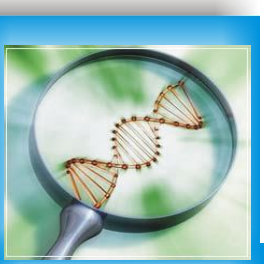

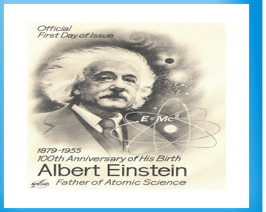

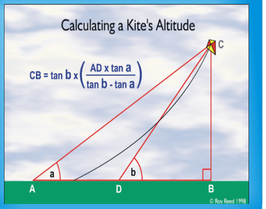

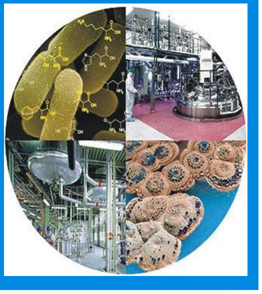

CSIR NET - ENGINEERING SCIENCE

SAMPLE THEORY

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1. ENGINEERING MATHEMATICS & APTITUDE

CAUCHY'S THEOREM

If f (z) is an analytic function of z and if f ' (z) is continuous at each point within and on a closed contour C, then $\int_C f(z)dz = 0$.

Proof. Let D be the region which consists of all points within and on the contour C. If P (x, y) ,Q (x, y), $\frac{\partial Q}{\partial x}, \frac{\partial P}{\partial y}$ are all continuous functions of x and y in the region D, then Green's theorem states that $\int_C (Pdx + Qdy) = \int_D\int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy.$

Since f (z) = u + iv is continuous on the simple curve C and f' (z) exists and is continuous in D, therefore u, v, u_x, u_y, v_x, v_y, are all continuous in D. The conditions of Green's theorem are thus satisfied. Hence

$$\int_C f(z)dz = \int_C (u + iv)(dx + idy)$$

$$= \int_C (udx - vdy) + i\int (v\ \ dx + u\ dy)$$

$$= -\int\int_D \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy + i\int\int_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dxdy = 0 \qquad \text{[By Green's theorem]}$$

$$= -\int\int_D \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy + i\int\int_D \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dxdy \qquad \text{[By Cauchy-Riemann equations]}$$

Hence $\int_C f(z)dz = 0$.

POLYMERS

Polymers is a material exist in countless forms and numbers because of a very large number and types of atoms present in their molecules.

Polymer is classified on the basis of structure, physical properties (mechanical behavior, thermal characteristic etc.

Natural and Synthetic Polymers

1. Those isolated form natural materials are called natural polymers. Typical examples are. cotton, silk, wool and rubber. Cellophane, cellulose rayon, leather and so on are, in fact chemical modifications of natural polymers.

2. Polymers synthesized form low molecular weight compounds are called synthetic polymers. Typical examples are: polyethylene, PVC, nylon and terylene.

Organic and Inorganic Polymers

1. A polymer whose backbone chain is essentially made of carbon atoms is termed an organic polymer.

2. The molecules of inorganic polymers, on the other hand, generally contain no carbon atom in their chain backbone. Glass and silicone rubber are examples of inorganic polymers.

Thermoplastic and Thermosetting Polymers

1. The process of heating reshaping and retaining the same on cooling can be repeated several time. Such polymers, that soften on heating and stiffen on cooling, are termed ‘thermoplastics’. Polyethylene, PVC nylon and sealing wax are examples of thermoplastic polymers.

2. Some polymers, on the other hand, undergo some chemical change on heating and convert themselves into an infusible mass.. They are like the yolk of the egg, which on heating sets into a mass, and, once set, cannot be reshaped. Such polymers, that become an infusible and insoluble mass on heating, are called ‘thermosetting’ polymers.

Plastics, Elastomers, Fibers and Liquid Resins:

1. When, for instance, a polymer is shaped into hard and tough utility articles by the application of heat and pressure, it is used as a ‘plastic. Typical examples are polystyrene, PVC and polymethyl methancrylate.

2. When vulcanized into rubbery products exhibiting good strength and elongation, polymers are used as ‘elastomers.’ Typical examples are natural rubbery, synthetic rubber, silicone rubber.

3. If drawn into long filament-like materials, whose length is at least 100 times its diameter, polymers are nylon and terylene.

4. Polymers used as adhesives, potting compounds, sealants, etc., in a liquid form are described as liquid resins.

2. COMPUTER SCIENCE & INFORMATION TECHNOLOGY

DEADLOCK

A set of precesses is deadlocked if each process in the set is waiting for an event that can only be caused by another process in the set. The events that we are mainly concerned with are resource acquisition and release. The resources may be physical resources (e.g. CPU, I/0 devices, memory) or logical resources (e.g. files semaphores or monitors).

