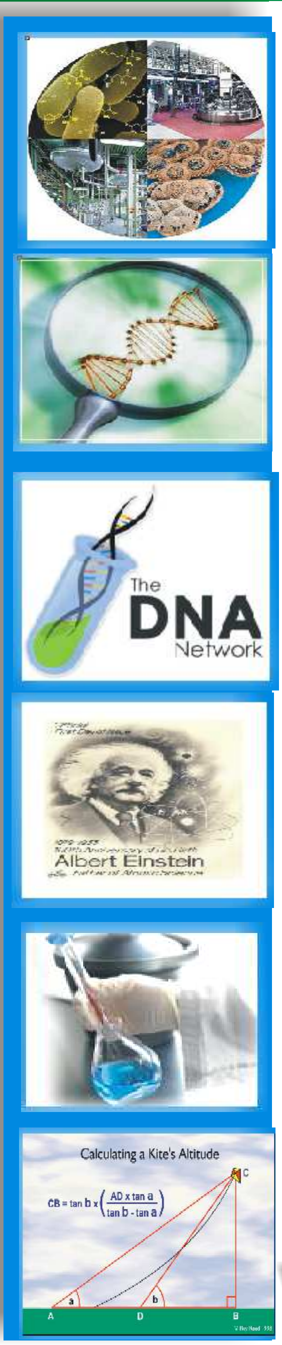


BHU-MCA

MOCK TEST PAPER

- *Attempt all the questions*
- *This paper consists of 150 objective type questions.*
- *Each of these question carries 1 marks. 1/3 mark for each wrong answer.*
- *Pattern of questions : MCQs*
- *Total marks : 150*
- *Duration of test : 3 Hours*

VPM CLASSES
For IIT-JAM, JNU, GATE, NET, NIMCET and Other Entrance Exams
1-C-8, Sheela Chowdhary Road, Talwandi, Kota (Raj.) Tel No. 0744-2429714
Web Site www.vpmclasses.com E-mail-vpmclasses@yahoo.com

1. For the smallest positive root of transcendental equation $x - e^{-x} = 0$, interval is

(A) (0,1)

(B) (–1, 0)

(C) (1, 2)

(D) (2, 3)

2. A root of the equation $x^3 - x - 1 = 0$ lies between 1 and 2. Its approximate value as obtained by applying Bisection method 3 times is

(A) 1.375

(B) 1.625

(C) 1.125

(D) 1.25

3. Performing 3 iterations of bisection method the smallest positive approximate root of equation

$x^3 - 5x + 1 = 0$ is

(A) 0.25

(B) 0.125

(C) 0.50

(D) 0.0625

4. The number of positive roots of the equation

$x^3 - 3x + 5 = 0$ is:

(A) 1

(B) 2

(C) 3

(D) None of these

5. The equation of tangent at (–4, –4) on the curve $x^2 = -4y$ is

(A) $2x + y + 4 = 0$

(B) $2x - y - 12 = 0$

(C) $2x + y - 4 = 0$

(D) $2x - y + 4 = 0$

6. At what point on the curve $x^3 - 8a^2y = 0$, the slope of the normal is $\frac{-2}{3}$

(A) (a, a)

(B) (2a, –a)

(C) (2a, a)

(D) None of these

7. If $z = uv$

$$u^2 + v^2 - x - y = 0$$

$$u^2 - v^2 + 3x + y = 0,$$

then $\frac{\partial z}{\partial x}$ is equal to–

(A) $u + v$

(B) $\frac{2u^2 - v^2}{2uv}$

(C) $\frac{3u^2 + v^2}{2uv}$

(D) $\frac{u^2 - 3v^2}{2uv}$

8. The point (0, 5) is closest to the curve $x^2 + 2y$ at

(A) $\left(2, \sqrt{2}, 0\right)$

(B) (0, 0)

(C) (2, 2)

(D) None of these

9. Find the point on the curve $4x^2 + a^2y^2 = 4a^2$, $4 < a^2 < 8$ that is farthest from the point (0, – 2)

(A) (0, 1)

(B) (–1, 0)

(C) (0, 2)

(D) (–2, 0)

10. Let $f(x,y) = (x - 2)^2 (y + 3)$. Then

(A) (2, - 3) is not a stationary point of f

(B) f has a local maximum at (2, - 3)

(C) f has a local minimum at (2, - 3)

(D) f has neither a local maximum nor a local minimum at (2, -3)

11. Solve $(D^2 – 2kD + k^2)\, y = e^x$.

(A) $y = (C_1 + C_2x)e^x + \frac{e^x}{2}$

(B) $y = (C_1 + C_2x)e^{kx} + \frac{e^x}{(K+1)^2}$

(C) $y = (C_1 + C_2x)e^{kx} + \frac{e^x}{(K-1)^2}$

(D) $y = (C_1 + C_2x)e^{kx} + e^x$

12. Find P. I. of $(D + I)^2 (D^2 + D + 1)^2y = ex$.

(A) $\frac{-e^x}{2}$

(B) $\frac{1}{9}e^x$

(C) $\frac{1}{18}e^x$

(D) $\frac{1}{36}e^x$

13. Let $f, g : R^3 \to R$ be defined by $f(x, y) = x^4 + y^2$; $g(x, y)= x^4 + y^2 - 10x^2y$.

Then at (0, 0)

(A) f has a local minimum but not g

(B) g has a local minimum but not f

(C) Both f and g have a local minimum

(D) Neither f nor g has a local minimum

14. Calculate – $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6 + y^2}$

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 0

15. If (2, 3) is a Critical point of f(x, y) and $f_{xx}(2, 3)\, f_{yy}(2, 3) - [f_{xy}(2, 3)]^2 = 0$, then

(A) (2, 3) is a Saddle point

(B) (2, 3) is a point of local maximum

(C) (2, 3) is a point of local minimum

(D) Further investigation is required to determine the nature of the point.

16. If u = f(y – z, z – x, x – y), then

(A) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

(B) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

(C) $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$

(D) $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

17. If $f(x, y, z) = 3x^2yz + 5xy^2z + 4z^4$ then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+z\frac{\partial t}{\partial z}$

(A) f

(B) 2f

(C) 3f

(D) 4f

18. If $f(x, y) = \begin{cases} y\dfrac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0 , & (x,y)=(0,0)\end{cases}$ Then

(A) $f_x(0, 0) = 1, f_y(0, 0) = -1$

(B) $f_x(0, 0) = 0, f_y(0, 0) = 0$

(C) $f_x(0, 0) = 0, f_y(0, 0) = -1$

(D) $f_x(0, 0) = 1, f_y(0, 0) = 0$

19. The volume of the solid bounded by the plane

$$4x + 2y + z = 10, \quad y = 3x, z = 0, x = 0$$

(A) $\int_0^1 \int_0^{3x} \int_0^{10-2y-4x} dz\, dy\, dx$

(B) $\int_0^1 \int_{3x}^{-2x+5} \int_0^{10-2y-4x} dz\, dy\, dz$

(C) $\int_0^3 \int_0^{y/3} \int_0^{10-2y-4x} dz\, dy\, dx$

(D) $\int_0^3 \int_0^1 \int_0^{10-2y-4x} dz\, dx\, dy$

20 Evaluate $\iint_D e^{x^2+y^2}\, dA$, D is the unit circle centered at the origin.

(A) $2\pi (e + 1)$

(B) $\pi (e - 1)$

(C) $\pi (e^2 - 1)$

(D) $2\pi (e - 1)$

21. Integrate r sin θ over the area of the Cardioid r = a (1 + cos θ) about the initial line.

(A) $\frac{2}{3} a^3$

(B) $\frac{4}{3} a^3$

(C) $\frac{4\pi}{3} a^3$

(D) $3\frac{2\pi}{3}a^3$

22. The integral $\int_0^1 \frac{\sec x}{x} dx$

(A) Converges to 0

(B) Converges to – 1

(C) Converges to 1

(D) Diverges

23. Let $I = \int_0^a \int_{y^2/2a}^{a-\sqrt{a^2-y^2}} dy\, dx + \int_a^{2a} \int_{y^2/2a}^{2a} V\, dy\, dx + \int_0^a \int_{a+\sqrt{a^2-y^2}}^{2a} V\, dy\, dx$ on changing order of integration the integral I equal to

(A) $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V\, dx\, dy$

(B) $\int_1^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V\, dx\, dy$

(C) $\int_{-a}^{a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V\, dx\, dy$

(D) $\int_a^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V\, dx\, dy$

24 A cubic f (x) vainshes at x = – 2 and has relative minimum/maximum at x = – 1 and x = $\frac{1}{3}$ if $\int_{-1}^{1} f(x)dx = \frac{14}{3}$, find the cubic f (x).

(A) $f(x) = x^3 + x^2 - 8$

(B) $f(x) = x^2 + x^3 - x + 8$

(C) $f(x) = x^3 + x^2 - x + 2$

(D) $f(x) = x^2 + x^3 - 2$

25. The function f(x) = $\sqrt{(ax^3 + bx^2 + (cx + d)}$ has its non-zero local minimum and maximum values at x = –2 and x = 2 respectively if a is the root of $x^2 - x - 6 = 0$ then

(A) a = – 2 b = 24 c = 24

(B) a = –2 b = 0 c = 24

(C) a = –2 b = 12 c = 24

(D) a = –2 b = 0 c = 0

26. If f (x) = $\begin{cases} e^x , & 0 \le x \le 1 \\ 2 - e^{x-1}, & 1 < x \le 2 \\ x - e , & 2 < x \le 3 \end{cases}$

and g(x) = $\int_0^x f(t)dt, x \in [1, 3]$, then

(A) g(x) has local maxima at x = 1 + $\log_e$ 2 and local minima at x = o

(B) f(x) has local maxima at x = 1 and local minima at x = 2

(C) g(x) has no local minima

(D) f(x) has no local maxima

27. Using taylor's polynomial of $\sqrt{(4{\cdot}01)(3{\cdot}98)}$ the first degree

The approximate value of is equal to

(A) 4·02

(B) 3·98

(C) 3·89

(D) 3·88

28. If z = f(x, y) and x = $e^u + e^{-v}$, y = $e^{-u} + e^v$ then $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} =$

(A) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$

(B) $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$

(C) $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}$

(D) $x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial yz}$

29. If z = f(x, y) where x = φ(ω), y = ψ(ω) φ(1) = 2 φ'(1) = 7 ψ(1) = – 3 ψ'(1) = 2 f_x (2, – 3) = – 8 f_y(2 – 3) = – 3

Then $\frac{dz}{d\omega}$ at ω = 1 is

(A) 42

(B) – 42

(C) 62

(D) – 62

30. Solve the following L.P.P..

Max. $z = 5x_1 + 7x_2$

s. t. $x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 34$

and $x_1 \ge 0, x_2 \ge 0$

(A) $x_1 = 1{\cdot}6$ $x_2 = 2{\cdot}4$ $z = 24{\cdot}8$

(B) $x = 1{\cdot}9$ $x_2 = 2{\cdot}0$ $z = 23{\cdot}5$

(C) $x_1 = 2{\cdot}1$ $x_2 = 1{\cdot}2$ $z = 18{\cdot}9$

(D) None of these

31. The sol^n of lpp max $z = 2x_1 - 10x_2$

subject to $x_1 - x_2 \geq 0$

and $x_1 - 5x_2 \leq -5$

$x_1, x_2 \geq 0$ is

(A) Is unique

(B) Is bounded

(C) Is unbounded

(D) Does not exist

32. Let $S_1 = \{(x_1, x_2) : 3x_1 + 5x_2 = 2, x_1, \geq 0, x_2 \in R\}$

$S_2 = \{(x_1, x_2) : 3x_1 + 5x_2 = 2, x_1 \in R\ x_2 \in R\}$

Then the set $S_1 \cap S_2$ is

(A) Convex and unbounded

(B) Not convex but bounded

(C) Both convex and bounded

(D) Neither convex nor bounded

33. The Lpp formulation of the unconstrained optimization problem

$$\text{Maximize } y = \min\{|2x_1 + 5x_2|, |2x_1 - 5x_2|\}\ x_1, x_2 \geq 0$$

(A) Max y such that $2x_1 + 5x_2 - y \geq 0$, $2x_1 - 5x_2 - y \geq 0$, $2x_1 - 5x_2 + y \leq 0$, $x_1, y, x_2 \geq 0$

(B) Max y such that $2x_1 + 5x_2 + y \leq 0$, $2x_1 - 5x_2 - y \geq 0$, $2x_1 - 5x_2 + y \leq 0$, $x_1, x_2, y \geq 0$

(C) Max y such that $2x_1 + 5x_2 - y \geq 0$, $2x_1 - 5x_2 + y = 0$, $2x_1 - 5x_2 + y \leq 0$, ; $x_1\ x_2, y \geq 0$

(D) Max y such that $2x_1 + 5x_2 - y \geq 0$, $2x_1 - 5x_2 - y = 0$, $2x_1 - 5x_2 - y \leq 0$, $x_1\ x_2, y \geq 0$

34. If X and Y are two non-empty sets where $f : X \to Y$, is a function defined such that

$$f(c) = \{f(x) : x \in C\} \text{ for } C \subseteq X$$

and $$f^{-1}(D) = \{x : f(x) \in D\} \text{ for } D \subseteq Y,$$

for any $A \subseteq Y$ and $B \subseteq Y$, then

(A) $f^{-1}\{f(A)\} = A$

(B) $f^{-1}\{f(A)\} = A$ only if $f(X) = Y$

(C) $f\{f^{-1}(B)\} = B$ only if $B \subseteq f(x)$

(D) $f\{f^{-1}(B)\} = B$

35. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0

(B) $2^9 - 1$

(C) 168

(D) 2

36. Let p be an odd prime number and T_p be the following set of 2×2 matrices

$$T_p = \left\{ A \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c \in \{0, 1, 2, ..., p-1\} \right\}$$

The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) is divisible by p is

(A) $(p - 1)^2$

(B) $2(p - 1)$

(C) $(p - 1)^2 + 1$

(D) $2p - 1$

37. Let W be the Subspace of R^4 given by

W = {(x, y, z , w) : x + y + z + w = 0} Then the dimension of W is

(A) 1

(B) 2

(C) 3

(D) 4

38. Eigen vectors of the matrix $\begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$ corresponding to the eigen values 3 and 1 are respectively

(A) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}$

39. Let A be a real symmetric matrix of order 4 Then which of the following statement is true about eigen values of A

(A) All eigen values are real

(B) All eigen values are purely imaginary

(C) All eigen values are either real or imaginary

(D) At least one eigen value has non-zero imaginary part

40. The nullity of the matrix $\begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$ is

(A) 1

(B) 2

(C) 3

(D) 4

41. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Then $\vec{\nabla}[\vec{\nabla}(\vec{\nabla}\cdot\vec{r}).\vec{r}].\vec{r}$ is

(A) 1

(B) 3

(C) 3^2

(D) 3^3

42. For any two unit vector $\vec{a}$ and $\vec{b}$ $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 =$

(A) 0

(B) 1

(C) –1

(D) |a| |b|

43. The unit tangent vector to the curve $3x^2 y + y^3 - 3x^2 - 3y^2 + 2 = 0$ at the point (1, 0) are

(A) i + j = 0

(B) 2i + 2j + 1 = 0

(C) 2i – j – 2 = 0

(D) j + 2 = 2i

44. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the unit vectors such that $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{c}) = 1$ and $\vec{a}\cdot\vec{b} = \frac{1}{2}$ then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(B) $\vec{a}, \vec{b}, \vec{d}$ are non-coplanar

(C) $\vec{b}, \vec{d}$ are non-parallel

(D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

45. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circle with

(A) variable radii and a fixed centre at (0, 1)

(B) variable radii and fixed centre at (0, –1)

(C) fixed radius 1 and variable centres along the x-axis

(D) fixed radius 1 and variable centres along the y-axis

46. If y = y (x) = and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$, y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ equals

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $-\frac{1}{3}$

(D) 1

47. If x dy = y (dx + y dy), y (1) = 1 and y(x) > 0. Then, y (–3) is equal to

(A) 3

(B) 2

(C) 1

(D) 0

48. Integrating factor of

$\{x(x-1)\frac{dy}{dx} - (x-2)y\} = x^3(2x-1)$ is given by —

(A) $\frac{x-1}{x^3}$

(B) $\frac{x^2}{x-1}$

(C) $\frac{x-1}{x^2}$

(D) $x^3/2x - 1$

49. Given Lpp. Max $(x_1 + 3x_2)$

S. to $3x_1 + x_2 \leq 3$

$x_1 \geq 0, x_2 \geq 0$

The optimal value of the objective function will be

(A) 1

(B) 3

(C) 9

(D) 10

50. If A and B are two independent events such that P (A) > 0, and P(B) ≠ 1, then $P(\bar{A}/\bar{B})$ is equal to

(A) 1 – P(A / B)

(B) $1 - P(A/\bar{B})$

(C) $\frac{1-P(A \cup B)}{P(B)}$

(D) $\frac{P(\bar{A})}{P(\bar{B})}$

51. There are n urns each containing (n + 1) balls such that the ith urn contains 'i' white balls and (n + 1 – 1i) red balls. Let u_i be the event of selecting ith urn, i = 1,2,3,...., n and W denotes the event of getting a white balls.

