{dy}{dx} \right)^2, x > 0$$

$$\text{or, } y = \frac{2}{3}P - \frac{2}{3x}P^2 \quad \left[\because P = \frac{dy}{dx} \right]$$

$$\text{or, } \frac{2}{3x}P^2 - \frac{2}{3}xP + y = 0$$

which is quadratic in P .

\therefore Singular solution is given $B^2 - 4AC = 0$

$$\Rightarrow \left(\frac{2}{3}x\right)^2 - 4\left(\frac{2}{3x}\right)(y) = 0$$

$$\Rightarrow \frac{4}{9}x^2 - \frac{8y}{3x} = 0$$

$$\Rightarrow \frac{4}{9}x^2 = \frac{8y}{3x}$$

$$\Rightarrow \frac{x^2}{3} = \frac{2y}{x}$$

$$\Rightarrow x^3 = 6y \text{ or, } y = \frac{x^3}{6}$$

43.(A) Here, given parabolas is

$$y^2 = 4a(x + a) \quad \dots (i)$$

Now, differentiating it w.r.t. x, we get

$$2y \frac{dy}{dx} = 4a$$

or, $y \frac{dy}{dx} = 2a \quad \dots (ii)$

Now, eliminating a from Eqs. (i) and (ii), we get

$$y^2 \left(\frac{dy}{dx}\right)^2 + 2xy \frac{dy}{dx} - y^2 = 0$$

or, $y^2 p^2 + 2xP - y^2 = 0$

$$\Rightarrow yP^2 + 2xP - y = 0$$

which is self orthogonal as by putting

$$P = \frac{1}{p}, \text{ we get the same equation}$$

$$\text{as } \Rightarrow y\left(\frac{1}{p}\right)^2 + 2x\frac{1}{p} - y = 0$$

$$\Rightarrow y \cdot \frac{1}{p^2} + \frac{2x}{p} - y = 0$$

$$\Rightarrow y + 2xp - yp^2 = 0$$

$$\Rightarrow yP^2 - 2xP - y = 0$$

\therefore The orthogonal trajectories of system of parabolas $y^2 = 4a(x + a)$ is itself.

i.e., $y^2 = 4a(x + a)$

44.(A) Here, it is given that the function $f : n \rightarrow Z$ defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -n/2, & \text{when } n \text{ is even} \end{cases}$$

Let n_1, n_2 (odd) $\in N$ then $f(n_1) = f(n_2)$

$$\Rightarrow \frac{n_1 - 1}{2} = \frac{n_2 - 1}{2}$$

$$\Rightarrow n_1 = n_2$$

and n_1, n_2 (even) $\in N$,

then $f(n_1) = f(n_2)$

$$\Rightarrow \frac{n_1}{2} = -\frac{n_2}{2}$$

$$\Rightarrow n_1 = n_2$$

\therefore f is one-one.

Again let t (odd) $\in Z$, then

$$f(n) = t \Rightarrow \frac{n-1}{2} = t$$

$$\Rightarrow n = 2t + 1$$

clearly $n = 2t + 1 \in \mathbb{N} \forall t \in \mathbb{Z}$

and $t(\text{even}) \in \mathbb{Z}$, then

$$f(n) = t \Rightarrow \frac{-n}{2} = t$$

$$\Rightarrow n = -2t$$

clearly $n = -2t \notin \mathbb{N} \forall t \in \mathbb{Z}$.

$\therefore f$ is not onto

\therefore Hence, f is one-one but not onto.

45.(D) Here it is given that a, b are roots of the equation $ax^2 + bx + c = 0$

$$\therefore a + b - \frac{b}{a} = \text{and } \alpha\beta = \frac{c}{a}$$

$$\therefore \lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$$

It is in $\frac{0}{0}$ form

So, by L'Hospital's rule

$$\lim_{x \rightarrow \alpha} \frac{(2ax + b) \sin(ax^2 + bx + c)}{2(x - \alpha)}$$

$$\Rightarrow \lim_{x \rightarrow \alpha} \frac{(2ax + b)^2 \cos(ax^2 + bx + c) + 2a \sin(ax^2 + bx + c)}{2}$$