Resources:

(a) There are three kinds of resources

– Sharable resources

– Consumable resource

(b) sharable resources can be used by more than one process at a time

(c) a consumable resource can only be used by one process and the resource gets “ used up”

(d) Deadlock can arise if 4 condition hold simultaneously

Mutual exclusion: only one process at a time can use a resource.

Hold and wait: a process holding at least one resource is waiting to acquired additional resources held by other processes.

No preemption: a resource can be released only voluntarily by the process holding it, after that process has completed it‘s task.

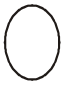
Circular wait: each process in a set is waiting for resource held by the next process in the set.

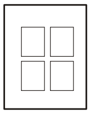
(e) Deadlock can be more precisely described in terms of directed graph called Resource Allocation Graph. This graph consists of two sets of nodes.

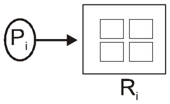
1. The set of all active processes in the system {P1, P2 ....Pn}

2. The set of all resource types in the system (R1 , R_2,....Rn)

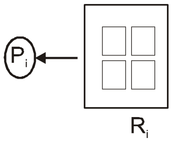
(f) Processes are represented by circles and resources are represented by squares.

Process 

Resource type with 4 instances 



P_i requests instances of R_j

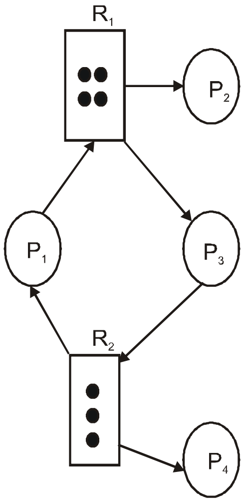


P_i is holding an instance of R_i

Resource Allocation Graph with a cycle but no deadlock:

Cycle $P_1 – R_1 – R_1 – P_3 – R_2 – P_1$

No deadlock, Process P4 may releases its instances of R_2. R_2 can then the allocated to P_3 thus breaking the cycle.



Basic facts:

If graph contains no cycle ⇒ no deadlock

If graph contains a cycle

– If there is only one instance per resource type, then deadlock occurs.

– If there are several instances per resource type, deadlock may occur.

Methods of handling deadlock:

Ensure that the system will never enter a deadlock state allow the system to enter a deadlock state and then recover ignore the problem. Assume that deadlock rarely occurs in the system so need to address it.

This method is used by many OS including UNIX

Deadlock prevention:

Ensure that at least one of the necessary condition does not hold.

Mutual exclusion: this must hold for non sharable resources.

Hold and wait: must guarantee that whenever a process requests a resource, it does not hold other resources.

No preemption: If process that is holding some resources requests another resource that cannot be immediately allocated to it, then all resources currently being held are released.

Circular wait: One way to ensure that this condition never holds is to impose a total ordering of all resource types and to require that each process requests resources in an increasing order of enumeration.

Deadlock avoidance:

Simplest and most useful model, requires that each process declares the maximum number of resources of each type that it may need.

The deadlock –avoidance algorithm dynamically examines the resource allocation state to ensure that there can never be a circular wait condition.

Resource allocation state is defined by the number of available and allocated resources and the maximum demands of the process.

Safe state:

When a process requests an available resource, the system must decide if immediate allocation leaves the system in a safe state.

A state is a safe if the system can allocate resources to each process in some order and still avoid a deadlock.

More formally a system is a in safe state if there exists a safe sequence of all processes.

3. ELECTRONICS

Multiplexers [Data Selectors]

The term ‘multiplex’ means “many into one.” Multiplexing is the process of transmitting a large number of information over a single line. A digital multiplexer (MUX) is a combinational circuit that selects one digital information from several sources and transmits the selected information on a single output line.

The multiplexer has several data-input lines and a single output line. The selection of a particular input line is controlled by a set of selection lines. The block diagram of a multiplexer with n input lines, m select signals and one output line is shown in Fig. The selection lines decide the number of input lines of a particular multiplexer. If the number of n input lines is equal to 2^m, then m select lines, two select lines are required; to select 1 of 8 input lines, three select line are required and so on.

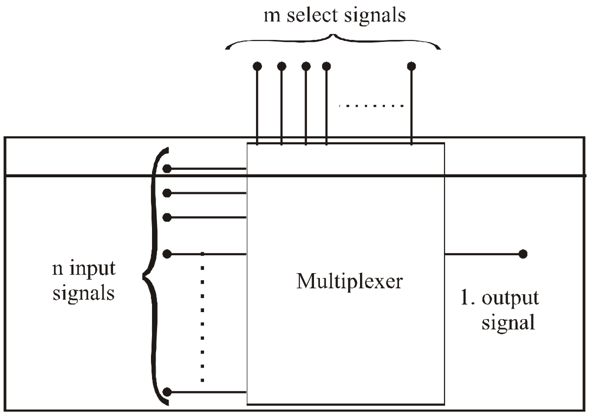


Fig. Block diagram of multiplexer

Basic 4-input MUX

The logic symbol of a 4-to -1 multiplexer is shown in Fig.(a) It has four data input lines (D_0 – D_3) , a single outputs line (Y) and two select lines (S_0 and S_1) to select one of the four input lines. The truth table for a 4-to-1 multiplexer is shown in Table.