If $P(u_i) \propto i$, where i = 1, 2, 3, ..., n, then $\lim_{n\to\infty} P(W)$ is equal to

(A) 1

(B) $\frac{2}{3}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

52. Find the missing entry in the following table :

x	0	1	2	3	4
y_x	1	3	9	–	81

(A) 30

(B) 29

(C) 31

(D) 28

53. A company manufactures two types of telephone sets A and B. The A type telephone set requires 2 hour and B type telephone requires 4 hours to make. The company has 800 work hour per day. 300 telephone can pack in a day. The selling prices of A and B type telephones are Rs. 300 and 400 respectively. For maximum profits company produces x telephone of A type and y telephones of B types. Then except $x \geq 0$ and $y \geq 0$, linear constraints and the probable region of the L.P.P is of the type

(A) $x + 2y \leq 400; x + y \leq 300; \text{Max} z = 300x + 400y$, bounded

(B) $2x + y \leq 400; x + y \geq 300; \text{Max} z = 400x + 300y$, unbounded

(C) $2x + y \geq 400; x + y \geq 300; \text{Max} z = 300x + 400y$, parallelogram

(D) $x + 2y \leq 400; x + y \geq 300; \text{Max} z = 300x + 400y$, square

54. The probability that at least one of the events A and B occurs is $\frac{3}{5}$. A and B occur simultaneously with probability $\frac{1}{5}$, then $P(\bar{A}) + P(\bar{B})$ is

(A) $\frac{2}{5}$

(B) $\frac{4}{5}$

(C) $\frac{6}{5}$

(D) $\frac{7}{5}$

55. Consider two events A and B such that $P(A)=\frac{1}{4}, P\left(\frac{B}{A}\right)=\frac{1}{2}, P\left(\frac{A}{B}\right)=\frac{1}{4}$. For each of the following statements, which is true

I. $P(A^c/B^c)=\frac{3}{4}$

II. The events A and B are mutually exclusive

III. $P(A/B) + P(A/B^c) = 1$

(A) I only

(B) I and II

(C) I and III

(D) II and III

56. Consider the following primal Linear Programming Problem (LPP).

Maximize $z = 5x_1 + 2x_2$

subject to $x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

The dual of this problem has

(A) infeasible optimal solution

(B) unbounded optimal objective values

(C) a unique optimal solution

(D) infinitely many optimal solutions

57. Which of the following sets are convex

(A) $x = \{[x_1, x_2]; x_1x_2 \le 1, x_1 \ge 0\ x_2 \ge 0\}$

(B) $x = \{[x_1, x_2]; x_2 - 3 \ge -\ x_1^2,\ x_1 \ge 0, x_2 = 0\}$

(C) $x = \{[x_1, x_2]; x_1 \ge 2, x_2 \le 3\}$

(D) $x = \{[x_1, x_2];\ x_1^2 + x_2^2 \ge 1,\ x_1^2 + x_2^2 \le 4\}$

58. A, B, C are three events for which

P(A) = 0.6, P(B) = 0.4 and P(B) = 0.5,

$P(A \cup B) = 0.8, P(A \cap C) = 0.3$ and $P(A \cap B \cap C) = 0.2$

If $P(A \cup B \cup C) \ge 0.85$ then the interval of values of $P(B \cap C)$ is

(A) [0.2, 0.35]

(B) [0.55, 0.7]

(C) [0.2, 0.55]

(D) none of these

59. For any two independent events E_1 and E_2 in a space S, $P\left[(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)\right]$ is equal to

(A) $\le \frac{1}{4}$

(B) $> \frac{1}{4}$

(C) $\geq \frac{1}{2}$

(D) $> \frac{1}{2}$

60. Let p be the probability that a man aged y will get into an accident in a year. What is the probability that a man among n men of all aged y will get into an accident first ?

(A) $\frac{1}{n}\left(1-(1-p)^n\right)$

(B) $\frac{1}{n}\left(1-(1+p)^n\right)$

(C) $n\left(1-(1+p)^n\right)$

(D) None of these

61. $(1 + \Delta)^{-1/2}\, \Delta$ is equal to (where Δ is backward difference operator)

(A) E

(B) $1 + \delta^2$

(C) δ

(D) Δ

62. If R and R' are symmetric relations

(not disjoint) on a set A, then the relation $R \cap R'$ is

(A) reflexive

(B) symmetric

(C) transitive

(D) none of these

63. Which of the following is not correct

(A) $n(A \cap B') = n(A) - n(A \cap B)$

(B) $A - B = (A \cup B) - A$

(C) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(D) $A \cap (B - C) = A \cap B - A \cup C$

64. For evaluating the integral $\int_{0\cdot2}^{1\cdot4} y\,dx$ by Trapezoidal rule the error for $0\cdot2 < x < 1\cdot4$ is

(A) $-\frac{h^2}{10}y''(x)$

(B) $-\frac{h^2}{12}y''(x)$

(C) $-\frac{h^3}{10}y'(x)$

(D) $-\frac{h^3}{12}y'(x)$

65. Find dy/dx at x = 1 from the following table by constructing a central difference table.

X	0·7	0·8	0·9	1·0	1·1
Y	0·644218	0·717356	0·783327	0·841471	0·891207

X	1.2	1·3
y	0·932039	0·963558

(A) 0·6407

(B) 0·5130

(C) 0·5431

(D) 0·5403

66 By applying Newton's method find the real root near 2 of the equation $x^4 - 12x + 7 = 0$.

(A) 2.5706

(B) 2.7715

(C) 2.7670

(D) 2.6706

67. Consider the following forward difference table

x	y = f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$
1	4			
		9		
2	13		12	
		21		c
3	34		B	
		a		6
4	73		24	
		63		
5	136			

The values of a, b, c respectively

(A) 36, 12, 6

(B) 39, 18, 6

(C) 37, 17, 7

(D) 39, 12, 7

68. If A = {1, 2}, B = {2, 5}, C = {5, 7} then

(A × B) ∩ (A × C) is equal to –

(A) {(2, 5), (1, 5)}

(B) {(2, 2), (5, 5)}

(C) {(2, 7), (1, 5)}

(D) None of these

69 The odds that person X speaks the truth are 3 : 2 and the odds that person Y speaks the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on an identical point.

(A) $\frac{1}{2}$

(B) $\frac{19}{20}$

(C) $\frac{21}{40}$

(D) $\frac{19}{40}$

70. The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \geq 1)$.

(A) 0·9986

(B) 1·0101

(C) 1·0

(D) None of the

71. The random variable x follows the poisson distribution with variance 2 Then $P(x = 2 \mid x > 1)$ is

(A) $\frac{3}{e^2 - 3}$

(B) $\frac{2}{e^2 - 3}$

(C) $\frac{e^{-2}}{2 - 3e^{-2}}$

(D) $\frac{e^{-2}-2}{3e^{-2}}$

72. Two events A and B are such that $P(A^c) = 0{\cdot}3$ $P(B) = 0{\cdot}4$ and $P(A \cap B^c) = 0{\cdot}5$ The $P[B \mid (A \cup B^c,] =$

(A) $\frac{2}{3}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

73. If A = {a, b, c, d} and B = {a, b}, then number of relations in A × B is—

(A) 63

(B) 255

(C) 15

(D) 7

74. The total no of generator of the cyclic group

$(G = \{0, 1, 2, 3, 4, 5\}, +_6)$ is

(A) 1

(B) 2

(C) 4

(D) 6

75. The number of trivial subject of a cyclic group of order 8

(A) 0

(B) 1

(C) 2

(D) 3

76. Let S denote group of all permutations on the finite set {a, b, c, d} under operation of permutation multiplication. Then the order of the sub group of S generated by

$$\begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix} \text{ is}$$

(A) 0

(B) 1

(C) 2

(D) 3

77. The number of distinct group homomorphism from $(z_{12}\ x_{12})$ to $(z_{30}\ x_{30})$ are

(A) 5

(B) 6

(C) 4

(D) infinite

78. Find the remainder when 2^{50} is divided by 9

(A) 0

(B) 1

(C) 2

(D) 3

79. Calculate u_{82}, when $u_{75} = 2459$, $u_{80} = 2018$, $u_{85} = 1180$ and $u_{90} = 402$.

(A) 1800

(B) 1705

(C) 1685

(D) 1601

80. If $M = \left\{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle|\right.$ a,b are real number} + and ·are usual matrix addition and matrix multiplication respectively then (M, + , ·) is

(A) a non commutating ring

(B) an integral domain but Not a field

(C) a Skew field but Not a field

(D) a field

81. Suppose U and W are distinct four-dimensional subspace of a vector space V where dim V = 6 then which of the following is the possible dimension of $U \cap W$

(A) 3

(B) 4

(C) 5

(D) 6

82. Consider the following sets

$B_1 = \{(1, 1, 1), (1, 0, 1)\}$

$B_2 = \{(1, 1, 1), (1, 2, 3), (2, -1, 1)\}$

$B_3 = \{(1, 2, 3), (1, 3, 5), (1, 0,1), (2, 3, 0)\}$

B_4 = {(1, 1, 2), (1, 2, 5), (5, 3, 4)}

Which of the above sets are basis for R^3

(A) B_2, B_4

(B) B_2 only

(C) B_1, B_3

(D) none of these

83. The total number of linear maps from the vector space R^5(R) to the vector space P_3(t) (set of polynomial of degree 3) are

(A) 12

(B) 15

(C) 18

(D) 20

84. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55

(B) 66

(C) 77

(D) 88

85. Let E^c denotes the complement of an even E. Let E, F, G be pairwise independent events with P (G) > 0 and $P(E \cap F \cap G) = 0$. Then, $P(E^c \cap F^c | G)$ equals

(A) $P(E^c) + P(F^c)$

(B) $P(E^c) - P(E^c)$

(C) P (E^c) – P (F)

(D) P (E) – P (F^c)

86. The solution of the differential equation $\frac{d^2y}{dx^2} - 4y = 1 + x^2$ is—

(A) $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{3}{8} + \frac{1}{4}x^2$

(B) $y = c_1 \cos(2x) + c_2 \sin(2x) + \frac{3}{8} + \frac{1}{4}x^2$

(C) $y = c_1 \cos(2x) + c_2 \sin(2x) - \frac{3}{8} + \frac{1}{4}x^2$

(D) $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{3}{8} + \frac{1}{4}x^2$

87. The differential equation of all circles passing through the origin and having their centres on the x-axis is —

(A) $2xy\frac{dy}{dx} + x^2 - y^2 = 0$

(B) $2xy\frac{dy}{dx} + x^2 + y^2 = 0$

(C) $2xy\frac{dy}{dx} + 2x^2 - y^2 = 0$

(D) $2xy\frac{dy}{dx} + x^2 - 2y^2 = 0$

88. Solve (D^2 16) y = sin 2x, given that y = 0 dy /dx = (5/6) when x = 0.

(B) $y = \frac{1}{6}\sin 4x + \frac{1}{12}\ \sin 2x$

(B) $y = \frac{1}{6}\sin 4x + \frac{1}{12}\ \cos 2x$

(C) $y = \frac{1}{3}\sin 4x + \frac{1}{6}\sin 2x$

(D) $y = \frac{1}{8}\sin 2x + \frac{1}{6}\sin 4x$

89. Find the PI of differential equation

$$(D^2 + 4D - 12)y = (x - 1)e^x$$

(A) $(4x^2 - 9x)\,\frac{e^{2x}}{4}$

(B) $\frac{1}{64}(4x^2 - 9x + 6)e^{2x}$

(C) $(4x^2 - 9x + 6)\,\frac{e^{2x}}{4}$

(D) $\frac{1}{64}(4x^2 - 9x)e^{2x}$

90. Let a and b be positive integers, and suppose Q is defined recursively as follows :

$$Q(a, b) = \begin{cases} 0 & \text{if } a < b \\ Q(a) - b, b) + 1 & \text{if } b < a \end{cases}$$

Find : (i) Q(2, 5), (ii) Q(12, 5).

(A) (1, 1)

(B) (0, 0)

(C) (0, 2)

(D) (0, 4)

91. Let a = i + j and b = 2i – k, then the point of intersection of the lines r × a = b × a and r × b = a × b is

(A) (– 1, 1, 1)

(B) (3, – 1, 1)

(C) (3, 1, –1)

(D) (1, –1, –1)

92. If $\phi(x, y, z) = xy^2z$ and $A = xz\, i - xy^2\, j + yz^2 k$ find $\frac{\partial^3}{\partial x^2 \partial z}$ (ϕA) at the point (2,–1,1).

(A) 4i – 2j

(B) 2i – 4j

(C) $i + 2j + \hat{k}$

(D) 3i – 6

93. Compute the directional derivative of $f = x^2 + y^2 + z^2$ at (1, 2, 3) in the direction of the line x/3 = y/4 = z/5. Find the maximum rate of increase of f at (1, 2, 3).

(A) $\sqrt{14}$

(B) $2\sqrt{14}$

(C) $3\sqrt{14}$

(D) $4\sqrt{14}$

94. If the volume of a parallelopiped with edges a = 2i – j – k, b = 3i + 2j + 2k and

c = 5i – λj + 3λ k be 4 units, determine the value of λ.

(A) λ = 1

(B) λ = 1/2

(C) λ = 1/5

(D) λ = 1/7

95. Function f, where

$$f(x, y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } x^2 + y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$$

is

(A) f_n, f_y does not exist

(B) f_x, f_y is continuous at

(C) $f_x \neq f_y$

(D) f is differentiable at (0, 0)-

96. A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the disguise is 12. If he chose three numbers with replacement, then the probability that he will laugh atleast once, is

(A) $1 - \left(\frac{31}{45}\right)^3$

(B) $1 - \left(\frac{43}{45}\right)^3$

(C) $1 - \left(\frac{42}{43}\right)^3$

(D) $1 - \left(\frac{41}{45}\right)^3$

97. If X follows a binomial distribution with parameters n = 8 and lp = 1/2, then p (| x – 4 | ≤ 2) is equal to

(A) 121/128

(B) 119/128

(C) 117/128

(D) 115/128

98. If two events A and B are such that P(A) > 0 and P(B) ≠ 1, then is equal to

(A) 1 – P(A/B)

(B) $1 - P(\bar{A}/B)$

(C) $\frac{1 - P(A \cup B)}{P(\bar{B})}$

(D) $\frac{P(A)}{P(\bar{B})}$

99. Suppose X is a binomial variate B (5, p) and P(X = 2) = P(X = 3), then p is equal to

(A) 1/2

(B) 1/3

(C) 2/3

(D) None of these

100. One mapping is selected at random from all the mappings of the set A = {1, 2, 3, ..., n} into itself. The probability that the mapping selected is one to one is given by

(A) $\frac{1}{n^n}$

(B) $\frac{1}{n!}$

(C) $\frac{(n-1)!}{n^{n-1}}$

(D) none of these

101. A supplies 20 men who work for 8 hrs a day for 6 days. B supplies 15 men working at 9 hours a day for 7 days and C supplies 10 men working 6 hours a day for 8 days to do a certain job. If Rs. 636 is paid for all the labour, what is C's share ?

(A) 129

(B) 128

(C) 130

(D) 127

102. A and B complete a piece of work in 80 and 120 days respectively . They together start the work but A left after 20 days. After another 12 days C joined B and now they complete the work in 28 more days. In how many days C can complete the work, working alone ?

(A) 114

(B) 116

(C) 112

(D) 113

103. 2, 6, 30, 60, 130, ...

(A) 210

(B) 216

(C) 200

(D) 160

104. A man travels 7 km towards East, then he turn left and travels 8 kms, again he turns left and travels 10 kms. Finally, he turns left and travels 2 kms. In which direction is he from his starting point ?

(A) North - west

(B) West

(C) East

(D) North - East

105. Shyam walks 5 km towards East and then turns left and walks 6 km. Again he turns right and walks 9 km. Finally he turns to his right and walks 6 km. How far is he from the starting point?

