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## PHYSICAL SCIENCE

## SAMPLE THEORY

* BLACK - BODY AND BLACK - BODY RADIATION
* PLANCK'S RADIATION LAW

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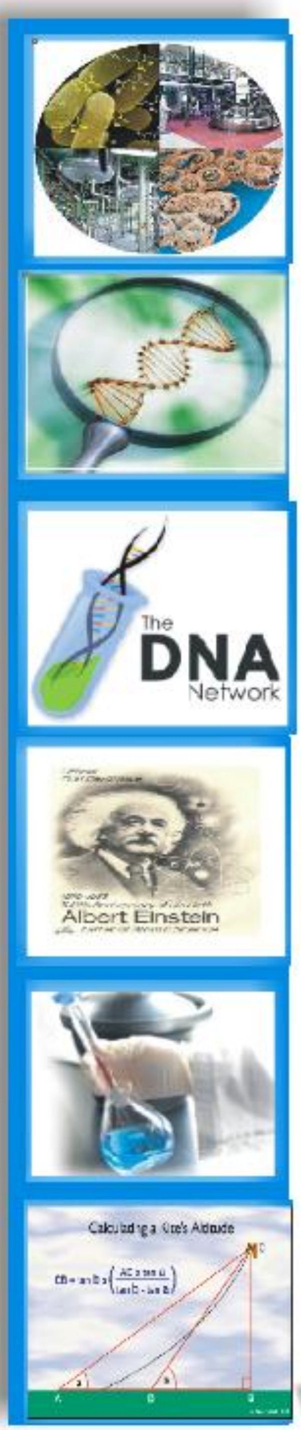


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For IIT-JAM, JNU, GATE, NET, NIMCET and Other Entrance Exams

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**Black - Body AND Black - Body Radiation**

A perfectly black body is one which absorbs all the heat radiations, of whatever wavelength, incident on it. It neither reflects not transmits any of the incident radiations and, therefore, appears black whatever be the color of incident radiation.

Let a black - body be placed in an isothermal enclosure. The body will emit the full radiation of the enclosure after it is in thermal equilibrium with the enclosure. These radiations are independent of the nature of the substance. Clearly the radiation from an isothermal enclosure is identical with that from a black - body at the same temperature. Therefore, the heat radiations in an isothermal enclosure are termed as black body radiation.

In practice, no substance possesses strictly the properties of a black - body. Lamp - black and the platinum black are the nearest approach to a black - body. However, the bodies showing close approximation to a perfectly black - body can be constructed.

**1. Ferry's black-body.** It consists of a hollow copper sphere blackened inside with a small fine hole O in the surface. When the radiations enter the hole, they suffer a number of reflections at the inner walls of the sphere until it is completely absorbed.

To avoid direct reflection of the radiation from the inner surface, a pointed projection is made in front of the hole as in **Fig. 1**(a). Thus the small hole acts as a black-body absorber. When this sphere is placed in a bath at a fixed temperature, the heat radiations come out of the hole as shown in **Fig. 1**(b). It is to be noted that only the hole and not the walls of the sphere, acts as a black-body radiator.

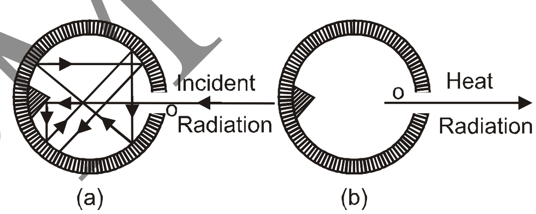


**Fig. 1**

**2. Wien's black-body.** Wien constructed a black body in the form of cylinder. This black body is commonly used in these days. It consists of a hollow metallic cylinder fitted with a heating coil wound around it. The inner surface of the cylinder is coated with black lamp. The cylinder is placed in concentric porcelain tubes. The temperature is measured with the help of thermocouple arrangement. Heat radiation emerges out of holes. This hole will act as black body radiator.

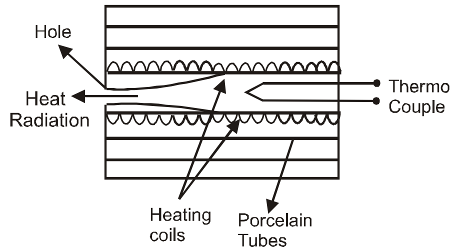


**Fig. 2**

**PLANCK'S RADIATION LAW**

Planck found an empirical formula to explain the experimentally observed distribution of energy in the spectrum of a black - body. The formula may be deduced using the following postulates.

1. A black body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators of the molecular dimensions; which cannot have any value of energy; but only energies given by

$$E = n h\nu$$

Where $\nu$ is the frequency of the oscillator, h is the Planck's constant and n is a number that can take only integral values, i.e

$$n = 0, 1, 2, 3.........$$

2. The oscillators cannot radiate or absorb energy continuously; but an oscillator of frequency $\nu$ can only radiate or absorb energy in units or quanta of magnitude $h\nu$. This assumption is the most revolutionary in character. In simple words this states that the exchange of energy between radiation and matter cannot take place continuously but are limited to discrete set of values 0, $h\nu$, $2h\nu$, ....$nh\nu$, i.e. , in multiples of some small unit, called the **quantum**.