From the truth table of Table a logical expression for the output in terms of the data input and the select inputs can be derived as follows:

The data output Y = data input D_0. if and only if $S_1 = 0$ and $S_0 = 0$

Therefore, $Y = D_0\bar{S}_1\bar{S}_0 = D_0\bar{0}.\bar{0} = D_0 1.1 = D_0$

The data output $Y = D_1$, if and only if $S_1 = 0$ and $S_0 = 1$.

Therefore, $Y = D_1\bar{S}_1S_0 = D_1$, when $S_1S_0 = 01$

Similarly, $Y = D_2S_1\bar{S}_0 = D_2$, when $S_1S_0 = 10$ and

$Y = D_3S_1S_0 = D_3$, when $S_1S_0 = 11$

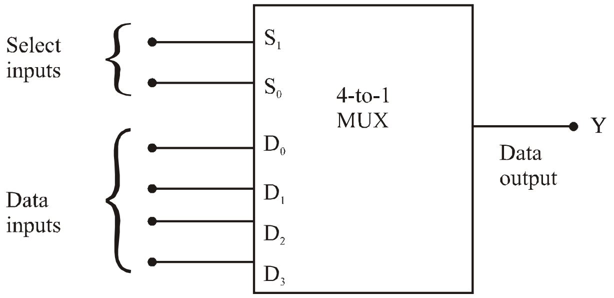
If the above terms are ORed, then the final expression for the data output is given by

$$Y = D_0\bar{S}_1\bar{S}_0 + D_1\bar{S}_1S_0 + D_2S_1\bar{S}_0 + D_3S_1S_0$$

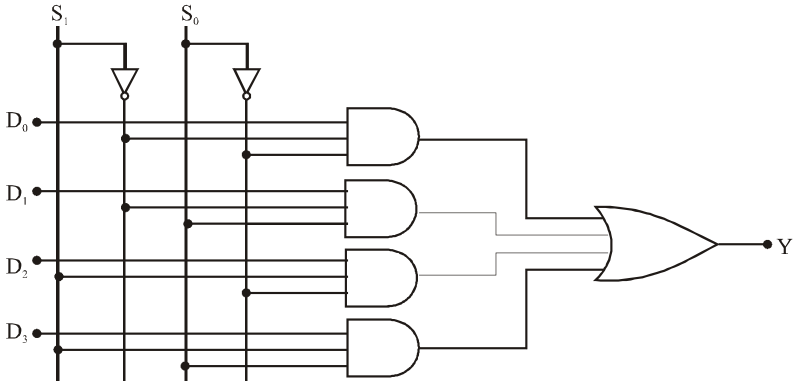
Using the above expression, the 4-to-1 multiplexer can be implemented using two NOT gates, four 3-input AND gates and one 4-input OR gate as shown in Fig. (b). Here, each of the four data lines is applied to any input of an AND gate and the AND gate outputs are connected with the inputs of OR gate to generate the output Y.

Table Truth table 4-to-1 multiplexer

Data select inputs		Output
S_1	S_0	Y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3



(a) Logic symbol



(b) Logic diagram

Fig. 4-to-1 multiplexer

4. ELECTRICAL ENGINEERING

NETWORK THEORY

NODE ANALYSIS

Node analysis is mainly based on Kirchoff's Current Law (KCL). This method uses the analysis of the different nodes of the network. In nodal analysis one of the nodes is assumed as reference node whose potential is assumed to be zero. At other voltage at different nodes are to be measured with respect to this reference node. The reference node should be given a number zero and then the equations are to be written for all other nodes by applying KCL.

Node 1. Equation is

$$I_1 - \frac{V_1}{R_1} - \frac{(V_1 - V_2)}{R_2} = 0 \qquad \text{...(i)}$$

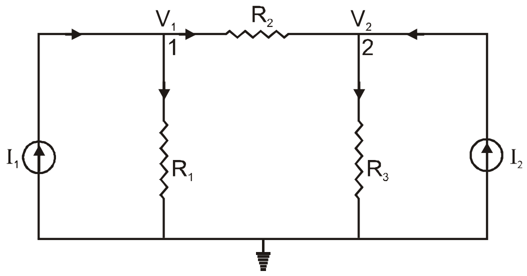


Fig.

Node 2. Equation is

$$\frac{(V_1 - V_2)}{R_2} - \frac{V_2}{R_3} + I_2 = 0 \qquad \text{... (ii)}$$

Rearranging equations (i) and (ii), we get

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] = I_1 \text{ and } -V_1\left[\frac{1}{R_1}\right] + V_2 \cdot \left[\frac{1}{R_2} + \frac{1}{R_3}\right] = I_2$$

Above equations can be solved simultaneously to get the unknown node voltages from which any branch current can be calculated.

Important Points for Nodal Analysis.