(A) 26 KM

(B) 21 KM

(C) 14 KM

(D) 9 KM

106. Statement: Should an organization like UNO be dissolved?

Arguments:

1. Yes. With cold war coming to an end, such organizations have no role to play

2. No, In the absence of such organizations there may be a world war.

(A) Only argument I is strong

(B) Only argument II is strong

(C) Either I or II is strong

(D) Neither I nor II is strong

107. Statement: Should there be no place of interview in selection?

Arguments:

1. Yes, it is very subjective in assessment.

2. No. It is the only instrument to judge candidates' motives and personality.

(A) Only argument I is strong

(B) Only argument II is strong

(C) Either I or II is strong

(D) Neither I nor II is strong

108. Question: What is the shortest distance between Devipur and Durgapur ?

Statements:

1. Durgapur is 20 kms away from Rampur.

2. Devipur is 15 kms away from Rampur.

(A) I alone is sufficient while II alone is not sufficient

(B) II alone is sufficient while I alone is not sufficient

(C) Either I or II is sufficient

(D) Neither I nor II is sufficient

109. In A Certain Code language "APPROACH" is written as 'YQNSMBAI" then "VERBAL" will be written as

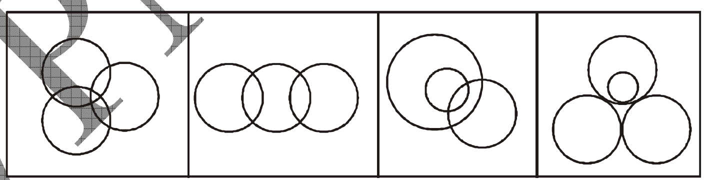
(A) TFPCYN

(B) TFPCYM

(C) TFPYCM

(D) TFPCNY

110. Which diagram represents the relationship among female, mothers and doctors ?



(A) (B) (C) (D)

Direction – (Q 111 and 112) Which one of the given responses would be a meaningful order of the following words ?

111. 1. Accident 2. Judge

3. Doctor 4. Lawyer

5. Police

(A) 1, 3, 4, 2, 5

(B) 1, 3, 5, 4, 2

(C) 1, 2, 3, 4, 5

(D) 1, 2, 5, 4, 3

112. 1. Fruit 2. Flower

3. Seed 4. Pollination

5. Bud

(A) 1, 2, 3, 4, 5

(B) 4, 2, 5, 3, 1

(C) 5, 2, 4, 3, 1

(D) 5, 2, 4, 1, 3

113. Which one set of letters when sequentially placed at the gaps in the given letter series shall complete it ?

–b b m – a m b – m – a – b b

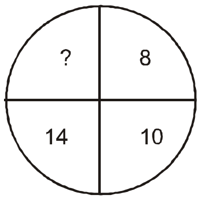
(A) m b a b m

(B) a b m a b

(C) m a ba m

(D) a m b b m

Direction – (Q. 114 and 115) Find the missing number from the given alterna tives.

114. 

(A) 20

(B) 16

(C) 12

(D) 18

115.

2	7	4
5	2	3
1	?	6
10	42	72

(A) 2

(B) 4

(C) 5

(D) 3

116. In a certain language, BUTTER is coded as CVUUFS, BREAD is coded as CSFBE, then how COFFEE is coded ?

(A) DPGGFF

(B) GGDPFF

(C) GDPGFF

(D) FFDPGG

117. If CLOUD can be coded as 59432 and RAIN as 1678, how can AROUND be coded ?

(A) 614832

(B) 614382

(C) 641382

(D) 461382

Directions – (Q-118-121) Select the one which is different from the other three responses.

118. (A) Paper : Pencil

(B) Head : Cap

(C) Ink : Inkpot

(D) Present : Wrapper

119. (A) Sky- Stars

(B) Moon- Planets

(C) Stadium- Players

(D) University – Students

120. (A) BFJQ

(B) RUZG

(C) GJOV

(D) ILQX

121. (A) 117-39

(B) 164–41

(C) 198–66

(D) 213-71

122. B is D's mother and C is D's brother. H is E's daughter whose wife is D. How are E and C related ?

(A) Father-in-law

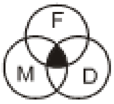
(B) Brother-in-law

(C) Uncle

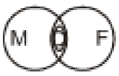
(D) Brother

123. Among the members of the club, some are lady doctors. Indicate which diagram does not imply this statement.

(Note : M = Member; F = Female and D = Doctors)

(A) 

(B) 

(C) 

(D) 

124. From the alternatives, select the set which is most like the given set :

Given set : (4, 10, 15)

(A) (3, 6, 12)

(B) (2, 8, 10)

(C) (5, 12, 18)

(D) (7, 10, 18)

125. A compass (or mariner compass) is a navigational instrument for finding directions on the Earth. It consists of a magnetized pointer free to align itself accurately with Earth's magnetic field, which is of great assistance in navigation.

Inference: Most modern ships use a compass to navigate their way around oceans.

(A) The inference is definitely true, i.e., it directly follows from the statement of facts given

(B) The inference is probably true, though not directly true, in the light of the statement of facts given.

(C) The inference is uncertain, i.e., data is insufficient to decide whether the inference is true or false

(D) The inference is probably false, though not definitely false, in the light of the statement of facts given

Directions – (Q. 126-132) Select the related letters/word/number from the given alternatives.

126. Camera : Lens :: Flash ?

(A) Bulb

(B) Night

(C) Light

(D) Shutter

127. House : Rent : : Capital ?

(A) Interest

(B) Investment

(C) Country

(D) Money

128. NUMBER : UNBMRE : : GHOST: ?

(A) HOGST

(B) HOGTS

(C) HGSOT

(D) HGOST

129. SKIP : RIFL : : KYKZ : ?

(A) WJHV

(B) WJVH

(C) JWVH

(D) JWHV

130. HKNQ : GDAX : : SVYB : ?

(A) TQMK

(B) ROLI

(C) JWVH

(D) ADGL

131. 19 : 37 : : 26 : ?

(A) 52

(B) 51

(C) 46

(D) 43

132. CE : 70 : : DE : ?

(A) 90

(B) 60

(C) 120

(D) 210

Direction : (Q-133 - 134) A series is given with one term missing. Choose the correct alternative from the given ones that will complete the series.

133. DIB, HMF, LQJ, ?

(A) OTM

(B) QBQ

(C) PVO

(D) PUN

134. 1, 2, 4, 8, ?

(A) 8

(B) 9

(C) 16

(D) 32

135. Statement: My first and foremost task is to beautify this city if city X and Y can do it__ why can't we do it. __ Statement of Municipal Commissioner of city Z after taking over charge.

Conclusion: I. The people of city Z are not aware about the present state of ugliness of their city.

II. The present Commissioner has worked in city X and Y and has good experience of beautifying cities.

(A) If only conclusion I follows

(B) If only conclusion II follows

(C) If either I or II follows

(D) If neither I nor II follows

question 136 to 140 based on the following information:

Aliya is the youngest member of the family. Her cousin Boman's paternal grandmother, Chaaru, is her maternal grandmother, while Bomans' maternal grandfather, Dinkar, is her paternal grandfather. Mother of Aliya's Father, Fenil and Boman's mother, Geet is Esha. Hitharth if Fenil and Geet's father-in-law. Ilesh Geet's husband is very fond of jugal, his only brother-in-law's son. Kajri warns her brother Ilesh not to spoil her son Jugal by pampering him too much.

136. How is Dinkar's daughter related to Jugal's father?

(A) Sister-in-law

(B) Sister

(C) Wife

(D) Daughter

137. What is the total number of females in the family?

(A) 4

(B) 5

(C) 6

(D) Cannot be determined

138. What is the relationship of Hitarth with Boman and Kajri respectively?

(A) Father, Paternal grandfather

(B) Maternal grandfather, Father

(C) Maternal grandfather, Father-in-law

(D) Paternal grandfather, Father

139. How many grandsons does Esha have?

(A) 1

(B) 2

(C) 3

(D) Cannot be determined

140. Who is Boman's aunt's in-laws?

(A) Chaaru and Hitarth

(B) Chaaru and Dinkar

(C) Esha and Dinkar

(D) Hitarth and Esha

Instruction for question 141 to 142:

Find the odd man out

141. (A) Cobbler

(B) Student

(C) Plumber

(D) Carpenter

142. (A) Chocolate

(B) Books

(C) Sugar

(D) Honey

Direction for question 143 and 144:

Astronomers were trying to find the weights of 6 planets Mars, Venus, Pluto, Jupiter, Mercury and Saturn. They found that the number of planets lighter than Mars was equal to the number of planets heavier than Venus. Saturn was heavier than Mars and Mercury was heavier than Pluto. Venus was lighter than Mars. Saturn was not the heaviest planet.

143. Which is the third lightest planet among the given 6 planets?

(A) Mars

(B) Jupiter

(C) Saturn

(D) Venus

144. If Jupiter is the heaviest planet, then which is the lightest planet among the 6 planets?

(A) Venus

(B) Mercury

(C) Pluto

(D) Mars

Directions for questions 145 and 146:

Five friends: Ajay, Binoy, Charak, Deepak and Goldy had recently written a high school examination. The following statements are known about their results:

1. Ajay did not secure 1st rank. Binoy did not secure 2nd rank.

2. Deepak did not secure 2nd rank. Goldy did not secure 3rd rank.

3. Charak had secured a rank among top three. Deepak did not secure rank among top three.

4. Ajay had secured rank among top three. Charak did not secure rank among top three.

5. Deepak had secured rank among top three. Goldy had secured rank among top three.

In each of the five statements above one statement is true and the other one is false, not necessarily in that order.

145. Who among the following secured 3rd rank?

(A) Ajay

(B) Biony

(C) Charak

(D) Deepak

146. Who among the following had secured 1st rank?

(A) Ajay

(B) Binoy

(C) Charak

(D) Deepak

147. The stated aims of the United Nations are to maintain international peace and security, to safeguard human rights, to provide a mechanism for international law, to promote social and economic progress, to improve living standards, and to fight diseases. It provides the opportunity to countries to balance global interdependence and national interests when addressing international problems. Most nations have now joined the UN.

Inference: A dispute between two nations is usually solved by the United Nations.

(A) The inference is definitely true, i.e., it directly follows from the statement of facts given

(B) The inference is probably true, though not directly true, in the light of the statement of facts given.

(C) The inference is uncertain, i.e., data is insufficient to decide whether the inference is true or false

(D) The inference is probably false, though not definitely false, in the light of the statement of facts given

(E) The inference is definitely false, i.e., it cannot possibly be inferred from the statement of facts given

148. All the government banks, which currently shut down at 12 P.M. should be open to the public till at least 3 P.M. every day.

A. No: This would increase the risk of investing in the stock market.

B. Yes: Since they are open only till 12 P.M. government run banks are losing customers to privately owned banks.

C. No: This would lead to a reduction in efficiency of people working in government run banks.

D. Yes: India has a population of more than 1 billion and there is a huge number of banking customers.

(A) Only arguments C and D are weak

(B) Only arguments B and C are strong

(C) Only arguments B is weak

(D) All arguments are weak

149. Indian students should pursue higher education in India rather than going abroad.

A. Yes: This would save the students and their parents a lot of money.

B. No: The quality of higher education is much better abroad than in India.

C. Yes: India has some of the top MBA colleges in the world.

D. No: India has more coaching institutes than any country in the world.

(A) Only arguments C and D are weak

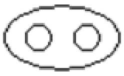
(B) Only arguments A and D are strong

(C) Only arguments D is strong

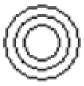
(D) All arguments are weak

150. Which of the following diagrams indicates the best relation between Football, Player and Field ?

(A) 

(B) 

(C) 

(D) 

ANSWER KEY

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	D	A	D	C	B	D	C	A	C	D	D	C	D	B	D	C	B	B
Ques.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	B	D	A	C	B	B	B	B	D	A	C	A	A	D	A	D	C	A	A	B
Ques.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	D	B	C	C	C	A	A	C	C	B	B	C	A	C	A	C	C	A	A	A
Ques.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	C	B	B	A	D	D	B	A	D	A	B	B	B	B	C	D	B	C	B	A
Ques.	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Ans.	A	B	D	C	C	D	A	A	D	C	C	A	B	D	D	B	B	C	A	C
Ques.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	B	C	A	A	C	B	A	D	B	A	B	D	C	A	D	A	B	A	B	A
Ques.	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
Ans.	B	B	B	C	C	A	A	C	D	B	B	A	D	C	D	B	D	D	D	C
Ques.	141	142	143	144	145	146	147	148	149	150										
Ans.	B	B	D	C	C	A	B	B	A	C										

HINTS AND SOLUTION

1.(A) Condition for root to lie between (a, b) is f(a) is negative and f(b) is positive

Given function $f(x) = x - e^x$

At x = 0, f(0) = –1(–ve)

and at x = 1, $f(1) = 1 - e^{-1} = (+ve)$, $\frac{e-1}{e}$ therefore smallest positive root will lie in interval (0, 1).

2.(A) If root of f(x) lies between x_1 and x_2 then $f(x_1)$ will be negative and $f(x_2)$ will be positive

Let $f(x) = x^3 - x - 1$

Bisected value of x	Sign of f(x)	Conclusion
at x = 1 at x = 2	f(1) is negative f(2) is positive	Root lies between 1 & 2
at x = $\frac{1+2}{2} = 1.5$	f(1.5) is Positive	Root lies between 1 and 1.5
at x = $\frac{1+1.5}{2} = 1.25$	f(1.25) is negative	Root lies between (1.25, 1.5)
at x = $\frac{1.25+1.5}{2}$	x = 1.375	Approximate value of x is (1.375)

3.(D) For smallest positive root, start from 0

x	Sign of f(x)	Conclusion
at x = 0 at x = 1	$f(0) = 0^3 - 0 + 1$ = 1(+ve) f(1) = 1 – 5 + 1 = –3(–ve)	∵ f(0) is + ve and f(1) is –ve ∴ root will lie between 0 and 1
at x = $\frac{0+1}{2}$ I iteration	f(0.5 = (–ve)	Root will lie between 0 and 0.5
At x = $\frac{0+0.5}{2} = 0.25$ II iteration	f(0.25) $= (0.25)^2 - 5(0.25) + 1$ = (–ve)	Root will lie between 0 and 0.25
at x = $\frac{0+0.25}{2} = 0.125$ III iteration	$f(0.125) = (0.125)^3$ –5(0.125) + 1 + ve	Root lies between 0 and 0.125

∴ Required root = $\frac{0+0.125}{2} = 0.0625$

Hence (D) is correct

4.(A) Let $f(x) = x^3 - 3x + 5$. Since there are two changes of signs in f(x), therefore (x) has at most two positive roots.

we have $f'(x) = 3x^2 - 3$. Therefore

$f'(x) = 0 \Rightarrow x = \pm 1$

The signs of f(x) at x = –∞, –1, 1, ∞ are:

x	$-\infty$	-1	1	∞
f(x)	–	+	+	+

5.(D) $x^2 = -4y \Rightarrow 2x = -4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{(-4,-4)} = 2.$

We know that equation of tangent is ,

$(y - y_1) = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1) \Rightarrow y + 4 = 2(x + 4)$

$\Rightarrow 2x - y + 4 = 0.$

6.(C) $x^3 - 8a^2 y = 0 \Rightarrow 3x^2 - 8a^2 \cdot \frac{dy}{dx} = 0$

$3x^2 = 8a.\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{3x^2}{8a^2}$

$\therefore$ Slope of the normal $= -\frac{1}{\left(\frac{dy}{dx}\right)} = -\frac{1}{\frac{3x^2}{8a^2}} = -\frac{8a^2}{3x^2}$

Given $\frac{-8a^2}{3x^2} = \frac{-2}{3}$ $\therefore$ (x, y) = (2a, a).

7.(B) Given that $z = uv$ ... (i)

$u^2 + v^2 - x - y = 0$ ... (ii)

$u^2 - v^2 + 3x + y = 0$ ... (iii)

Solving (ii) and (iii), we get

$u^2 = -x,\ v^2 = 2x + y$

$\therefore \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$

$$= v\left(-\frac{1}{2v}\right) + u\cdot\left(\frac{1}{v}\right)$$

$$= \frac{2u^2 - v^2}{2uv}$$

8.(D) Let a point on the curve be (h, k)

Then h = 2k ... (i)

Distance = $D = \sqrt{h^2 + (k-5)^2}$

By (i) ; $D = \sqrt{2k + (k-5)^2}$

$$\frac{dD}{dk} = \frac{1}{2\sqrt{2k + (k-5)^2}} \times 2(k-5) + 2 = 0 \Rightarrow k = 4$$

So, at k = 4 function D must be minimum.