$$\Rightarrow \frac{(2a\alpha + b)^2}{2}$$

$$\Rightarrow \frac{a^2}{2} \left(2\alpha + \frac{b}{a} \right)^2$$

$$\Rightarrow \frac{a^2}{2} [2\alpha - (\alpha + \beta)]^2 \quad \left[\because \alpha + \beta = \frac{-b}{a} \right]$$

$$\Rightarrow \frac{a^2}{2} (\alpha - \beta)^2$$

46.(B) Here, it is given that

$$f(x) = \begin{cases} x^\alpha \cos 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$

\therefore We have

$$\begin{aligned} \text{R} \lim_{\alpha \rightarrow 0} f(0+h) &= \lim_{h \rightarrow 0} h^\alpha \cos \frac{1}{h} \\ &= \lim_{h \rightarrow 0} h^{\alpha-1} h \cos \frac{1}{h} \end{aligned}$$

$= 0$, if $\alpha - 1 \geq 0$ i.e., $\alpha \geq 1$ or $\alpha \geq 0$

Similarly,

$$\text{R} \lim_{h \rightarrow 0} f(0-h) = 0$$

and $f(0) = 0$

$\therefore f(x)$ is continuous at $x = 0$, if $\alpha > 0$

47. 3

$$\lim_{n \rightarrow \infty} \frac{2^{n+1} + 3^{n+1}}{2^n + 2^n} = \lim_{n \rightarrow \infty} \frac{3^{n+1} \left[\left(\frac{2}{3}\right)^{n+1} + 1 \right]}{3^n \left[\left(\frac{2}{3}\right)^n + 1 \right]} = 3$$

48.(A) Probability distribution function of exponential distribution is given by

$$f(x, \theta) = \begin{cases} \theta e^{-\theta x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

and its mean = $\frac{1}{\theta}$

$$\Rightarrow \theta = \frac{1}{2}$$

so pdf $f\left(x, \frac{1}{2}\right) = \begin{cases} \frac{1}{2} e^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

$$P(Y \geq t) = \int_t^{\infty} f\left(x, \frac{1}{2}\right) dx = \int_t^{\infty} \frac{1}{2} e^{-x/2} dx = \frac{1}{2} \left[\frac{e^{-x/2}}{-1/2} \right]_t^{\infty} = e^{-t/2} \text{ ans.}$$

49. 2

$$E(Y) = \frac{1}{2} \int_0^{\infty} x e^{-x/2} dx$$

$$= \frac{1}{2} \left[\frac{x e^{-x/2}}{-1/2} - 4e^{-x/2} \right]_0^{\infty}$$

$$= \frac{4}{2} = 2$$

50.(D) Let X and Y be independent poisson variates with parameters λ_1 and λ_2

Then X + Y is also poisson variate with parameter $\lambda_1 + \lambda_2$

$$\begin{aligned}
 p[X = r | (X + Y = n)] &= \frac{p(X = r \cap X + Y = n)}{p(X + Y = n)} \\
 &= \frac{p(X = r \cap Y = n - r)}{p(X + Y = n)} \\
 &= \frac{p(X = r)p(Y = n - r)}{p(X + Y = n)} \quad [\text{since } X, Y \text{ independent}] \\
 &= \frac{e^{-\lambda_1} \frac{\lambda_1^r}{r!} \cdot e^{-\lambda_2} \frac{(\lambda_2)^{n-r}}{(n-r)!}}{e^{-(\lambda_1 + \lambda_2)} \frac{(\lambda_1 + \lambda_2)^n}{n!}}
 \end{aligned}$$

where $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ $q = 1 - p$

Hence the conditional distribution of X given X + Y = n is a binomial distribution with parameters n and $p = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