(i) While assuming branch currents, make sure that each unknown branch current is considered atleast once.

(ii) Convert the voltage source present into their equivalent current sources for node analysis wherever possible.

(iii) Follows the same sign convention, currents entering at node are to be considered positive, while currents leaving the node are to be considered as negative.

CONTROL SYSTEM

ROUTH-HURWITZ CRITERION

In order to determine the existence of a root having positive real part for a polynomial equation given by

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + ...+ a_n = 0 \qquad ...(1)$$

Two rows consisting of alternate coefficients of polynomial (1) are formed as follows

$$\begin{array}{llll} a_0 & a_2 & a_4 & a_6 ... \\ a_1 & a_3 & a_5 & .... \end{array}$$

and the following array is formed (illustrated for a 6th order polynomial)

$$\begin{array}{lllll} s^6 & a_0 & a_2 & a_4 & a_6 \\ s^5 & a_1 & a_3 & a_5 & 0 \\ s^4 & b_1 & b_3 & b_5 & 0 \\ s^3 & c_1 & c_3 & c_5 & 0 \\ s^2 & d_1 & d_3 & 0 & 0 \\ s^1 & e_1 & 0 & 0 & 0 \\ s^0 & f_1 & 0 & 0 & 0 \end{array} \qquad ...(2)$$

In the Routh array (1) the coefficients b_1, b_3, c_1 etc. are related to the coefficients of the polynomial (1) as under

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}, \; b_3 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_5 = \frac{a_1 a_6 - a_0 \times 0}{a_1} = a_6$$

$$c_1 = \frac{b_1 a_3 - a_1 b_3}{b_1}, \; d_1 = \frac{c_1 b_3 - b_1 c_3}{c_1} \text{ etc.}$$

From the array (2) so formed, it is observed that the elements of the first column are related to Hurwitz determinants [for example $b_1 = \left(\frac{D_2}{D_1}\right)$]. The number of changes of sign in the first column elements of Routh array gives the number of positive real part roots of the polynomial. Therefore, for a stable system there should be not change of sign in the first column of Routh array formed from the coefficients of the characteristic equation expressed in polynomial form.

While applying Routh-Hurwitz criterion it is imperative that no powers of s in the characteristic equation be absent. Any absence of such powers indicate the presence of at least one positive real part root and confirms system instability by inspection.

However, if the characteristic equation contains either only odd or even powers of s, this indicates that the roots have no real parts and possess only imaginary parts and therefore the system has sustained oscillations in its output response.

Following difficulties are faced while applying Routh-Hurwitz criterion

Where the angle at ω_1, the gain cross - over frequency, is measured negatively.

Gain margin (GM) and phase margin (PM) are frequently used for frequency response specifications by designers. It is important to note once again that these measures of stability are valid for open - loop stable systems only. A large gain margin or a large phase margin indicates a very stable feedback system but usually a very sluggish one. A GM close to unity or a PM close to zero corresponds to a highly oscillatory system. Usually a GM of about 6 db or a PM of 30 - 35° result in a reasonably good degree of relative stability. In most practical systems a good gain margin automatically guarantees a good phase margin and vice versa . However, the cases where the specification on one does not necessarily satisfy the other, also exist as shown in Figs.A and B

In a second - order system with $G(j\omega)H(j\omega) = K / j\omega(j\omega T + 1)$ whose polar plot is shown in Fig. C. GM always remains fixed at infinite value as the plot always reaches the real axis at the origin, while the PM reduces continuously with increasing system gain. In this case, as shall be seen below, PM is the correct measure of relative stability.

Usually it is the phase margin which is specified as a measure of system performance in design.

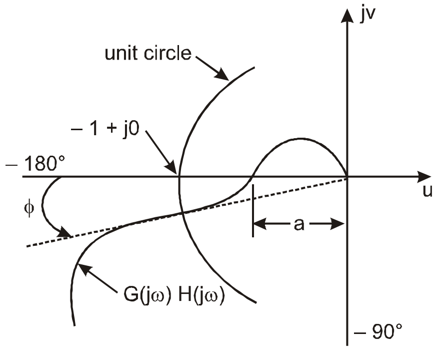


Fig. A Polar plot of a system with good GM and Poor PM.

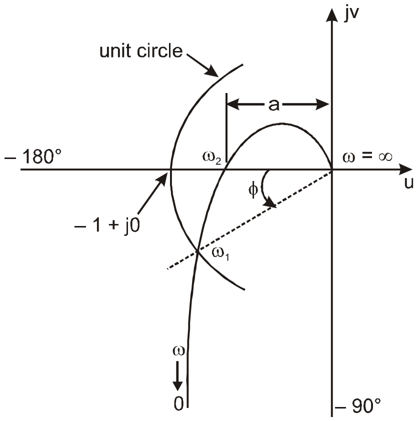


Fig. B Polar plot of a system with good PM and Poor GM.