Then point will be $\left(\pm 2\sqrt{2}, 4\right)$.

9.(C) Given curve is $4x^2 + a^2y^2 = 4a^2$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{4} = 1 \quad ...(1)$$

Let point P(a cos ϕ, 2 sin ϕ) be on (1), also given a point Q (0, –2).

Let u = $(PQ)^2$

$= (a \cos \phi)^2 + (2 \sin \phi + 2)^2$

Differentiating both sides w.r.t. ϕ, we have

$$\frac{du}{d\phi} = \cos\phi\{(8 - 2a^2)\sin\phi + 8\}$$

For the extremum value of u, $\frac{du}{d\phi} = 0$

$\Rightarrow \phi = \frac{\pi}{2}$ and $\sin\phi = \frac{4}{a^2 - 4}$

$\because\ 4 < a^2 < 8$ $\qquad\qquad \Rightarrow 0 < a^2 - 4 < 4$

Or $\qquad \frac{a^2 - 4}{4} < 1$ $\qquad\qquad$ or $\frac{4}{a^2 - 4} > 1$

$\therefore\ \sin\phi > 1$ (impossible) $\qquad \therefore \qquad \phi = \pi/2$

Again, $\frac{d^2u}{d\phi^2} = (8 - 2a^2)\cos^2\phi + (2a^2 - 8)\sin^2\phi - 8\sin\phi$

$\therefore\ \left.\frac{d^2u}{d\phi^2}\right|_{\phi=\pi/2} = 0 + (2a^2 - 8) - 8$

$= 2(a^2 - 8) < 0$ $\qquad\qquad (\because 4 < a^2 < 8)$

$\therefore$ u is maximum at $\phi = \pi/2$

So, $\sqrt{PQ}$ is also maximum at $\phi = \pi/2$

Hence co-ordinates of required point P are (0, 2).

10.(A) $\because\ f(x, y) = (x - 2)^2 (y + 3)$

then $\qquad f_x = 2(x - 2)(y + 3)$, $f_x = 0$ x = 2, y = –3

$f_y = (x - 2)2$, fy = 0 $f_y = 0$ x = 2, 2

$f_{xy} = 2(x - 2)$

and $f_{xx} = 2(y + 3)$ and $f_{yy} = 0$

then the discriminant $f_{xx}f_{yy} - fxy^2 = 2(y + 3)(0) - [2(x - 2)]^2$

$= -4(x - 2)^2 < 0$

Hence the points of stationary are only one which is (2, –3) at (2, –3)

f_{xx} = 2(–3 + 3) = and

fxx.fyy – fxy^2 = 0

Since at (2, –3) fxx and $f_{xx}.f_{yy}$ – fxy^2 both are zero So, it is a doubtful case, and so requires further examination.

again $f(x, y) = (x – 2)^2 (y + 3)$, f(2, –3) = 0

then

$f(x, y) – f(2, –3) = (x – 2)^2 (y + 3)$

> 0 if $y > –3$

< 0 if $y < –3$

Thus, f(x, y) – f(2, –3) does not keep the same sign near the origin. Hence f has neither a maximum nor a minimum value at the origin.

11.(C) Here the auxiliary equation is $m^2 – 2km + k^2 = 0$

or $(m – k)^2 = 0$ or m = k, k

∴ C. F. = $(C_1x + C_2) e^{hx}$, where C's are arbitrary constants

and P.I. = $\frac{1}{(D^2 - 2kD + k^2)} e^x = \frac{1}{(1 - 2k + k^2)} e^x$

$= [1/(k – 1)^2]e^x$

∴ The complete solution of the given equation is

$y = (C_1x + C_2) e^{kx} + [e^x/(k – 1)^2]$

12.(D) Here the auxiliary equation is

$m^2(m + 1)^2(m^2 + m + 1)^2 = 0$

Its roots are 0, 0, –1, –1, $\frac{1}{2}$ $[-1 \pm i\sqrt{3}]$ twice each.

$\therefore$ C.F. = $(C_1x + C_2)\, e^{0x} + (C_3x + C_4)e^{-x} + e^{-x/2}[(C_5x + C_6)\cos(\frac{1}{2}\sqrt{3x}) + (C_7x + C_8)$ $\sin(\frac{1}{2}\sqrt{3x})]$,

where C's are arbitrary constants.

And P.I. = $\frac{1}{D^2(D+1)^2(D^2+D+1)^2}e^x$

$= \frac{1}{1^2(1+1)^2(1^2+1+1)^2}e^x = \frac{1}{36}e^x$

$\therefore$ Required complete solution is

y = C. F. + P.I. where C.F. and P.I. are given above.

13.(D) $f(x\ y) = x^4 + y^2$

$g(x, y) = x^4 + y^2 - 10x^2y$.

$f_x = 4x^3$ $\quad f_{xx} = 12x^2$ $\quad f_{xy} = 0$

$fy = 2y$ $\quad f_{yy} = 2$

$f_{xx}\, f_{yy} - f_{xy} = 0$ $\quad$ at (x y) = (0, 0)

$\Rightarrow$ f has no extremum at (0, 0)

$g_x = 4x^3 - 20xy$ $\quad g_{xx} = 12x^2 - 20\, y$

$g_y = 2y - 10x2$ $\quad g_{yy} = 2$ $\quad g_{xy} = 0$

$\Rightarrow g_{xx}\, g_{yy} - g_{xy} < 0$

$\Rightarrow$ g has extrema

But $g_{xx} = 0$ at (0 0)

$\Rightarrow$ does not have minimum.

$$f(x\,y) = \begin{cases} x^2 + y^2 & \text{if x and y are rational} \\ 0 & \text{otherwise} \end{cases}$$

14.(C) First, we will use the path y = x. Along this path we have,

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2} = \lim_{(x,x)\to(0,0)} \frac{x^3x}{x^6+x^2} = \lim_{(x,x)\to(0,0)} \frac{x^4}{x^6+x^2} = \lim_{(x,x)\to(0,0)} \frac{x^2}{x^4+1} = 0$$

Now, let's try the path $y = x^3$. Along this path the limit becomes,

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2} = \lim_{(x,x^3)\to(0,0)} \frac{x^3x^3}{x^6+(x^3)^2} = \lim_{(x,x^3)\to(0,0)} \frac{x^6}{2x^6} = \lim_{(x,x^3)\to(0,0)} \frac{1}{2} = \frac{1}{2}$$

15.(D) If (2, 3) is a critical point of f(x, y) and $f_{xx}(2, 3)\, f_{yy}(2, 3) - [f_{xy}(2, 3)]^2 = 0$

$\Rightarrow$ (2, 3) is not a saddle point and further investigation is required to determine the nature of the point.

16.(B) Let $y - z = t_1$, $z - x = t_2$ and $x - y = t_3$ ...(1)

then $u = f(t_1, t_2, t_3)$, where each t_1, t_2, t_3 are function of x,y and z.

Now by the formula,

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t_1}\cdot\frac{\partial t_1}{\partial x} + \frac{\partial u}{\partial t_2}\cdot\frac{\partial t_2}{\partial x} + \frac{\partial u}{\partial t_3}\cdot\frac{\partial t_3}{\partial x}$$

$$= \frac{\partial u}{\partial t_1}(0) + \frac{\partial u}{\partial t_2}(-1) + \frac{\partial u}{\partial t_3}(1) \quad \text{[by differentiation of (1)}$$

$$= -\frac{\partial u}{\partial t_2} + \frac{\partial u}{\partial t_3} \qquad ...(2)$$

Similarly,

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial t_1}\cdot\frac{\partial t_1}{\partial y} + \frac{\partial u}{\partial t_2}\cdot\frac{\partial t_2}{\partial y} + \frac{\partial u}{\partial t_3}\cdot\frac{\partial t_3}{\partial y}$$

$$= \frac{\partial u}{\partial t_1}(1) + \frac{\partial u}{\partial t_2}(0) + \frac{\partial u}{\partial t_3}(-1)$$

$$= \frac{\partial u}{\partial t_1} - \frac{\partial u}{\partial t_3} \qquad ...(3)$$

and

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t_1}\cdot\frac{\partial t_1}{\partial z} + \frac{\partial u}{\partial t_2}\cdot\frac{\partial t_2}{\partial_z} + \frac{\partial u}{\partial t_3}\cdot\frac{\partial t_3}{\partial z}$$

$$= \frac{\partial u}{\partial t_1}(-1) + \frac{\partial u}{\partial t_2}(1) + \frac{\partial u}{\partial t_3}(0)$$

$$= -\frac{\partial u}{\partial t_1} + \frac{\partial u}{\partial t_2} \qquad ....(4)$$

Now on adding (2), (3) and (4), we get the required result.

17. (D) Here f(x, y, z) is a homogeneous function of degree 4,

therefore we have to verify that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial t}{\partial z} = 4\,f$$

Now differentiating f partially wrt x, y, z respectively,

$\partial f/\partial x = 6xyz + 5y^2z$; $\partial f/\partial y = 3x^2z + 10xyz$; $\partial f/\partial z = 3x^2y + 5xy^2 + 16z^3$

$$\therefore \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z}$$

$= x(6xyz + 5y^2z) + y\,(3x^2z + 10xyz) + z(3x^2y + 5xy^2 + 16z^3)$

$= 6x^2yz + 5xy^2z + 3x^2yz + 10xy^2z + 3x^2yz + 6xy^2z + 16z^4$

$= 4\,(3x^2yz + 5xy^2z + 4z^4) = 4\,f$

Therefore Euler's theorem is verified for the given function.

18. (C) Given $f(x, y) = \begin{cases} y\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}$

$$f_x(0, 0) = \lim_{h\to 0}\frac{f(0+h, 0)-f(0,0)}{h}$$

$$= \lim_{h\to 0}\frac{0-0}{h}$$

$$= 0$$

$$f_y(0, 0) = \lim_{k\to 0}\frac{f(0, 0+k)-f(0,0)}{k}$$

$$= \lim_{k\to 0}\frac{k\left(\dfrac{-k^2}{k^2}\right)-0}{k}$$

$$= -1$$

19.(B) The first plane 4x + 2y + z = 10 is the top of the volume and so we have to calculate volume under z = 10 – 4x – 2y and above the region D in the xy-plane

The second plane y = 3x gives one of the sides of the volume

The region D will be the region in the xy-plane bounded by (y = 3x, x = 0 and z + 4x + 3y = 10)

So $0 \le z \le 10 - 4x - 2y$

$3x \le y \le -2x + 5$

$0 \le x \le 1$

Then $V = \int_0^1\int_{3x}^{-2x+5}\int_0^{10-2y-4x} dz\, dy\, dx$

20.(B) The region U is defined by,

$0 \le \theta \le 2\pi$

$0 \le r \le 1$

In terms of polar coordinates the integral is then,

$$\iint_D e^{x^2+y^2}\, dA = \int_0^{2\pi}\int_0^1 r\, e^{r^2}\, dr\, d\theta$$

Notice that the addition of the r gives us an integral that we can now do. Here is the work for this integral.

$$\iint_D e^{x^2+y^2}\, dA = \int_0^{2\pi}\int_0^1 r\, e^{r^2}\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} e^{r^2}\Big|_0^1 d\theta$$

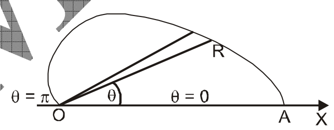
$$= \int_0^{2\pi} \frac{1}{2}(e-1)d\theta$$

$$= \pi(e-1)$$

21.(B) $\iint_A r \sin\theta dA$

$$= \int_0^{\pi}\int_0^{a(1+\cos\theta)} r \sin\theta . r d\theta dr$$

$$= \int_0^{\pi}\left[\int_0^{a(1+\cos\theta)} r^2 \sin\theta\, dr\right] d\theta$$



$$= \int_0^{a} \sin\theta \left[\tfrac{1}{3} r^2\right]_0^{a(1+\cos\theta)} d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi} \sin\theta(1+\cos\theta)^3\, d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi} 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta\left(2\cos^2\frac{\theta}{2}\right)^3 d\theta$$

$$= \frac{16a^3}{3}\int_0^{\pi}\sin\frac{1}{2}\theta\cos^7\frac{1}{2}\theta\, d\theta$$

$$= \frac{16a^3}{3}\int_0^{\pi/2}\sin\phi\cos^7\phi . 2\, d\phi, \text{ where } \theta = 2\phi$$

$$= \frac{32a^3}{3}\left[-\frac{\cos^8\phi}{3}\right]_0^{\pi/2} = \frac{4}{3}a^3$$

22.(D) Since | sec x | ≥ 1 for all values of x, we have

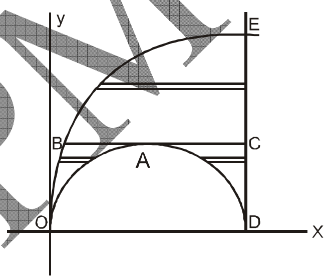
$$\left|\frac{\sec x}{x}\right| \geq \frac{1}{x}$$

and the integral $\int_0^1 \frac{1}{x} dx$ is known to diverge. Hence the given integral is also divergent.

23.(A) Here the integration extends to all points of the space bounded by the circle

$y = \sqrt{(2ax - x^2)}$

i.e., $x^2 + y^2 - 2ax = 0$; the parabola $y^2 = 2ax$; the straight line x = 0 i.e., y axis and the line x = 2a.



Let A be the point of contact of the tangent BC to the semi-circle which is parallel to x-axis. In changing the order of integration, the given integral breaks up in three integrals : first corresponding to the area O AB, second to the area BCE and third to the area ACD.

Now solving $x^2 - 2ax + y^2 = 0$ for x, we have $x = a \pm \sqrt{(a^2 - y^2)}$.

Clearly lower and upper limits of x for the area O AB are $y^2/2a$ and $a - \sqrt{(a^2 - y^2)}$ and those of y for this area are 0 and a.

The limits of x for the area BCE are from $y^2/2a$ to 2a and those of y are from a to 2a. Again the limits of x for the area ACD are from $a + \sqrt{(a^2 - y^2)}$ to 2a and those of y from 0 to a.

Hence, we have

$$\int_0^{2a}\int_{\sqrt{(2ax-x^2)}}^{\sqrt{(ax)}} V\,dx\,dy = \int_0^{a}\int_{y^2/2a}^{a-\sqrt{(a^2-y^2)}} V\,dy\,dx + \int_a^{2a}\int_{y^2/2a}^{2a} V\,dy\,dx + \int_0^{a}\int_{a+\sqrt{(a^2-y^2)}}^{2a} V\,dy\,dx.$$

24. (C) Since f (x) being a cubic function, f' (x) is a quadratic function. f (x) has relative minimum and maximum at

$x = -1$ and $x = \frac{1}{3}$, so

$$f'(-1) = f'\left(\frac{1}{3}\right) = 0$$

Then $f'(x) = a(x + 1)\left(x - \frac{1}{3}\right)$

$$= a\left(x^2 + \frac{2}{3}x - \frac{1}{3}\right) \qquad \text{where a is constant}$$

Integrating w.r.t. x, we get

$$f(x) = a\left(\frac{x^3}{3} + \frac{x^2}{3} - \frac{x}{3}\right) + b \qquad ...(1)$$

where b is constant of integration and $f(-2) = 0$

then $a\left(-\frac{8}{3} + \frac{4}{3} + \frac{2}{3}\right) + b = 0$ $\qquad \therefore b\ \frac{2}{3} = a$

$\therefore$ From (1), $f(x) = \frac{a}{3}(x^3 + x^2 - x + 2)$ ...(2)

Also $\int_{-1}^{1} f(x)dx = \frac{14}{3}$

$\Rightarrow \frac{a}{3}\int_{-1}^{1}(x^3 + x^2 - x + 2)dx = \frac{14}{3}$

$\Rightarrow \frac{a}{3}\int_{-1}^{1}(x^3 + x)dx + \frac{a}{3}\int_{-1}^{1}(x^2 + 2)dx = \frac{14}{3}$

$\Rightarrow 0 + \frac{2a}{3}\int_{0}^{1}(x^2 + 2)dx = \frac{14}{3}$ ($\because$ First integral is odd and II integral is even)

$\Rightarrow \frac{2a}{3}\left\{\frac{x^3}{3} + 2x\right\}_0^1 = \frac{14}{3}$ $\Rightarrow \frac{2a}{3}\left(\frac{1}{3} + 2\right) = \frac{14}{3}$

$\therefore$ a = 3

From (2), $f(x) = x^3 + x^2 - x + 2$

25.(B) Since f (x) is minimum at x = – 2 and maximum at x = 2, let $g(x) = ax^3 + bx^2 + cx + d$

$\therefore$ g (x) is also minimum at x = – 2 and maximum at x = 2

$\therefore$ a < 0

$\because$ a is a root of $x^2 - x - 6 = 0$ i.e., x = 3, – 2

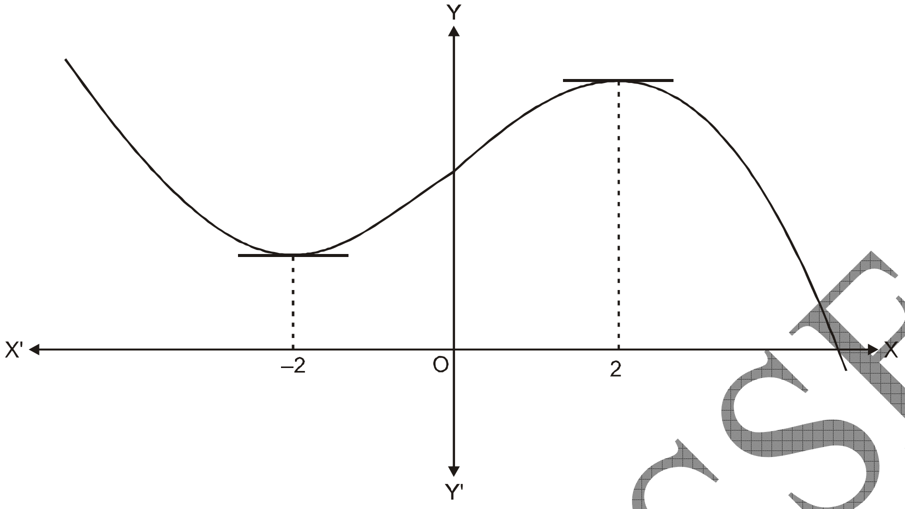
$\therefore$ a = – 2

Then $g(x) = -2x^3 + bx^2 + cx + d$

$\therefore g'(x) = -6x^2 + 2bx + c = -6(x + 2)(x - 2)$

{$\because$ g (x) is minimum at x = – 2 and maximum at x = 2}

On comparing we get



b = 0 and c = 24

Since minimum and maximum values are positive

$\therefore$ g (– 2) > 0 $\Rightarrow$ 16 – 48 + d < 0 $\Rightarrow$ d > 32

and g (2) > 0 $\Rightarrow$ – 16 + 48 + d > 0 $\Rightarrow$ d > – 32

It is clear d > 32.