The average energy of a Planck's oscillator is given by

$$\bar{\varepsilon} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

The number of resonators per unit volume in the frequency range □ and $\nu + d\nu$ is given by

$$N = \frac{8\pi\nu^2}{c^3} d\nu$$

The energy density belonging to range $d\nu$ can be obtained by multiplying the average energy of a Planck's oscillator by the number of resonators per unit volume, in the frequency range $\nu$ and $\nu + d\nu$ , i.e

$$E_\nu \, d\nu = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1} d\nu = \frac{8\pi \, h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1} d\nu$$

Where $E_\nu \, d\nu$ is the energy density, i.e total energy per unit volume belonging to the range $d\nu$.

The above law was found by Planck empirically and after his name is called Planck's radiation law.

The energy density $E_\lambda \, d\lambda$ belonging to range $d\lambda$ can be obtained by using the relation $\nu = \frac{c}{\lambda}$ and hence $|d\nu| = \left| -\frac{c}{\lambda^2} d\lambda \right|$,i.e

$$E_\nu \, d\nu = \frac{8\pi \, h}{c^3} \left(\frac{c}{\lambda}\right)^3 \cdot \left(\frac{1}{e^{hc/\lambda kT} - 1}\right) \left(-\frac{c}{\lambda^2} d\lambda\right) = \frac{8\pi \, hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$$

which gives the energy density for wavelength range $\lambda$ and $\lambda + d\lambda$ in the spectrum of a black body.

**Wein's Law and Rayleigh - Jean's Law in Relation to Planck's Law**

Planck's formula is given by

$$E_\nu \, d\nu = \frac{8\pi \, hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$$

**For shorter wavelength** $e^{hc/\lambda kT}$ becomes large compared to unity and hence the Planck's law reduces to unity and hence the Planck's law reduces to

$$E_\nu \, d\nu = \frac{8\pi \, hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} d\lambda = \frac{8\pi \, hc}{\lambda^5} . e^{-hc/k\lambda T} d\lambda$$

Which is **Wein's law.**

**For longer wavelengths** $e^{hc/\lambda kT}$ may be approximated to $\left(1 + \frac{hc}{\lambda kT}\right)$ and hence Plank's law reduces to

$$E_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{(1+hc/\lambda kT-1)}d\lambda = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{hc/\lambda kT}d\lambda$$

$$= \frac{8\pi hc}{\lambda^5}\cdot\frac{\lambda kT}{hc}d\lambda$$

$$= \frac{8\pi kT}{\lambda^5}d\lambda$$

Which is **Rayleigh jean's law.**

**SOLVED EXAMPLES**

**1.** For Black-body radiation what is the Heat Capacity of system at constant volume–

(1) $\frac{32\pi^5K^4T^3V}{15(hc)^3}$

(2) $\frac{8\pi^5(KT)^4}{15(hc)^3}$

(3) $\frac{32\pi^5K^4T^3}{15(hc)^3}$

(4) $\frac{32\pi^4K^4T^3V}{45(hc)^3}$

**1.(1)** We know by stefan's law the radiation energy density is $E = \frac{8\pi^5(KT)^4}{15(hc)^3}$

$\therefore$ So radiation energy (heat) $U = VE = \frac{8\pi^5(KT)^4V}{15(hc)^3}$

$V \to$ volume of system (black body)

$\therefore$ We know that at constant volume heat capacity is

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{8\pi^5K^4V(4T^3)}{15(hc)^3}$$

$$C_V = \frac{32\pi^5K^4VT^3}{15(hc)^3}$$

**2.** Black –body radiation at temperature T, i.e. in thermal equilibrium with the walls of a cavity at temperature T, is caused to expand isothermally from a volume $V_i$ to a volume $V_f$ then what is the expression of heat supplied to the walls

(1) $\frac{8}{15}\frac{\pi^5(KT)^4}{(hc)^3}\left(V_f/V_i\right)$

(2) $\frac{8}{15}\frac{\pi^5(KT)^4}{(hc)^3}\left(V_f - V_i\right)$

(3) $\frac{16}{45}\frac{\pi^5(KT)^4}{(hc)^3}\left(V_f + V_i\right)$

(4) $\frac{32\pi^5(KT)^4}{45(hc)^3}\left(V_f - V_i\right)$

**2.(4)** From first law of the thermodynamics

$dQ = dU + PdV$

du is internal energy of black body

So, $dU = u(T)\,dV = u(T)\,(V_f - V_i)$

$u(T) \rightarrow$ energy radiated from black body per unit volume

$\therefore$ radiation pressure $p = \frac{1}{3}u(T)$

So, $dQ = u(T)\,(V_f - V_i) + \frac{1}{3}u(T)(V_f - V_i)$

So, $dQ = u(T)\,(V_f - V_i) +$

$dQ = \frac{4}{3}u(T)(V_f - V_i)$

from Stefan's law radiation energy

$u(T) = \frac{8\pi^5(KT)^4}{15(hc)^3}$

So, heat supplied to the walls is

$dQ = \frac{32\pi^5(KT)^4}{45(hc)^3}(V_f - V_i)$

**3.** The maximum value of the emissive power of a black body corresponds to a frequency $v_1$ at $T_1$ K and $v_2$ at $T_2$K. It follows from Planck's law of radiations that:

(1) $\frac{T_1}{T_2} = \frac{v_1}{v_2}$

(2) $\frac{T_2}{T_1} = \frac{v_1}{v_2}$

(3) $\frac{T_1^2}{T_2^2} = \frac{v_1}{v_2}$

(4) $\frac{T_2^2}{T_2^2} = \frac{v_1}{v_2}$

**3.(1)** From Wien's displacement law,

$\lambda_m T$ = constant

$\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$ ...(1)

Again, $\frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1}$ ...(2)

From (1) and (2), we have $\frac{T_1}{T_2} = \frac{v_1}{v_2}$