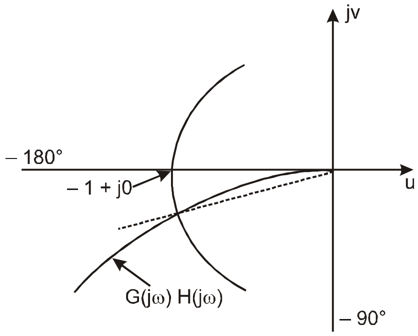


Fig. C. Polar plot of a second - order system with GM always equal to infinity while PM reduces continuously with increasing gain.

Correlation Between phase Margin and Damping Factor

Consider a unity feedback second - order system with an open - loop transfer function

$$G(s)H(s)=\frac{K}{s(\tau s+1)}=\frac{\omega_n^2}{s(s+2\varsigma\omega_n)}$$

where $\omega_n=\sqrt{(k/\tau)}$ and $2\varsigma\omega_n=1/\tau$.

Replacing s by $j\omega$ for obtaining the polar plot, we have

$$G(j\omega)\,H(j\omega)=\frac{\omega_n^2}{j\omega(j\omega+2\varsigma\omega_n)} \qquad (A)$$

At the gain cross - over frequency $\omega = \omega_1$, the magnitude / $G(j\omega)H(j\omega)$/ = 1. Therefore from eqn., we have

$$\frac{\omega_n^2}{\omega_1\sqrt{\left(\omega_1^2+4\varsigma^2\omega_n^2\right)}}=1 \quad \text{or} \quad \left(\omega_1^2\right)^2+4\varsigma^2\omega_n^2\left(\omega_1\right)^2-\omega_n^4=0$$

$$\left(\omega_1/\omega_n\right)^2=\sqrt{\left(4\varsigma^4+1\right)}-2\varsigma^2$$

The phase margin of this system is given by

$$\phi=-90^\circ-\tan^{-1}\left(\frac{\omega_1}{2\zeta\omega_n}\right)+180^\circ$$

$$=-90^\circ-\tan^{-1}\left[\frac{1}{2\zeta}\left\{\left(4\zeta^4+1\right)^{1/2}-2\zeta^2\right\}^{1/2}\right]$$

$$= \;=-\tan^{-1}\left[2\zeta\left\{\frac{1}{\left(4\zeta^4+1\right)^{1/2}-2\zeta^2}\right\}^{1/2}\right] \qquad \text{(B)}$$

Equation (B) gives a relationship between ζ and ϕ for an underdamped second - order system. The ζ - ϕ relationship is plotted in Fig.. In range $\zeta \le 0.7$. a reasonably good linear approximation for ζ - ϕ relationship is given below and is shown by the dotted line in the same figure.

$$\zeta \approx 0.01\,\phi \qquad \text{(C)}$$

Where ϕ is in degrees.

Equation (B) and its approximation of eqn. (C) hold good for second - order systems, but can be used for higher - order systems as well , provided the transient response of the system is primarily contributed by a pair of dominant underdamped roots.

5. MATERIAL SCIENCE

Corrosion

Corrosion, oxidation and degradation commonly spoil the aesthetics, leads to component weakening, the requirement for remedial actions and, in the worse case, failure of materials in products. Often this is directly due to the aggressive environment that a product is exposed to over its operational life. In the case of plastics or paint coatings it can be embrittlement or discolouration, paint delamination or solvent swelling. Metals can be attacked by aqueous based corrosion or oxidised at elevated temperature.

Aqueous Corrosion

In order for aqueous corrosion to occur the following have to be present:-

- Anodic area where components degradation occurs
- Cathodic area
- Electrolyte to transport corrosive species and various ions
- Electron conduction path connecting anode and cathode

The corrosion rate is dependent on many factors, some of which are listed below:-

- Relative areas of the anode and cathode
- Electrolyte conductivity and whether the metal is immersed in water, underground or just corroding as a result of surface moisture related to the humidity
- Availability of fresh reactants; corrosive species such as oxygen and chloride ions

Mechanisms to minimise the impact of the environmental attack are often in place, but have failed to be effective for a range of possible reasons:

- Unusual or unexpected exposure conditions (chemical, humidity, pH, turbulence from flowing water etc, differential aeration etc.)
- Galvanic cell set up with another nearby metal component electrically in contact with the corroding part, with the circuit completed by an electrolyte, which can be as little as surface adsorbed water
- Ineffective or insufficient biocides or inhibitors in a closed system like a heating system as microbiological attack can be very aggressive

The protection system designed to slow down or prevent corrosion can be

- Passivation having or deliberately growing a protective oxide. Stainless steel has a protective surface oxide and aluminium and titanium are often anodised to grow thicker oxides
- Choice of material / alloy to insure the material is passive or immune in the environmental conditions the component is likely to see over life
- Avoid two phase alloys which can provide the anode and cathode locally close together and give phenomena such as dezincification of brasses where one phase can be preferentially eaten away

- Painting, spraying or dip coating to provide a diffusion barrier to slow down or stop the ingress of corrosive species such as oxygen or chloride ions
- Avoid rough surfaces as points protruding into the electrolyte will preferentially corrode
- Cracks or small pits should be avoided as, from the catchment area principle, the anode at the crack tip (because of a lower availability of oxygen) is small with respect to the cathode and attack can be relatively fast giving pits. Aluminium and stainless steels are susceptible to pitting corrosion for example.