Hence a = – 2, b = 0, c = 24, d > 32.

26.(B) Given,

$$f(x) = \begin{cases} e^x, & 0 \le x \le 1 \\ 2 - e^{x-1}, & 1 < x \le 2 \\ x - e, & 2 < x \le 3 \end{cases}$$

and g(x) $= \int_0^x f(t)\,dt \Rightarrow g'(x) = f(x)$

Put g'(x) $= 0 \Rightarrow x = 1 + \log_e 2$ and $x = e$.

Also, $g''(x) = \begin{cases} e^x, & 0 \le x \le 1 \\ -e^{x-1}, & 1 < x \le 2 \\ 1\ , & 2 < x \le 3 \end{cases}$

At $x = 1 + \log_e 2$,

$g''(1 + \log_e 2) = -e^{\log_e 2} < 0$, g(x) has a local maximum.

Also, at $x = e$,

$g''(e) = 1 > 0$, g(x) has a local minima.

$\because$ f(x) is discontinuous at x = 1, then we get local maxima at x = 1 and local minima at x = 2

Hence, (b) is the correct answer.

27.(B) here $f(x, y) = \sqrt{xy}$

$$\frac{\partial f}{\partial x} = \frac{\sqrt{y}}{2\sqrt{x}} = \frac{1}{2}\sqrt{\frac{y}{x}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}\sqrt{\frac{x}{y}}$$

here Let h = 0·01 k = –0·02

a = 4 b = 4

$$\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(x,y) = h\frac{\partial f}{\partial x} + k\frac{\partial f}{\partial y}$$

$$= \frac{0{\cdot}01\,x - 0{\cdot}02\,y}{2\sqrt{xy}}$$

$$= \frac{-0{\cdot}01 \times 4}{8}$$

$$= -0{\cdot}005$$

By Taylor's polynomial

$$f(4\cdot01, 3\cdot98) = \sqrt{4\cdot01 \times 3\cdot98} = f(4,4) + \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(4, 4) + \frac{1}{2!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(4, 4) + ...$$

$$= 4 + (-0\cdot005) \times 4$$

$$= 3\cdot98$$

28.(B) Here $\frac{\partial x}{\partial u} = e^u, \ \frac{\partial y}{\partial u} = -e^{-u}$ ...(1)

$\frac{\partial x}{\partial v}$ and $\frac{\partial x}{\partial v} = -e^{-v}, \ \frac{\partial y}{\partial v} = -e^v$ ...(2)

Now taking z as a composite function of x and y, we have

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= \frac{\partial z}{\partial x}e^u - \frac{\partial z}{\partial y}e^{-v} \quad \text{[by(1)]} \quad ...(3)$$

and

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$$

$$= \frac{\partial z}{\partial x}(-e^{-v}) + \frac{\partial z}{\partial y}(-e^{-v}) \quad \text{[by(2)]} \quad ...(4)$$

Subtracting (4) from (3), we obtain

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}(e^u + e^{-v}) - \frac{\partial z}{\partial y}(e^{-u} - e^v)$$

$$= x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

29. (D) We have z = f(x, y)

and x = 2 y = – 3 at w = 1

$\frac{\partial x}{\partial w} = 7$ $\frac{\partial y}{\partial w} = 2$ at w = 1

$$\left.\frac{\partial z}{\partial x}\right|_{(2,-3)} = -8 \quad \left.\frac{\partial z}{\partial y}\right|_{(2,-3)} = -3$$

$$\left(\frac{dz}{d\omega}\right)_{\omega=1} = \left(\frac{\partial z}{\partial x}\right)_{\omega=1}\left(\frac{dx}{d\omega}\right)_{\omega=1} + \left(\frac{\partial z}{\partial y}\right)_{\omega=1}\left(\frac{dy}{d\omega}\right)_{\omega=1}$$

$$= -8 \times 7 - 3 \times 2$$

$$= -56 - 6$$

$$= -62$$

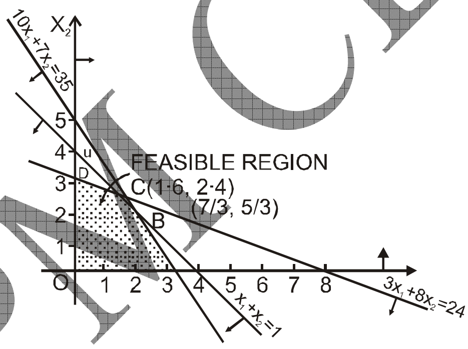
30. (A) Let us draw the lines :

$$x_1 + x_2 = 4$$

$$3x_1 + 8x_2 = 24$$

$$10x_1 + 7x_2 = 35$$

and $x_1 = 0, x_2 = 0$



which correspond to the inequalities of the given constraints. On considering the solution space for each of the given inequality, we find that the common solution space, represented by the shaded area OABCD, is the feasible region.

Now to search the maximum value of z which is at one of the corners of the polygon OABCD, we find that

At A (3.5, 0) ; $z = 5 \times 3.5 + 7.0 = 17.5$

At B $\left(\frac{7}{31}, \frac{5}{3}\right)$; $2 = 5\frac{7}{8} + 7\frac{5}{3} = 23.3$

At C (1.6, 2.4) ; $z = 5 \times 1.6 + 7 \times 2.4 = 24.8$

At D (0, 3) ; $z = 5.0 + 7.3 = 21$

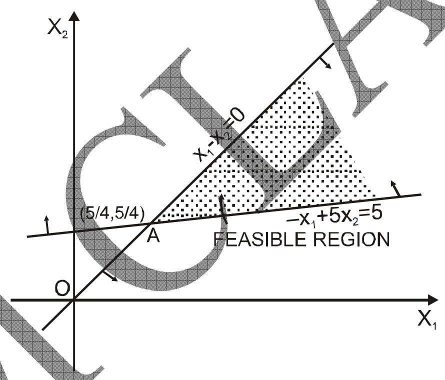
Thus z is maximum at C where $x_1 = 1{\cdot}6$, $x_2 = 2{\cdot}4$ and max. $z = 24{\cdot}8$.

31.(C) The bounding lines corresponding to the inequalities of the given constraints are :

$$x_1 - x_2 = 0$$

$$x_1 - 5x_2 = -5$$

and $$x_1 = 0, x_2 = 0.$$



Draw these lines in a two dimensional space and consider the solution space for each given inequality. We find that the feasible region i.e. their common solution space is unbounded from one side.

But it is clear from the figure that the objective function z attains its minimum value at the point A which is the intersection of the two lines $x_1 - x_2 = 0$ and $-x_1 + 5x_2 = 5$. Solving them we find that $x_1 = x_2 = \frac{5}{4}$. but optional sol^n is unbounded.

32.(A) here $S_1 = \{(x_1, x_2) : 3x_1 + 5x_2 = 2, x_1, \geq 0, x_2 \in R\}$

$S_2 = \{(x_1, x_2) : 3x_1 + 5x_2 = 2, x_1, \in R\ x_2 \in R\}$

$S_1 \cap S_2 = S_1$

and S_1 represents a line in 2D in upperhalf plane

and S_1 be convex and unbounded

33.(A) Given lpp Maximize $y = \min \{|2x_1 + 5x_2|, |2x_1 - 5x_2|\}$; $x_1, x_2 \geq 0$

it can be written as

Maximize y

Subject to constraint $|2x_1 + 5x_2| \geq y$

and $|2x_1 - 5x_2| \geq y$

{if $|x| \geq a$ then $x \leq -a$ or $x \geq a$}

So maximize y

Subject to constraints $2x_1 + 5x_2 \geq y$

$2x_1 + 5x_2 \leq -y$

$2x_1 - 5x_2 \geq y$

$2x_1 - 5x_2 \geq -y$

and $x_1\ x_2 \geq 0$

but $2x_1 + 5x_2 \leq -y$ can not hold for $x_1, x_2 \geq 0$

$\Rightarrow$ Lpp Max y

S to $2x_1 + 5x_2 \geq y$

$2x_1 - 5x_2 \geq y$

$2x_1 - 5x_2 \leq -y$

and $x_1\ x_2 \geq 0$

$\Rightarrow$ Max y

S. to $2x_1 + 5x_2 - y \geq 0$

$2x_1 - 5x_2 - y \geq 0$

$2x_1 - 5x_2 + y \leq 0$

34.(D) Since, only (c) satisfy given definition

ie, $f\{f^{-1}(B)\} = B$

Only, if $B \subseteq f(x)$

35.(A) Since, $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is linear equation in three variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

36.(D) Given, A =, $\begin{bmatrix} a & b \\ c & a \end{bmatrix}$

a, b, c $\in$ {0, 1, 2, ..., p – 1}

If A is skew-symmetric matrix, then a = 0 , b = – c

$\therefore$ $|A| = -b^2$.

Thus, p divides | A | only when b = 0 ...(i)

Again, if A is symmetric matrix, then b = c and $|A| = a^2 - b^2$.

This, p divides | A | if either p divides (a – b) or p divides (a + b).

p divides (a – b), only when a = b

ie, $a = b \in \{0, 1, 2, ..., (p-1)\}$

ie, p choices ...(ii)

p divides (a + b).

$\Rightarrow$ p choices, including a = b = 0 included in (i)

$\therefore$ Total number of choices are (p + p – 1) = 2p – 1.

Hence, (d) is the correct option.

37.(C) Since $W \neq R^4$ eg $(1, 2, 3, 4) \notin W$

Thus dim W < 4

now u_1 (1, –1, 0, 0) and $u_2 = (0, 0, -1, 1)$

$u_3 = (0, 1, -1, 0)$

are three independent vectors in W

Thus dim W = 3

So (u_1, u_2, u_3) can form a basis of W

38.(A) Given Matrix $A = \begin{bmatrix} -3 & 12 \\ -2 & 7 \end{bmatrix}$

if $[v_1\ v_2]^T$ be the eigenvector corresponding to eigenvalue $\lambda = 3$

Then $$\begin{bmatrix} -6 & 12 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-6v_1 + 12v_2 = 0$

$\Rightarrow \quad v_1 = 2 \quad v_2 = 1$

So $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is one of the eigen vector corresponding to $\lambda = 3$

Now if $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ be the eigen vector corresponding to $\lambda = 1$

So $\begin{bmatrix} -4 & 12 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

given $w_1 = 3 \quad w_2 = 1$

So $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is one of the eigen vector corresponding to $\lambda = 1$

39.(A) By the properties of Eigen values of real symmetric matrix

All the eigen values of a real symmetric matrix are real.

40.(B) Given matrix A = $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_1 \to R_1 + 2R_3$

$R_2 \to R_2 - R_3$

$A \sim \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$C_2 \to C_2 + C_4$

$C_3 \to C_3 + C_4$

$$A \sim \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 + R_1$

$$A \sim \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_3$

$C_2 \leftrightarrow C_4$

$R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank A = 2

nullity = n – rank A

= 4 – 2

2

41.(D) Since $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{\nabla}\cdot\vec{r} = 3\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) . (x\hat{i} + y\hat{j} + z\hat{k})$

$= 3$

$\vec{\nabla}\,(\vec{\nabla}\cdot\vec{r})\vec{r} = 3(\vec{\nabla}\cdot\vec{r})$

$= 3 \times 3 = 3^2$

$\vec{\nabla}\,[\vec{\nabla}(\vec{\nabla}\cdot\vec{r}).\vec{r}] = \vec{\nabla}[3^2].\vec{r}$

$= 3^3$

42. (B) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$|\vec{a}| = 1 = a_1^2 + a_2^2 + a_3^2$

$|\vec{b}| = 1 = b_1^2 + b_2^2 + b_3^2$

$$|\vec{a}\times\vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = [a_2 b_3 - b_2 a_3] - [a_1 b_3 - b_1 a_3] + [a_1 b_2 - a_2 b_1]$$

$|\vec{a}\times\vec{b}| = |\vec{a}\times\vec{b}||\vec{a}\times\vec{b}|$

$= (a_2 b_3 - b_2 a_3)^2 + (a_1 b_3 - b_1 a_3)^2 + (a_1 b_2 - a_2 b_1)^2$

$= a_2^2 b_3^2 + b_2^2 a_3^2 + a_1^2 b_3^2 + b_1^2 a_3^2 + a_1^2 b_2^2 + a_2^2 b_1^2 - 2\, a_2 b_2 a_3 b_3 - 2a_1 b_1 a_3 b_3 - 2a_1 b_1 a_2 b_2$

$= (1 - a_1^2 - a_3^2) b_3^2 + b_2^2 a_3^2 + a_1^2 b_3^2 + (1 - a_1^2 - a_2^2) b_1^2 + (1 - a_2^2 - a_3^2)\, b_2^2 + a_2^2 b_1^2 - 2a_2 b_2 a_3 b_3$

$-2a_1b_1a_3b_3 - 2a_1b_1a_2b_2$

$= b_1^2 + b_2^2 + b_3^2 - a_1^2 b_1^2 - a_2^2 b_2^2 - a_3^2 b_3^2 - 2a_1b_1a_3b_3 - 2a_1b_1a_2b_2 - 2a_2b_2a_3b_3$

$= 1 - \left[a_1^2 b_1^2 = a_2^2 b_2^2 + a_3^2 b_3^2 + 2a_1 b_1 a_3 b_3 + 2a_1 b_1 a_2 b_2 + 2a_2b_2a_3b_3\right]$

$= 1 - \left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow \quad |\vec{a}\times\vec{b}| + (\vec{a}\cdot\vec{b})^2 = 1$

43. (C) Let $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2 = 0$ ...(i)

on diff w.r.to x

$$3x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} - 6x - 6y\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right) = \frac{6x}{3x^2+3y^2-6y}$$

$$\left(\frac{dy}{dx}\right)_{(1,0)} = \frac{6}{3}$$

$$= 2$$

Tangent vector is given by

$(j - 0) = 2\,(i - 1)$

$2i - j - 2 = 0$

44.(C) Let angle between $\vec{a}$ and $\vec{b}$ be θ_1, $\vec{c}$ and $\vec{d}$ be θ_2 and $\vec{a}\times\vec{b}$ and $c \times d$ be θ.