51.(C) Regression equation of X on X + Y is given by

$$= \bar{X} + r \frac{\sigma_x}{\sigma_{x+y}} (x + y - \overline{x + y})$$

Since X follows poisson distribution with parameter λ_1 so mean $\bar{X} = \lambda_1$

$$SD = X = \sqrt{\lambda_1}$$

Also X + Y follows poisson distribution with parameter $\lambda_1 + \lambda_2$

Then mean = $\lambda_1 + \lambda_2$

$$\text{SD pf } X + Y = \sqrt{\lambda_1 + \lambda_2}$$

and r is correlation coefficient between X and X + Y is given by

$$\begin{aligned} r &= \frac{\text{Cov}(x, x+y)}{\sigma_x \sigma_{x+y}} \\ &= \frac{\text{Cov}(x, x) + \text{cov}(x, y)}{\sigma_x \sigma_{x+y}} \end{aligned}$$

Since X, Y are independent

$$\Rightarrow \text{Cov}(x, y) = 0$$

$$\Rightarrow r = \frac{\text{Cov}(x, x)}{\sigma_x \sigma_{x+y}}$$

$$\begin{aligned} \text{now Cov}(x, x) &= E(x^2) - (E(x))^2 \\ &= \lambda_1 \end{aligned}$$

$$\text{so } r = \frac{\lambda_1}{\sqrt{\lambda_1} \sqrt{\lambda_1 + \lambda_2}}$$

$$\begin{aligned} \Rightarrow \text{regression} &= \lambda_1 + \frac{\lambda_1}{\sqrt{\lambda_1} \sqrt{\lambda_1 + \lambda_2}} \frac{\lambda_1}{\sqrt{\lambda_1 + \lambda_2}} (x + y(\lambda_1 + \lambda_2)) \\ &= \lambda_1 + \frac{\lambda_1}{\lambda_1 + \lambda_2} (x + y) - \frac{(\lambda_1 + \lambda_2)\lambda_1}{\lambda_1 + \lambda_2} \\ &= \frac{\lambda_1}{\lambda_1 + \lambda_2} (x + y) \end{aligned}$$

Hence the answer.

52. 2

Since $f(z) = \cos z - \frac{\sin z}{z}$

to find zeros of $f(z)$ put $f(z) = 0$

$$\Rightarrow \cos z - \frac{\sin z}{z} = 0$$

$$\Rightarrow \left[1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots \right] - \left[\frac{z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots}{z} \right] = 0$$

$$\Rightarrow \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots \right) - \left(1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \dots \right) = 0$$

$$\Rightarrow z^2 \left[\frac{1}{3!} - \frac{1}{2!} \right] + \dots = 0$$

so $z = 0$ is a zero of $f(z)$ of order 2

53. 3

$$\frac{g(z)}{zf(z)} = \frac{\sinh z}{z \cos z - \sin z}$$

To find poles $z \cos z - \sin z = 0$

$$\Rightarrow \left[z - \frac{z^3}{2!} + \frac{z^5}{4!} - \dots \right] - \left[z - \frac{z^3}{3!} + \dots \right] = 0$$

$$\Rightarrow z^3 \left[\frac{1}{3!} - \frac{1}{2!} \right] + z^5 \left[\frac{1}{5!} - \frac{1}{4!} \right] = 0$$

$\Rightarrow \frac{g(z)}{zf(z)}$ have pole at $z = 0$ of order 3

54.(A) Given $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \dots(1)$

it is a laplace eq. of two dimensional let is's sol. is

$$U(x, y) = X(x) Y(y) \quad \dots(2)$$

then from (1) and (2)

$$\frac{1}{X(x)} \frac{d^2 X}{dx^2} + \frac{1}{Y(y)} \frac{d^2 Y}{dy^2} = 0$$

$$\Rightarrow \frac{1}{X(x)} \frac{d^2 X}{dx^2} = -\frac{1}{Y(y)} \frac{d^2 Y}{dy^2}$$

Since L.H.S. and R.H.S. both are independent to each other then each equal to same constant say λ^2 then

$$\frac{d^2 X}{dx^2} = \lambda^2 X(x) \quad \dots(3)$$

and $\frac{d^2 Y}{dy^2} = -\lambda^2 Y(y) \quad \dots(4)$

sol. of (3) $X(x) = C_1 \cosh \lambda x + C_2 \sinh \lambda x \quad \dots(5)$

sol. of (4) $Y(y) = d_1 \cos \lambda y + d_2 \sin \lambda y \quad \dots(6)$

Now using the boundary conditions.