Stress Corrosion Cracking and Fatigue

Corrosion, with the exception of pitting, is generally a slow process. However, it can combine with mechanical loading to produce a particularly aggressive form. In stress corrosion cracking (SCC) a tensile stress or residual stress from component manufacture forces the crack tip open to allow the attacking elements in and concentrated chemical attack. The corrosion component of SCC allows areas crack growth to continue even when the stress is lowered.

Combining fatigue with corrosion is probably the most catastrophic form of attack affecting aluminium alloys in aircraft for example. SCC is distinct in SEM/EDX and AES (Auger).

High Temperature Oxidation

High temperature oxidation of metals occurs when the temperature is sufficient to allow interdiffusion of metal and oxygen. It can be deliberate:

- Furnace produced coloured metal for controlled heat transfer characteristics (emissivity)
- Growing a certain layer thicknesses of silicon oxide insulation or silicon oxynitride as a dielectric on semiconductor wafers
- Furnace oxidation method of cleaning contaminants off metals prior to thermal reduction

In other cases oxidation is to be avoided or only permitted until a protective oxide is produced. Areas of interest may be:-

- Inside a jet engine
- Automotive engine components
- Air leaks in a reducing or other gas furnace process such as hardening or annealing
- Welding under a protective inert gas shield

Surface oxidation can be evaluated by a combination of surface analysis techniques and SEM/EDX. The laboratory also has facilities to mimic thermal processes under a range of reducing and oxidizing gas environments.

Corrosion Protection

1. Cathodic protection

The principle of this method is to alter the electrode potential of the metallic structure so that they can lie in the immunity region. This is the region where the metal is in the stable state of the element and corrosion reactions are not possible. It is mostly used in steel structures in marine and under ground regions.

Two methods are used to apply the cathodic protection to a metal structure.

- **Impressed Current -** This method is used for the protection of pipelines and the hulls of ships in sea water. In this method, an electric current is applied to the metal surface by use of DC electrical circuit. The negative and positive terminal of the current source is connected to the metal requiring protection and an auxiliary anode respectively. The flow of electric current charges the structure with electrons and changes the electrode potential in the negative direction. This process continues till it reaches the immunity region. The current flows from anode to cathode. Thus it protects the metal surface from corrosion.
- **Sacrificial Anode -** This is especially used for ships, offshore oil and gas production platform etc. In this technique, the more reactive metal is used to alter the electrode potential and get the immunity region. Zinc is generally used as sacrificial anode. It generates the anodic dissolution current with more negative potential. The cathodic curve intersection is now at a more negative potential which is the immunity region. At this region, the corrosion rate of steel is negligible.

6. SOLID MECHANICS

STRESS AND STRAIN

When a member is subjected to loads, it develops resisting force. These resisting forces may be conveniently split into normal and parallel to the sectional plans. The resistance force parallel to the plane is called *shearing resistance*. The intensity of resisting force normal to the sectional plane is called *Normal stress* (σ) and parallel to the sectional plane is called *shearing stress*.

$$\text{Normal stress, } \sigma = \frac{dR}{dA}, \quad \text{and} \quad \text{Sheare stress, } \tau = \frac{dQ}{dA}$$

where R = normal resistance

Q = the shearing resistance.

Unit of stress : N/m^2.

Simple stresses

These are of two types

- **Normal stress :** When a bar is subjected to force P as shown in the figure in such a manner that the applied forces must be same, the resistance R acting on a section is given by

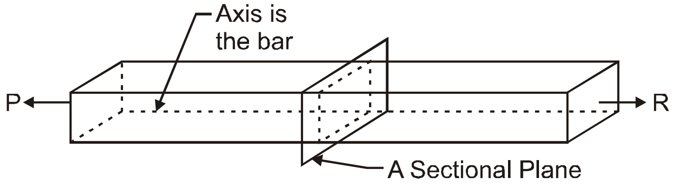


Fig. Bar subjected to tensile force

Fig. Free body diagram

$$R = \int \sigma dA = \sigma \int dA = \sigma A$$

where, A = cross sectional area.

From figure, $P = R$

$\therefore$ $P = \sigma A$

or $\sigma = \dfrac{P}{A}$

- **Shear stress :** The force parallel to the cross section of bar is shear force. Let Q be the shear stress acting on the section. Then with the usual assumptions that stresses are uniform, we get

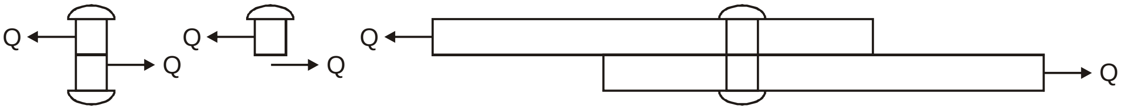


Fig. Rivet in direct shear

$$R = \int \tau dA = \tau \int dA = \tau A$$

For equilibrium, $Q = R = \tau.A$

$\therefore$ Shear stress, $\tau = \dfrac{Q}{A}$

Hence direct shear force is equal to the shearing force per unit area.