Since $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 1$

$\Rightarrow \quad \sin\theta_1 \cdot \sin\theta_2 \cdot \cos\theta = 1$

$\Rightarrow \quad \theta_1 = 90°,\ \theta_2 = 90°,\ \theta = 0°$

$\Rightarrow \quad \vec{a}\perp\vec{b}, \vec{c}\perp\vec{d}, (\vec{a}\times\vec{b}) \,||\, (\vec{c}\times\vec{d})$

So $\vec{a}\times\vec{b} = k(\vec{c}\times\vec{d})$ and $\vec{a}\times\vec{b} = k(\vec{c}\times\vec{d})$

$\Rightarrow \quad (\vec{a}\times\vec{b})\cdot\vec{c} = k(\vec{c}\times\vec{b})\cdot\vec{c}$ and $(\vec{a}\times\vec{b})\cdot\vec{d} = k(\vec{c}\times\vec{d})\cdot\vec{d}$

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 0$ and $[\vec{a}\,\vec{b}\,\vec{d}] = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ and $\vec{a}, \vec{b}, \vec{d}$ vectors so options (a) and (b) are incorrect.

Let $\vec{b} \parallel \vec{d}$ $\quad \Rightarrow \vec{b} = \pm\vec{d}$

As $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 1 \quad \Rightarrow \quad (\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{b}) = \pm 1$

$\Rightarrow [\vec{a}\times\vec{b}\,\vec{c}\,\vec{b}] = \pm 1$

$\Rightarrow [\vec{c}\,\vec{b}\,\vec{a}\times\vec{b}] = \pm 1$

$\Rightarrow \vec{c}\cdot[\vec{b}\times(\vec{a}\times\vec{b})] = \pm 1 \Rightarrow \vec{c}\cdot[\vec{a} - (\vec{b}\cdot\vec{a})\vec{b}] = \pm 1$

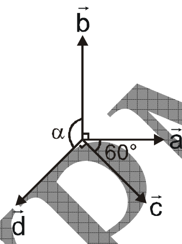
$\Rightarrow \qquad \vec{c}\cdot\vec{a} = \pm 1 \quad (\because \vec{a}\cdot\vec{b} = 0)$

which is a contradiction so option (c) is correct.

Let option (d) is correct

$\Rightarrow \qquad \vec{d} = \pm\vec{a}$

and $\qquad \vec{c} = \pm\vec{b}$



As $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = 1$

$\Rightarrow \quad (\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{a}) = \pm 1$

which is a contradiction so option (d) is incorrect.

Alternatively option (c) and (d) may be observed from the given figure

45. (C) Given, $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \int \frac{y}{\sqrt{1-y^2}} dy = \int dx$

$\Rightarrow -\sqrt{1-y^2} = x + c \Rightarrow (x + c)^2 + y^2 = 1$

Here, centre (–c, 0); radius = 1

46. (A) Given, $\frac{dy}{dx} = \frac{-\cos x(y+1)}{2+\sin x}$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x} dx$

On integrating both sides

$\int \frac{dy}{y+1} = -\int \frac{\cos x}{2+\sin x} dx$

$\Rightarrow \log (y + 1) = - \log (2 + \sin x) + \log c,$

When $x = 0, y = 1 \Rightarrow c = 4$

$\Rightarrow y + 1 = \frac{4}{2+\sin x}$

$\therefore y\left(\frac{\pi}{2}\right) = \frac{4}{3} - 1$

$\Rightarrow y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

47. (A) Given, x dy = y(dx + y dy), y > 0

$\Rightarrow x\,dy - y\,dx = y^2\,dy$

$\Rightarrow \quad \dfrac{x\,dy - y\,dx}{y^2} = dy$

$\Rightarrow \quad d\left(\dfrac{x}{y}\right) = -\,dy$

On integrating both sides, we get

$\dfrac{x}{y} = -y + c \quad$ ...(i)

Since, $\quad y(1) = 1 \Rightarrow x = 1, y = 1$

$\therefore \quad c = 2$

$\therefore$ Eq. (i) becomes, $\quad \dfrac{x}{y} + y = 2$

Again, for $x = -3$

$\Rightarrow \quad -3 + y^2 = 2y$

$\Rightarrow \quad y^2 - 2y - 3 = 0$

$\Rightarrow \quad (y + 1)(y - 3) = 0$

As $y > 0$ take $y = 3$, neglecting $y = -1$

48. (C) $\dfrac{dy}{dx} - \dfrac{x-2}{x(x-1)}y = \dfrac{x^2(2x-1)}{x-1}$

Here $\quad P = -\dfrac{x-2}{x(x-1)}$

$= -\dfrac{2}{x} + \dfrac{1}{x-1}$

$e^{\int P\,dx} = e^{-2\log x + \log(x-1)}$

$= e^{\log \frac{x-1}{x^2}}$

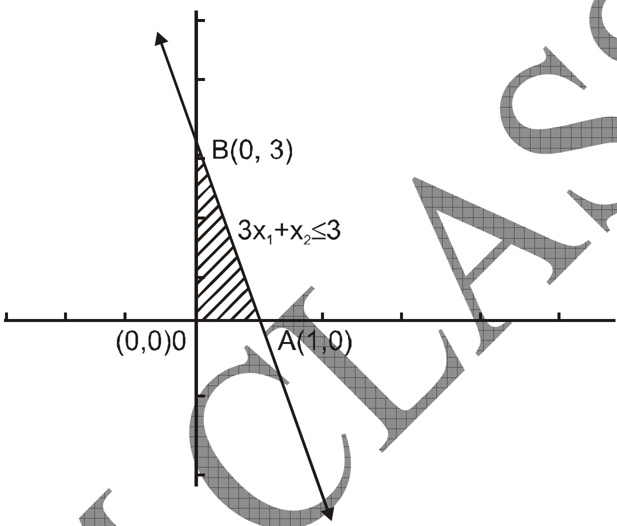
$= \frac{x-1}{x^2}$

49.(C) Given l.p.p. is Maximize $x_1 + 3x_2$

s.to $3x_1 + x_2 \le 2$

$x_1\ x_2 \ge 0$

Graphical representation.



The shaded region OAB is optimal region for given Lpp. and B(0, 3) gives optimum value of the objective function which is = 9

50.(B) Since, $P(A/\bar{B}) + P(\bar{A}/\bar{B}) = 1$

$\therefore \quad P(\bar{A}/\bar{B}) = 1 - P(A/\bar{B})$

51.(B) Here, $P(u_i) = ki$, $\sum P(u_i) = 1$

$\Rightarrow \quad k = \frac{2}{n(n+1)}$

$\therefore \quad \lim_{n\to\infty} P(W) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i^2}{n(n+1)^2}$

$= \lim_{n\to\infty} \frac{2n(n+1)(2n+1)}{6n(n+1)^2} = \frac{2}{3}$

52. (C) Given $y_0 = 1$, $y_1 = 3$, $y_2 = 9$, ..., $y_3 = ?$, $y_4 = 81$.

Four values of y are given. Let y be polynomial of degree 3 therefore, we have $\Delta^4 y_0 = 0$

or $(E - 1)^4 y_0 = 0$

or $(E^4 - 4E^3 + 6E^2 - 4E + 1)\, y_0 = 0$

or $E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4Ey_0 + y_0 = 0$

or $y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

Substituting the values of y_0, y_1, y_2 and y_4, we get

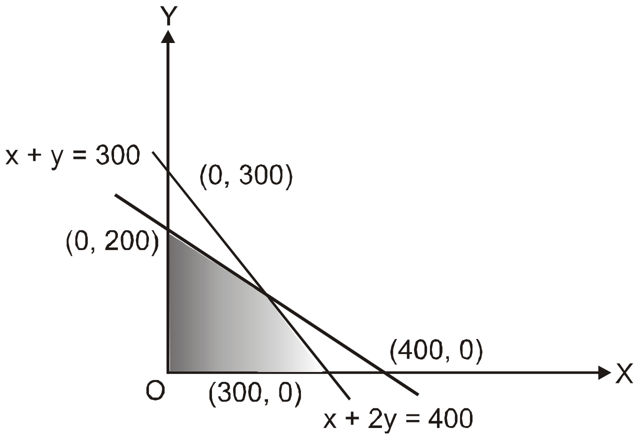
$81 - 4y_3 + 6 \times 9 - 4 \times 3 + 1 = 0$

$y_3 = 31.$

We now have two paths that give different values for the limit and so the limit doesn't exist.

53. (A) The linear constraints are $x + 2y \le 400,\ x + y \le 300$

and $x, y \ge 0$. Also Max z = 300x + 400y.



Hence the region is bounded.

54.(C) Here, $P(A \cup B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{5}$. So, from the addition theorem,

$$\frac{3}{5} = P(A) + P(B) - \frac{1}{5}$$

or $\frac{4}{5} = 1 - P(\bar{A}) + 1 - P(\bar{B})$

$\therefore \; P(\bar{A}) + P(\bar{B}) = 2 - \frac{4}{5} = \frac{6}{5}.$

Hence (c) is correct answer.

55.(A) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{1/4}$

$\Rightarrow P(A \cap B) = \frac{1}{8}$

Hence events A and B are not mutually exclusive.

$\therefore$ Statement II is incorrect.

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}\Rightarrow P(B)=\frac{1}{2}$$

$$\because P(A\cap B)=\frac{1}{8}=P(A).P(B)$$

$\therefore$ events A and B are independent events.

$$P\left(\frac{A^c}{B^c}\right)=\frac{P(A^c\cap B^c)}{P(B^c)}=\frac{P(A^c)P(B^c)}{P(B^c)}=\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{2}{1}=\frac{3}{4}$$

Hence statement I is correct.

$$\text{Again}\, P\left(\frac{A}{B}\right)+P\left(\frac{A}{B^c}\right)=\frac{1}{4}+\frac{P(A\cap B^c)}{P(B^c)}$$

$$=\frac{1}{4}+\frac{P(A)-P(A\cap B)}{P(B^c)}=\frac{1}{4}+\frac{\frac{1}{4}-\frac{1}{8}}{\frac{1}{2}}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

Hence statement III is incorrect.

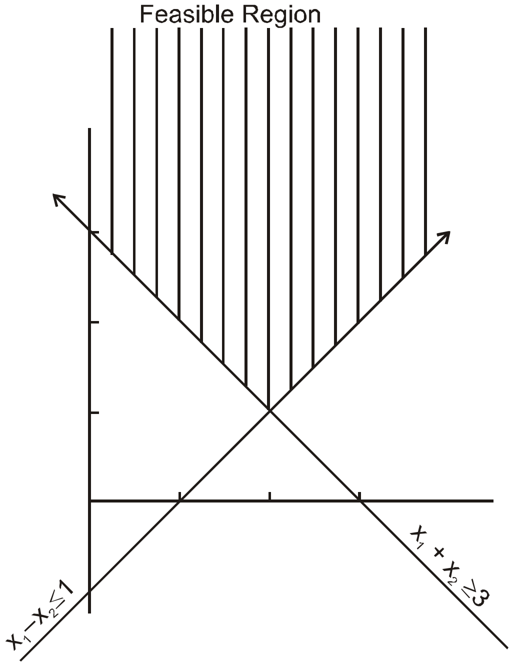
56.(C) Given L.P.P Max $Z = 5x_1 + 2x_2$

Subject to $x_1 - x_2 \le 1$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

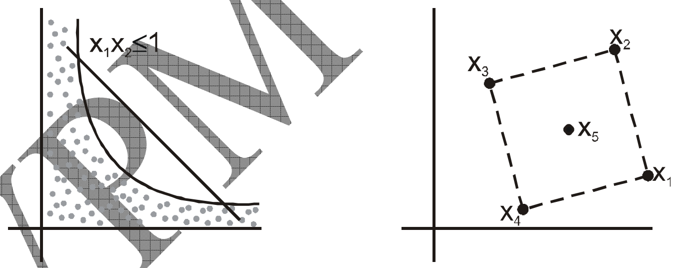
Graphical representation



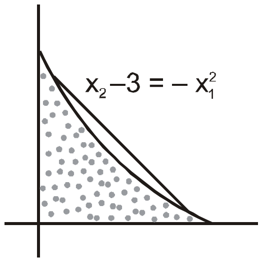
The region of feasible solutions is the shaded area

Hence z can be made arbitrarily large and the problem has no finite maximum value of Z. This problem said to have unbounded solutions.

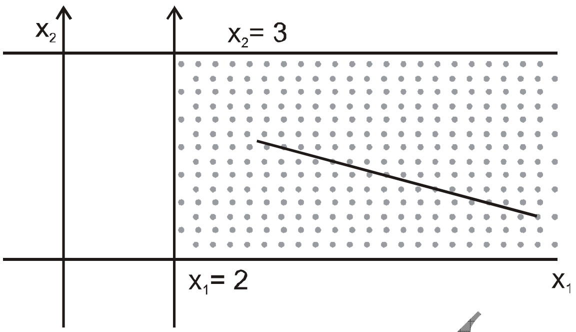
57. (C)1. $X = \{[x_1, x_2];\ x_1x_2 \leq 1\ x_1 \geq 0,\ x_2 \geq 0\}$



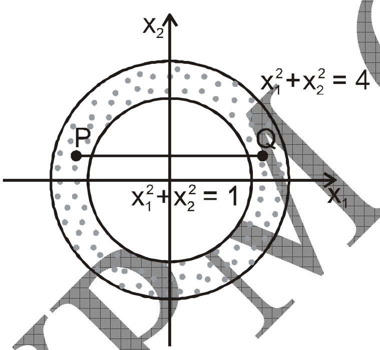
2. $X = \{[x_1, x_2];\ x_2 - 3 \geq -x_1^2,\ x_1 \geq 0,\ x_2 = 0\}$



3. $X = \{[x_1, x_2];\ x_1 \geq 2,\ x_2 \leq 3\}$



5. $X = \{[x_1, x_2];\ x_1^2 + x_2^2 \geq 1,\ x_1^2 + x_2^2 \leq 4\}$



Only [C] is convex set.

58.(A) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap) - P(B \cap C) - P(A \cap B) - P(C \cap A) + P(A \cap B \cap C)$

$= 0.6 + 0.4 + 0.5 - 0.2 - P(B \cap C) - 0.3 + 0.2 = 1.2 - P(B \cap C)$

because $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad 0.8 = 0.6 + 0.4 - P(A \cap B)$

But $0.85 \le P(A \cup B \cup C) \le 1$

$\therefore \quad 0.85 \le 1.2 - P(A \cup B \cup C) \le 1 \Rightarrow 0.2 \le P(B \cap C) \le 0.35$

Hence (a) is the correct answer.

59.(A) $P((E_1 \cup E_2)) \cap (\bar{E}_1 \cup \bar{E}_2)$

$= P\left((E_1 \cup E_2) \cap (\overline{E_1 \cup E_2})\right) = P(\phi) = 0 \le \frac{1}{4}.$

Hence (a) is the correct answer.

60. (A) Let E_i be the event of a person to get into an accident. Then $P(E_i) = p \ \forall i$ P(at least one man meet with an accident)

$= P(E_1 \cup E_2 \cup ... \cup E_n)$

$= 1 - P(\bar{E}_1 \cup \bar{E}_2 \cup ... \cup \bar{E}_n) = 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap ... \cap \bar{E}_n)$

$= 1 - P(\bar{E}_1)P(\bar{E}_2)...P(\bar{E}_n) = 1 - (1-p)(1-p)...(1-p)$

$= 1 - (1 - p)^n$

Hence P (atleast one man meets with an accident/a person is chosen)

$= \frac{1}{n}\left(1 - (1-p)^n\right).$

61.(C) Let P $(1 + \Delta)^{-1/2} \Delta$

if $E = 1 + \Delta \Rightarrow P = E^{-1/2} \Delta$

$= \delta$ (central difference operator)

62.(B) Since $R \cap R'$ are not disjoint, there is at least one ordered pair, say, (a, b) in $R \cap R'$.

but $(a, b) \in R' \Rightarrow (a, b) \in R$ and $(a, b) \in R'$

since R and R' are symmetric relations, we get

$(b, a) \in R$ and $(b, a) \in R'$

and consequently $(b, a) \in R \cap R'$

similarly any other ordered pair $(c, d) \in R \cap R'$, then we must also have, $(d, c) \in R \cap R'$.

Hence $R \cap R'$ is symmetric.

Hence (b) is the correct answer.