$$u(x, 0) = 0 \Rightarrow X(x) Y(0) = 0 \Rightarrow Y(0) = 0 \quad [X(x) \neq 0]$$

$$u(x, \pi) = 0 \Rightarrow X(x) Y(\pi) = 0 \Rightarrow Y(\pi) = 0 \quad [X(x) \neq 0]$$

$$u(0, Y) = 0 \Rightarrow X(0) Y(y) = 0 \Rightarrow X(0) = 0 \quad [Y(y) \neq 0]$$

if $Y(0) = 0 \Rightarrow \boxed{d_1 = 0}$

if $Y(\pi) = 0 \Rightarrow d_2 \sin \lambda \pi = 0$

$\Rightarrow d_2 \neq 0 \quad \sin \lambda \pi = 0$

$$\lambda\pi = n\pi$$

$$\lambda = n$$

So $Y_n(y) = d_n \sin ny$... (7)

again $X(0) = 0 \Rightarrow C_1 = 0$

So, $X_n(x) = C_n \sinh nx$ [$\lambda = n$]

Hence $U_n(x, y) = X_n(x)Y_n(y)$

$$U_n(x, y) = C_n \sinh nx \cdot d_n \sin ny$$

So, $U_n(x, y) = \sum_{n=1}^{\infty} u_n(x, y)$
 $= \sum_{n=1}^{\infty} a_n \sinh nx \sin ny$ [$a_n = c_n d_n$]

55.(C) Since $u_n(x, y) = \sum_n a_n \frac{e^{nx} - e^{-nx}}{2} \sin ny$

taking $n = 1$

$$a \left(\frac{e^x - e^{-x}}{2} \right) \sin y = u(x, y)$$

$$u_x(x, y) = \frac{a}{2} (e^x + e^{-x}) \sin y$$

$$u_x(\pi, y) = \frac{a}{2} (e^\pi + e^{-\pi}) \sin y = \sin y$$

$$\Rightarrow a_1 = \frac{2}{e^\pi + e^{-\pi}}$$

$$\text{so } u\left(x, \frac{\pi}{2}\right) = \frac{2}{e^{\pi} + e^{-\pi}} \frac{e^x - e^{-x}}{2} \sin \frac{\pi}{2} = \frac{e^x - e^{-x}}{e^{\pi} + e^{-\pi}}$$

Hence [C] is correct option.

- 56.(A)** By the alligation rule, we find that wine containing 30% of spirit and wine containing 12% spirit should be mixed in the ration 1 : 2 to produce a mixture containing 18% spirit.

This means that 1/3 of the butt of sherry was left; in other words the butler drew out 2/3 of the butt.

Hence 2/3 of the butt was stolen.

- 57.(A)** There are five prime number between 30 and 50.

They are 31, 37, 41, 43 and 47.

$$\therefore \text{ Required average} = \frac{(31+37+41+43+47)}{5} = \frac{199}{5} = 39.8$$

- 58.(D)** The pattern is $8^2, 12^2, 16^2, 20^2,$

$$\therefore \text{ Missing number} = 24^2 = 576.$$

- 59.(C)** Pattern is .3, 3., 1.5., 1, 1.5. 15., 1.5, 2.25. 2,

$$\therefore \text{ Missing number} = 120.6 = 720.$$

- 60.(D)** The meaning of Voracious is excessively greedy and grasping and the word ravenous provides the same meaning. So the synonym of Voracious is ravenous.

- 61.(C)** The meaning of Abortive is Failing to accomplish an intended result and the word unsuccessful provides the same meaning. So the synonym of abortive is unsuccessful.

- 62.(C)** Fragile is the opposite of hardy. Amateur is the opposite of professional.

- 63.(D)** Chapters make up a book. Rooms make up a house.

- 64.(A)** Bias means - A partiality that prevents objective consideration of an issue or situation.

and the word bias provides the same meaning. So the synonym of bias is prejudice.



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65.(A) Antonym of Coincidence is incidence.

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