Strain

It is defined as the change in length per unit length. The strain may be tensile or compressive depending upon whether the length increases (under tensile load) or decreases under (compressive load).

$$\text{Linear strain, } e = \frac{\text{Change in length}}{\text{Original length}}$$

$$\therefore \quad \text{Lateral strain} = \frac{\text{Change in the lateral dimension}}{\text{Original lateral dimension}}$$

Poisson's ratio

The ratio of lateral strain to the longitudinal strain in a constant quantity and is called the *Poisson's ratio* and is denoted by v or $\dfrac{1}{m}$

$$v = \frac{\text{Lateral strain}}{\text{Logitudinal strain}}$$

For the most metals its value is between 0.25 to 0.33. For cement concrete 0.15, for rubber 0.5, for steal 0.3. It value of same in tension as well as in compression.

Types of strain.

- **Normal strain :** It produces under the action of shear stress and it is measured by change in angle.
- **Shear strain :** It produces under the action of shear stress and it is measured by change in angle.

Thus, shear strain, $r = \tan \phi$

For small straining, then $\phi = \phi$

$$\therefore \quad \phi = \frac{dl}{l}$$

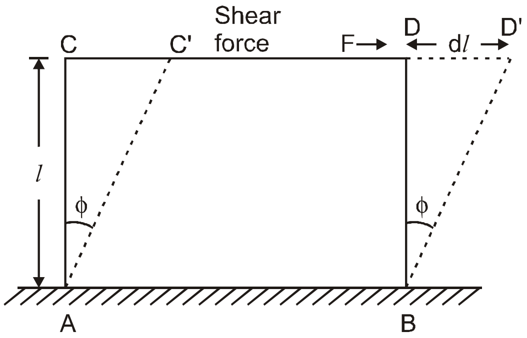


Fig.

- **Superficial strain :** It is the ratio of change in area of cross-section per unit original area.
- **Volumetric strain :** When a uniform stress is applied on all the three faces of the body, then all the three dimension will change resulting in change in volume. Thus

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volum}}$$

STRESS AND STRAIN RELATIONSHIP (Hooke's Law)

Hooke's law states that stress is directly proportional to strain upto the elastic limit.

Let σ be the stress and e be the strain, then

$$\sigma \propto e$$

or $$\sigma = E\,e$$

where, E = constant of proportionality of material and is known as modulus of elasticity.

Stress-Strain Relation in aluminium and high strength Steel

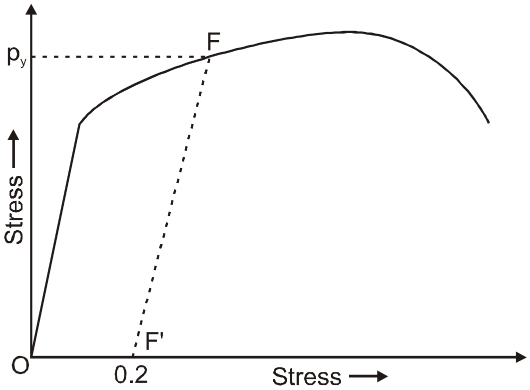


Fig. Stress strain relationship in Aluminium and high strength steel

Stress stain diagram is shown in the figure. This stress p at which if unloading is made there will be 0.2 percent permanent set is known as 0.2 percent proof stress and this point is treated as yield point for all practical purposes.

Stress-Strain Diagram (brittle material)

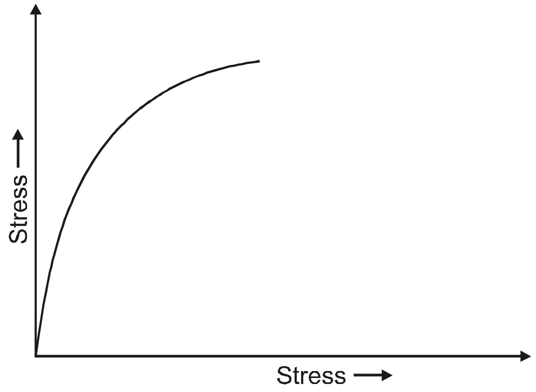


Fig. Stress Strain relations for brittle material.

In brittle materials there is no appreciable change in rate of strain. There is no yield point and no necking takes place. Ultimate point and breaking point are one and the same. The strain at failure is very small.

7. THERMODYNAMICS

Entropy

The second law leads to the definition of a new property called entropy.