63.(B) Statement B is not true

By venn diagram

now check if for $A \cup B - A$

we get the difference

$\Rightarrow A - B \neq (A \cup B) - A$

64.(A) Error term in trapezoidal rule is given by –

$$Er \leq -\frac{h^2(b-a)}{12} f''(\xi)$$

here b = 1·4 a = 0·2

$$Er \leq -\frac{1\cdot 2}{12} h^2 y''$$

$$Er \leq -\frac{h^2}{10} y''$$

65.(D) We form a difference table

X	x	y	$\Delta\begin{pmatrix} = \delta \\ x + \frac{1}{2}h \end{pmatrix}$	Δ^2	Δ^3	Δ^4

0·7	– 3	0·644218				
			73138			
0·8	– 2	0·717356		– 7167		
			65971		– 660	
0·9	– 1	0·783327		– 7827		79
			58144		– 581	
1·0	0	0·841471		– 8408		85
			48736		– 496	
1·1	1	0·891207		– 8904		87
			40832		– 409	
1·2	2	0·932039		– 9313		
			31519			
1·3	3	0·963558				

$$\frac{dy}{dx} = \frac{1}{2h}\left[y_1 - y_{-1} - \frac{1}{6}(\Delta^2 y_1 - \Delta^2 y_{-1}) + \frac{1}{30}(\Delta^4 y_1 - \Delta^4 y_{-1}) - \ldots\right]$$

$$\therefore \left(\frac{dy}{dx}\right) = \frac{1}{0\cdot 2}\left[0\cdot 891207 - 0\cdot 783327 + \frac{1}{6}(0\cdot 008904 - 0\cdot 007827) + \frac{1}{30}(0\cdot 000087 - 0\cdot 000079 + \ldots)\right]$$

$= 0\cdot 54030.$

66. (D) Let

$f(x_1) = x^4 - 12x + 7$

then we have $f'(x) = 4x^3 - 12$

Let $x_0 = 2$

From eqns. (i) and (ii), we get

$f(x_0) = f(2) = 2^4 - 12.2 + 7 = -1$

and $f'(x_0) = f'(2) = 4(2)^3 - 12 = 20$

Applying Newton's method, we get

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{(-1)}{20} = \frac{41}{20} = 2.05$$

and
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.05 - \frac{(2.05)^4 - 12(2.05) + 7}{4(2.05)^3 - 12} = 2.6706$$

Hence, the root of the equation is 2.6706

67.(B) Forward difference table is given by

x	y	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1	4				
		9			
2	13		12		
		21		6	
3	34		18		0
		39		6	
4	73		24		
		63			
5	136				

on Comparing we get

a = 39 b = 18 c = 6

68.(A) A × B = {(1, 2) (1, 5), (2, 2), (2, 5)}

A × C = {(1, 5), (1, 7), (2, 5), (2, 7)}

∴ (A × B) ∩ (A × C)

= {(1, 5), (2, 5)}

69. (D) Let us define the events :

A : X speaks the truth, B : Y speaks the truth

Then $\bar{A}$ and $\bar{B}$ represent the complementary events that X and Y tell a lie respectively. We are given :

$$P(A) = \frac{3}{3+2} = \frac{3}{5} \quad \Rightarrow \quad P(\bar{A}) = 1 - \frac{3}{5} = \frac{2}{5}$$

and $$P(B) = \frac{5}{5+3} = \frac{5}{8} \quad \Rightarrow \quad P(\bar{B}) = 1 - \frac{5}{8} = \frac{3}{8}$$

The event E that X and Y contradict each other on an identical point can happen in the following mutually exclusive ways :

(i) X speaks the truth and Y tells a lie, i.e., the event $A \cap \bar{B}$ happens,

(ii) X tells a lie and Y speak the truth, i.e., then event $\cap(\bar{A})$ B happens.

Hence by addition theorem of probability, the required probability is given by :

$$P(E) = P(i) + P(ii) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$[Since A and B are independent]

$$= \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8} = \frac{19}{40} = 0{\cdot}475$$

Hence A and B are likely to contradict each other on a identical point in 47·5% of the cases.

70.(A) Let X ~ B (n, p). Then we are given : Mean = np = 4 ...(1) and Var (X) = npq = $\frac{4}{3}$.

Dividing, we get q = $\frac{1}{3}$ $\Rightarrow$ p = $\frac{2}{3}$. Substituting in (1), we obtain n = $\frac{4}{p} = \frac{4 \times 3}{2}$ = 6.

$\therefore$ P (X ≥ 1) = 1 – P (X = 0) = 1 – q^n = 1 – $\left(\frac{1}{3}\right)^6$ = 1 – $\frac{1}{729}$ = 0·99863.

71.(B) since x ~ P(X)

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

here λ = 2

$$P(X = 2 \mid X > 1) = \frac{P(X = 2 \cap X > 1)}{P(X > 1)}$$

$$= \frac{P(X = 2)}{P(X > 1)}$$

$$= \frac{P(X = 2)}{1 - P(X < 1)} = \frac{\frac{e^{-2}\,2^2}{2!}}{1 - (e^{-2} + 2e^{-2})}$$

$$= \frac{2}{e^2 - 3}$$

72. (B) $P(A^c) = 0{\cdot}3$ ⇒ $P(A) = 0{\cdot}7$

$P(B) = 0{\cdot}4$ ⇒ $P(B^c) = 0{\cdot}6$

$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$

= 0·7 + 0·6 – 0·5

= 0·8

$$P[B|A \cap B^c] = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$$

$$= \frac{P(B \cap A)}{P(A \cup B^c)}$$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) - P(A \cap B^c)$

$= 0{\cdot}7 - 0{\cdot}5$

$= 0{\cdot}2$

$\Rightarrow P[B \mid A \cup B^c) = \frac{0{\cdot}2}{0{\cdot}8} = \frac{1}{4} = 0{\cdot}25$

73. (B) A × B has 4 × 2 = 8 elements. Hence number of possible subsets are $2^8 = 256$. But these subsets consist of empty subset also, hence number of relations = 256 – 1 = 255.

74.(B) We find that

1(0) = 0 $\Rightarrow$ O(0) = 1

1(1) = 1,2(1) = 2,3 (1) = 3,4 (1) = 4,5(1) = 5,6(1) = 0 $\Rightarrow$ O(1) = 6

1(2) = 2,2(2) = 4,3(2) = 0 $\Rightarrow$ O(2) = 3

1(3) = 3,2(3) = 0 $\Rightarrow$ O(3) = 2

1(4) = 4,2(4) = 2,3(4) = 0 $\Rightarrow$ O(4) = 3

1(5) = 5,2 (5) = 4,3(5) = 3,4(5) = 2,5(5) = 1,6(5) = 0 $\Rightarrow$ O(5) = 6

Observing the orders of all the elements of G, we find

O(1) = O(5) = 6 = O(G)

Therefore G = [1] = [5] i.e. 1 and 5 are two generators of G.

75. (C) Every cyclic group have two trivial subgroup ϕ and itself

76.(D) Let $\sigma = \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}$

We know that

order of subgroup = order of σ

$$\sigma = \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}\begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}$$

$$= \begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}\begin{pmatrix} a & c & d & b \\ a & d & b & c \end{pmatrix}$$

$$= \begin{pmatrix} a & b & c & d \\ a & d & b & c \end{pmatrix}$$

$$\sigma^3 = \sigma^2\sigma = \begin{pmatrix} a & b & c & d \\ a & d & b & c \end{pmatrix}\begin{pmatrix} a & b & c & d \\ a & c & d & b \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} a & b & c & d \\ a & d & b & c \end{pmatrix}\begin{pmatrix} a & d & b & c \\ a & b & c & d \end{pmatrix}$$

$$= \begin{pmatrix} a & b & c & d \\ a & b & c & d \end{pmatrix}$$

$$\sigma^3 = I$$

$\Rightarrow \quad O(\sigma) = 3$ = order of subgroup.

77. (B) Since a homomorphism ϕ is defined from $(z_{12}\ x_{12})$ to $(z_{30}\ x_{30})$ then no. of distinct homomorphism are given by

gcd (12, 30) = 6

78.(C) $2^7 = 1 \pmod 9$

$(2^7)^7 = 1 \pmod 9$

$2^{49} = 1 \pmod 9$

$2.2^{49} = 2 \pmod 9$

$2^{50} \quad = 2 \pmod 9$

79.(B) Here 'a' = 75, 'h' = 5, 'a + hu' = 82

$\therefore$ a + hu = 82 Þ 75 + 5u = 82 $\Rightarrow$ 4 = 7/5 = 1.4

$\therefore$ From the given data we have the following difference table :

x	u_x	Δu_x	$\Delta^2 u_x$	$\Delta^3 u_x$
75	2459			
		– 441		
80	2018		– 397	
		– 838		457
85	1180		60	
		– 778		
90	402			

From the above difference table, we find that $u_{75} = 2459$, $\Delta u_{75} = -441$, $\Delta^2 u_{75} = -397$, $\Delta^3 u_{75} = 457$

Also Newton-Gregory's formula is

$$y_{a+hu} = y_a + \frac{u^{(1)}}{1!}\Delta y_a + \frac{u^{(2)}}{2!}\Delta^2 y_a + \frac{u^{(3)}}{3!}\Delta^3 y_a, \text{ which here reduces to}$$

$$u_{82} = 2459 + \frac{1.4}{1!}(-441) + \frac{(1.4)(1.4-1)}{2!}(-397) + \frac{(1.4)(1.4-1)(1.4-2)}{3!}(457)$$

(Note)

$$= 2459 - \frac{(1.4)(441)}{1} - \frac{(1.4)(0.4)}{2}(397) + \frac{(1.4)(0.4)(-0.6)}{6}(457)$$

= 2459 – 617.4 – 111.16 – 25.592 = 1704.848 = 1705 nearly.

80.(A) $M = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \middle| \text{a,b are real number} \right\}$

Let $A = \begin{bmatrix} a_1 & b_1 \\ -b_1 & a_1 \end{bmatrix}$ $B = \begin{bmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{bmatrix}$ $C = \begin{bmatrix} a_3 & b_3 \\ -b_3 & a_3 \end{bmatrix}$

A, B, C, $\in$ M

(i) The sum and product of two matrices A and B with their elements as real number

i.e. M is closed with respect to addition and multiplication

(ii) A + (B + C) = (A + B) + C $\forall$ A, B, C $\in$ M

and A, (B, C) = (A, B), C $\forall$ A, B, C $\in$ M

Therefore addition and multiplication of matrices is associative

(iii) Let $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in$ M (null matrix)

i.e. A + O = O + A = A $\forall$ A $\in$ M

(iv) A $\in$ M then there exist – A $\in$ M then

(– A) + (A) = 0 (null matrix)

(v) A (B + C) = A B + A·C , $\forall$ A, B, C $\in$ M

Therefore matrix multiplication is distributive with respect to addition

(vi) Addition of matrices are commutative

But matrix multiplication is not commutative in general AB $\neq$ BA

$\therefore$ {M, +, ·} is a non commutative ring

81. (A) Since U and W are distinct V + W contains U and W properly

and dim (U + W) > 4

But dim (U + W) cannot be greater then 6

Since dim V = 6

Hence we have only two possibilities (a)

dim (U + W) = 5 or dim (U + W) = 6

$d(U \cap W) = \dim U + \dim V - \dim (U + W)$

$= 8 - \dim (U + W)$

$= 3$ or 2

82.(B) B_1 and B_3 are obviously not a basis for R^3 as dim $R^3 = 3$ and basis of R^3 must contain exactly 3 elements

B_2 gives the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix}$

on applying elementary row and column transformation the matrix reduces into echelon form which is

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

$\Rightarrow$ it forms a basis for R^3

But gives the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 5 & 3 & 4 \end{bmatrix}$

By elementary matrix transformations the given matrix reduces to $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ not linearly independent

$\Rightarrow$ does not form a basis

83. (D) Since the number of linear maps from U to V is given by dim [Hom (U, V)] = mn where dim U = m and dim U = x

here $\dim R^5(R) = 5$

$\dim (P_3(t)) = 4$

$\Rightarrow \dim (\text{Hom}(R^5, P_3(t)) = 20$

84. (C) There are two possible cases

Case 1 Five 1's, one 2's, one 3's

Number of numbers $= \frac{7!}{5!} = 42$

Case 2 Four 1's, three 2's

Number of numbers $= \frac{7!}{4!3!} = 35$

Total number of permutations = 42 + 35 = 77

85. (C) $P\left(\frac{E^c \cap F^c}{G}\right) = \frac{P(E^c \cap F^c \cap G)}{P(G)}$

$= \frac{P(G) - P(E \cap G) - P(G \cap F)}{P(G)}$

$= \frac{P(G)[1 - P(E) - P(F)]}{P(G)}$ $[P(G) \neq 0]$

$= 1 - P(E) - P(F) = P(E^c) - P(F)$

86.(D) The differential equation

$\frac{d^2y}{dx^2} - 4y = 1 + x^2$

Auxiliary equation is

$$m^2 - 4 = 0$$

$$(m + 2)(m - 2) = 0$$

$$m = 2, -2$$

$$\text{C.F.} = c_1 e^{2x} + c_2 e^{-2x}$$

$$\text{P.I.} = \frac{1}{D^2 - 4}(1 + x^2)$$

$$= -\frac{1}{4}\left(1 - \frac{D^2}{4}\right)^{-1}(1 + x^2)$$

$$= -\frac{1}{4}\left(1 + \frac{D^2}{4} \ldots\right)^{-1}(1 + x^2)$$

$$= -\frac{1}{4}\left(1 + x^2 + \frac{1}{2}\right)$$

$$= -\frac{3}{8} - \frac{x^2}{4}$$

∴ General solution

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{3}{8} + \frac{1}{4}x^2$$

87.(A) The equation of circle whose centre (a, 0) and radius a

$$(x - a)^2 + (y - 0)^2 = a^2$$

$$x^2 + a^2 - 2ax + y^2 = a^2$$

$$x^2 - 2ax + y^2 = 0 \qquad \ldots(1)$$

Differentiating w.r.t. to x

$$2x - 2a + 2y\frac{dy}{dx} = 0$$

$$2a = 2x + 2y\frac{dy}{dx}$$

The value of 2a put in (1)

$$x^2 + y^2 - x \left(2x + 2y\frac{dy}{dx}\right) = 0$$

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$-x^2 + y^2 - 2xy\frac{dy}{dx} = 0$$

$$\therefore \quad 2xy\frac{dy}{dx} + x^2 - y^2 = 0$$

which is required differential equation

88. (A) Here the auxiliary equation is $m^2 + 16 = 0$

$\therefore$ C.F. = $C_1 \cos 4x + C_2 \sin 4x$, where C_1 and C_2 are arbitrary constants and

$\therefore$ The complete solution is y = C.F. + P.I.

or $\quad y = C_1 \cos 4x + C_2 \sin 4x + (1/12) \sin 2x \quad$ ...(i)

Differentiating both sides of (i) with respect to x, we get

$dy/dx = -4C_1 \sin 4x + 4C_2 \cos 4x + \frac{1}{6} \cos 2x \quad$ ...(ii)

Given that y = 0, dy / dx = 5/6 when x = 0

$\therefore$ From (i) and (ii) we get $0 = C_1$ and $(5/6) = 4C_2 + \frac{1}{6}$

These give $C_1 = 0$, $C_2 = \frac{1}{6}$

$\therefore$ From (i) the required solution is

$y = \frac{1}{6} \sin 4x + (1/12) \sin 2x$ or $12y = 2 \sin 4x + \sin 2x$.

89.(D) P.I. $= \frac{1}{D^2+4D-12}(x-1)e^{2x}$

$= e^{2x}\frac{1}{(D+2)^2+4(D+2)-12}(x-1)$

$= e^{2x}\frac{1}{D^2+8D}(x-1)$

$= e^{2x}\frac{1}{8D\left(1+\frac{1}{8}D\right)}(x-1)$

$= \frac{e^{2x}}{8}\frac{1}{D}\left(1+\frac{1}{8}D\right)^{-1}(x-1)$

$= \frac{e^{2x}}{8}\frac{1}{D}\left[(x-1)-\frac{1}{8}\right]$

$= \frac{1}{64}[4x^2-9x]e^{2x}$

90.(C) (i) Q(2, 5) = 0 since 2 < 5.

(ii) Q(12, 5) = Q(7, 5) + 1

= [Q(2, 5) + 1] + 1 = Q(2, 5) + 2

= 0 + 2 = 2.