The Clausius Inequality

The first law is simply an energy balance. However, the second law leads to an inequality; an irreversible process is less efficient than a reversible process. Another important inequality in thermodynamics is the Clausius inequality:

$$\oint \frac{\delta Q}{T} \le 0$$

That is, the cyclic integral of δQ/T is always less than or equal to zero. This is valid for all cycles, reversible or irreversible.

For internally reversible cycles, it can be shown that:

$$\oint \frac{\delta Q}{T}\bigg|_{\text{int,rev}} = 0$$

The Clausius inequality forms the basis for the definition of a new property called entropy.

As can be seen in the equation above, for an internally reversible process the cyclic integral of δQ / T is zero. A quantity whose cyclic integral is zero depends on the state only and not the process path, and thus it is a property.

Clausius in 1865 realized that he discovered a new property and he called it entropy:

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int,rev}} \qquad \text{(kJ/K)}$$

Entropy per unit mass is designated by s (kJ/kg.K).

The entropy change of a system during a process can be calculated:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} = (\text{kJ}/\text{K})$$

To perform this integral, one needs to know the relation between Q and T during the process.

Note that the cyclic integral of δQ /T will give us the entropy change only if the integration carried out along an internally reversible path between two states.

For irreversible processes, we may imagine a reversible process between the two states (initial and final) and calculate the entropy change (since entropy is a property).

The Increase of Entropy Principle

Entropy change of a closed system during an irreversible process is greater that the integral of ΔQ / T evaluated for the process. In the limiting case of a reversible process, they become equal.

$$dS \geq \frac{\delta Q}{T}$$

The entropy generated during a process is called entropy generation, and is denoted by S_{gen},

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

Note that the entropy generation Sgen is always a positive quantity or zero (reversible process). Its value depends on the process, thus it is not a property of a system.

The entropy of an isolated system during a process always increases, or in the limiting case of a reversible process remains constant (it never decreases). This is known as the increase of entropy principle.

The entropy change of a system or its surroundings can be negative; but entropy generation cannot.

$$S_{gen} = \begin{cases} > 0 & \text{irreversible process} \\ =0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

1. A process must proceeds in the direction that complies with the increase of entropy principle, Sgen > 0. A process that violates this principle is impossible.

2. Entropy is a non–conserved property, and there is no such thing as the conservation of entropy. Therefore, the entropy of universe is continuously increasing.

3, The performance of engineering systems is degraded by the presence of irreversibility. The entropy generation is a measure of the magnitudes of the irreversibilities present during the process.

8. FLUID MECHANICS

The Navier-Stokes equations

The flow of an incompressible homogeneous fluid with density ρ_0 and viscosity μ is described by the incompressible Navier-Stokes equations,

$$\rho_0 (\vec{u}_t + \vec{u} \cdot \nabla \vec{u}) + \nabla p = \mu \Delta \vec{u},$$
$$\nabla \cdot \vec{u} = 0.$$

Here, $\vec{u}(\vec{x},t)$ is the velocity of the fluid, and $p(\vec{x},t)$ is the pressure. The first equation is conservation of momentum, and the second equation is conservation of volume.

Remark. It remains an open question whether or not the three-dimensional Navier-Stokes equations, with arbitrary smooth initial data and appropriate boundary conditions, have a unique, smooth solution that is dened for all positive times.

This is one of the Clay Institute Millenium Prize Problems.

Let U, L be a typical velocity scale and length scale of a fluid flow, and define dimensionless variables by

$$\vec{u}^* = \frac{\vec{u}}{U}, \quad p^* = \frac{p}{\rho U^2}, \quad \vec{x}^* = \frac{\vec{x}}{L}, \quad t^* = \frac{Ut}{L}.$$

Using these expressions in, and dropping the stars on the dimensionless variables,

we get

$$\begin{aligned} &\vec{u}_t + \vec{u}\cdot\nabla\vec{u} + \nabla p = \frac{1}{\mathrm{R}}\Delta\vec{u}, \\ &\nabla\cdot\vec{u} = 0, \end{aligned}$$

where R is the Reynolds number defined in.

Euler equations

The nondimensionalized equation suggests that for flows with high Reynolds number, we may neglect the viscous term on the right hand side of the momentum equation, and approximate the Navier-Stokes equation by the incompressible Euler equations

$$\begin{aligned} &\vec{u}_t + \vec{u}\cdot\nabla\vec{u} + \nabla p = 0, \\ &\nabla\cdot\vec{u} = 0. \end{aligned}$$

The Euler equations are difficult to analyze because, like the Navier-Stokes equations, they are nonlinear. Moreover, the approximation of the Navier-Stokes equation by the Euler equations is problematic. High-Reynolds number flows develop complicated small-scale structures (for instance, boundary layers and turbulence) and, as a result, it is not always possible to neglect the second-order spatial derivatives $\Delta\vec{u}$ in the viscous term in comparison with the first-order spatial derivatives $\vec{u}$ $\nabla\vec{u}$ in the inertial term, even though the viscous term is multiplied by a small coefficient.