91. (C) Let r = x i + y j + z k, then

r × a = b × a ⇒ (r – b) × a = 0

∴ z = – 1, x – y = 2

r × b = a × b

⇒ (r – a) × b = 0

∴ y = 1, x + 2z = 1

$\therefore$ x = 3, y = 1, z = – 1

$\therefore$ r = 3i + j – k

92. (A) $\phi A = (xy^2z)(xz\, i - xy^2\, j + yz^2\, k) = x^2y^2z^2\, i - x^2y^4 z\, j + xy^3z^3\, k$

$$\frac{\partial}{\partial z}(\phi A) = \frac{\partial}{\partial z}(x^2y^2z^2 i - x^2y^4z\, j + xy^2z^3\, k) = 2x^2y^2\, z\, i - x^2y^4\, j + 3xy^3z^2\, k$$

$$\frac{\partial^2}{\partial x \partial z}(\phi A) = \frac{\partial}{\partial x}(2x^2y^2z\, i - x^2y^4\, j + 3xy^3z^2\, k) = 4xy^2\, z\, i - 2xy^4\, j + 3y^3z^2\, k$$

$$\frac{\partial^3}{\partial x^2 \partial z}(\phi A) = \frac{\partial}{\partial x}(4xy^2z\, i - 2xy^4\, j +|\, 3y^3\, z^2\, k) = 4y^2z\, i - 2y^4\, j$$

If x = 2, y = –1, z = 1 this becomes $4(-1)^2(1)\, i - 2(-1)^4\, j = 4i - 2j$.

93.(B) The directional derivation is

$$\frac{\partial f}{\partial s} = l\frac{\partial f}{\partial x} + m\frac{\partial f}{\partial y} + n\frac{\partial f}{\partial z}.$$

Here $l = 3/5\sqrt{2}$, $m = 4/5\sqrt{2}$, $n = 5/5\sqrt{2}$ and

$$\frac{\partial f}{\partial x} = 2x = 2,\ \frac{\partial f}{\partial y} = 2y = 4,\ \frac{\partial f}{\partial z} = 2z = 6 \text{ at } (1,2,3).$$

$\therefore$ then required directional derivative is $df/ds = 52/5\sqrt{2}$.

Also, grad f $= i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z} = 2i + 4j + 6k$.

$\therefore$ Then maximum rate of increase of f is | grad f | = $2\sqrt{(14)}$.

94.(D) We have V = [a b c] = 4

$$\therefore \quad \begin{vmatrix} 2 & -1 & -1 \\ 3 & 2 & 2 \\ 5 & -\lambda & 3\lambda \end{vmatrix} = 4$$

or $28\lambda = 4$ and so $\lambda = 1/7$

95.(D) It may be easily shown that

$$f_x(0, 0) = 0 = f_y(0, 0)$$

Also when $x^2 + y^2 \neq 0$

$$|f_x| = \frac{|x^4y + 4x^2y^3 - y^5|}{(x^2+y^2)^2} \leq \frac{6(x^2+y^2)^{5/2}}{(x^2+y^2)^2}$$

$$= 6(x^2 + y^2)^{1/2}$$

Evidently

$$\lim_{(x,y)\to(0,0)} f_x(x, y) = 0 = f_x(0, 0)$$

Thus, f_x is continuous at (0, 0) and $f_y(0, 0)$ exists,

$\Rightarrow$ f is differentiable at (0, 0).

96.(B) Out of the numbers 10, 11, 12, 13, ..., 99 those numbers the product of whose digits is 12 are 26, 34, 43, 62 i.e., only 4.

$$\therefore \quad p = P(E) = \frac{4}{90} = \frac{2}{45}$$

$$q = P(\bar{E}) = 1 - P(E) = 1 - \frac{2}{45} = \frac{43}{45}$$

Hence, the probability that he will laugh atleast once

$$= 1 - q^3$$

$$= 1 - \left(\frac{43}{45}\right)^3$$

97.(B) Here, $p = \frac{1}{2}$, n = 8

$\therefore \quad q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ The binomial distribution is $\left(\frac{1}{2} + \frac{1}{2}\right)^8$

Also, $|x - 4| \le 2$

$\Rightarrow -2 \le x - 4 \le 2$

$\Rightarrow \quad 2 \le x \le 6$

$\therefore p(|x - 4| \le 2) = p(x = 2) + p(x = 3) + p(x = 4) + p(x = 5) + p(x = 6)$

$= {}^8C_2\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + {}^8C_3\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^3 + {}^8C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 + {}^8C_5\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 + {}^8C_6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6$

$= \frac{{}^8C_2 + {}^8C_3 + {}^8C_5 + {}^8C_6}{2^8}$

$= \frac{238}{256} = \frac{119}{128}$

98.(C) $P(A/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

$= \frac{P(\bar{A} \cup \bar{B})}{P(\bar{B})}$

$= \frac{1 - P(A \cup B)}{P(\bar{B})}$

99.(A) Since, $P(X = 2) = P(x = 3)$, we get

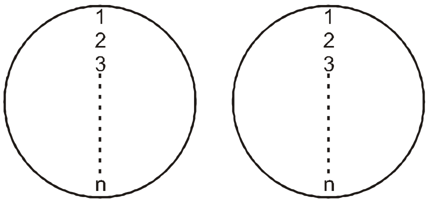
${}^5C_2 p^2 q^3 = {}^5C_3 p^3 q^2$

$\therefore \quad q = p \qquad (\therefore q = 1 - q)$

$1 - p = p$

$\therefore \quad p = 1/2$

100.(C) Total number of cases = n^n



We next find the number of favorable cases. For the first element we have n choices. For the second element we have (n – 1) choices and so on.

$\therefore$ The number of favorably cases

$= n (n-1) (n-2)... 2\cdot1$

$= n!$

$\therefore$ Required probability $= \frac{n!}{n^n}$

$= \frac{(n-1)!}{n^{n-1}}$

101.(B) Total man hours work done by men supplied by A, B, C = $(20 \times 8 \times 6)$, $(15 \times 9 \times 7)$, $(10 \times 6 \times 8)$

and the wages must be in the ration of work done.

So Rs. 636 has to be divided among A, B, C in the ratio

$(20 \times 8 \times 6) : (15 \times 9 \times 7) : (10 \times 6 \times 8) = 64 : 63 : 32$

$\therefore$ C's share = $\frac{32}{159} \times 636 = 128$

102.(C) A did the work for 20 days, B did the work for (20 + 12 + 28) = 60 days, C did the work for 28 days.

Let C alone can complete the work in x days.

Fraction of work did by A + Fraction of work did by B + Fraction of work did by C = 1

or $\frac{20}{80} + \frac{60}{120} + \frac{28}{x} = 1$

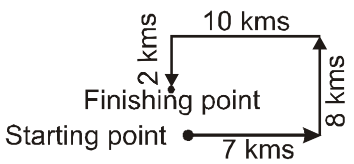
or $x = 28 \times 4 = 112$ days.

C alone can complete the work in **112 days**

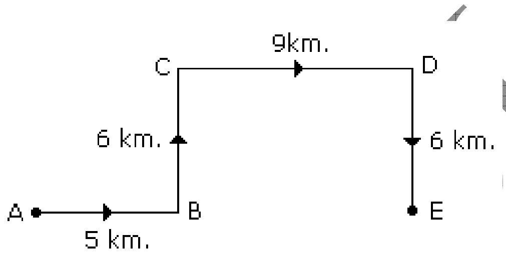
103.(A) Pattern is $1^3 + 1$, $2^3 - 2$, $3^3 + 3$, $4^3 - 4$, $5^3 + 5$, ... Missing number

= 216 - 6 = 210.

104.(A)



105.(C)



Required distance = AE

= 5 + 9

= 14 km.

106.(B) An organization like UNO is meant to maintain peace all over and will always serve to prevent conflicts between countries. So, its role never ends. So, argument I does not hold. Also, lack of such an organization may in future lead to increased mutual conflicts and international wars, on account of lack of a common platform for mutual discussions. So, argument II holds.

View Answer Workspace Report Discuss in Forum

107.(A) Clearly, besides interview, there can be other modes of written examination to judge candidates' motives. So argument II is not strong enough. However, the interview is a subjective assessment without doubt. So, argument I holds.

108.(D) Clearly, the distance of each village from Rampur is given in I and II. But nothing about their relative positions is mentioned. So, the distance between the two villages cannot be calculated.

109. (B) The pattern is :

A -2 = Y

P+1 = Q

P-2 = N

R+1 = S

O-2 = M

A+1 = B

C-2 = A

H+1 = I

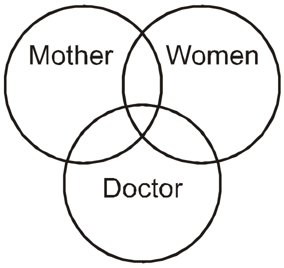
Similarly, “VERBAL” will be written as “TFPCYM”.

110.(A) Some women may be mother.

Some mothers may be doctor.

Some doctors may be women.

Therefore, the correct figure is :



111.(B) The proper order of the words is as follows :

accident, doctor, Police,
1 3 5
Lawyer, Judge
4 2

112.(D) The proper order of the words is as follows :

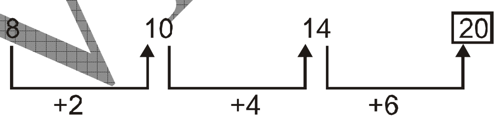
Bud, Flower, Pollination,
5 2 4
Fruit, Seed
1 3

113.(C) The order of the given series is as follows :

m b b / m a a / m b b / m / a a / m bb

Therefore, the required letters are 'm a b a m'.

114.(A) The order of the given numbers in the question figure is as follows :



∴ ? = 20

115.(D) As, $2 \times 5 \times 1 = 20$

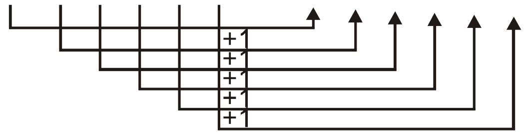
and $4 \times 3 \times 6 = 72$

Similarly, $7 \times 2 \times ? = 42$

$\therefore$ $? = \frac{42}{14} = 3$

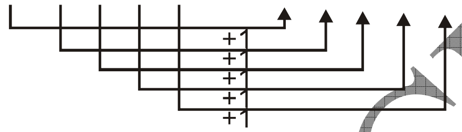
116.(A) As,

B U T T E R → C V U U F S



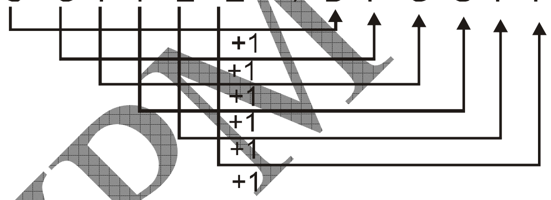
and

B R E A D → C S F B E



In the same way,

C O F F E E → D P G G F F



117.(B) On comparison

C L O U D and R A I N

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

5 9 4 3 2 1 6 7 8

A R O U N D

Then, ↓ ↓ ↓ ↓ ↓ ↓

6 1 4 3 8 2

118.(A) In all the rest, the first thing is kept inside the second, while pencil is used to write.

119.(B) There are the stars in the sky, players in the stadium and students in the university, But moon is not in the planets.

120.(A) According to question,

B F J Q
+4 +4 +7

R U Z G
+3 +5 +7

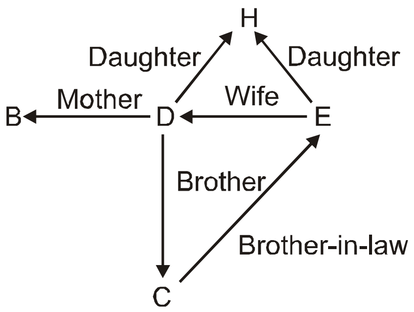
G J O V
+3 +5 +7

I L Q X
+3 +5 +7

Therefore, B F J Q is odd.

121.(B) In all the rest, the first number is thrice of the second. While in the alternative (B) it is four time.

122.(B) According to question



Therefore, 'E' is the brother-in-law of 'C'.

123.(B) Only alternative (B) diagram does not imply according to the given statement because it represents some female are only member and some female are only doctor, but some female are both doctor and as well as member, does not represent like this.

124.(C) As, from the given set

$4 \times 2 = 8 + 2 = 10$

and $4 \times 3 = 12 + 3 = 15$

In the same way,

$5 \times 2 = 10 + 2 = 12$

and $5 \times 3 = 15 + 3 = 18$

125.(C) There is no data about the use of a compass in modern ships. Therefore, we can only say that this statement is uncertain.

126.(A) As the lens is prime in the camera, In the same way bulb is prime in the flash.

127.(A) As house is given at rent, in the same way capital is given at interest.

128.(C) As

N U M B E R U N B M R E

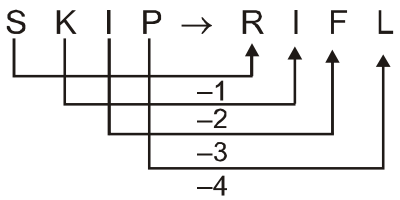
1 2 3 4 5 6 $\rightarrow$ 2 1 4 3 6 5

In the same way,

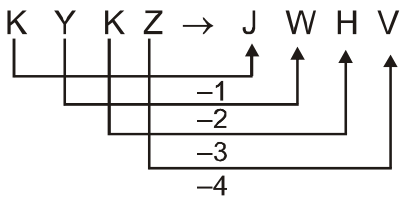
G H O S T → H G S O T

1 2 3 4 5 → 2 1 4 3 5

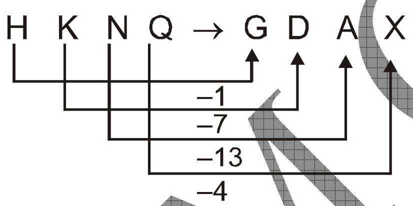
129.(D) As,



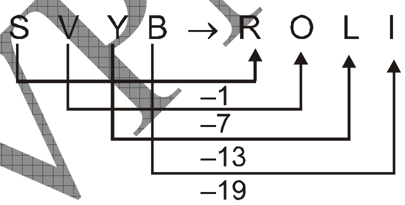
In the same way,



130.(B) As,



In the same way,



131.(B) As,

19 ⇒ 19 × 2 – 1 = 37

In the same way,

$26 \Rightarrow 26 \times 2 - 1 = \boxed{51}$

132.(A) As,

$CE \Rightarrow C \times E \Rightarrow 3 \times 5$

$\Rightarrow 15 \times 4 + 10 = 70$

In the same way,

$DE \Rightarrow D \times E \Rightarrow 4 \times 5$

$\Rightarrow 20 \times 4 + 10 = 90$

133.(D) The order of the given letter series is as follows :

$P \xrightarrow{+4} H \xrightarrow{+4} L \xrightarrow{+4} P$

$I \xrightarrow{+4} M \xrightarrow{+4} Q \xrightarrow{+4} U$

$B \xrightarrow{+4} F \xrightarrow{+4} J \xrightarrow{+4} N$

∴ ? = PUN

134.(C) The order of the given number series is as follows :

$1 \xrightarrow{\times 4} 2 \xrightarrow{\times 4} 4 \xrightarrow{\times 2} 8 \xrightarrow{\times 2} \boxed{16}$

∴ ? = 16

135.(D) The Commissioner only cites examples of cities X and Y and undertakes to beautify city Z. This does not imply that he has worked in cities X and Y. So, I do no follow. Also, nothing about people's response to the state of the city can be deduce from the statement. Thus, II also does not follow.

136.(B) Chaaru is Bomans's paternal grandmother and Aliya's maternal grandmother.

Therefore, chaaru is the mother of Boman's father and Aliya's mother.

Dinkar is Boman's maternal grandfather and Aliya's paternal grandfather.

Therefore, Dinkar is the father of Boman's mother and Aliya's father.

Now, Fenil is Aliya's father and Geet is Boman's mother. Esha is the mother of Fenil and Geet.

Therefore, Esha is Dinkar's wife.

Now, Hitarth is Fenil and Geet's father-in-law. So Hitarth is the father of Fenil's wife and Geet's husband.

Therefore, Hitarth is Chaaru's husband.

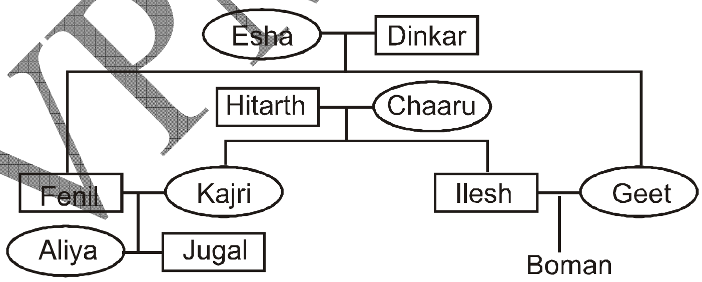
Ilesh is Geet's husband. So Fenil is Ilesh's brother-in-law. Jugal is Ilesh's brother-in-law's son.

Since Ilesh has only one brother in law, Jugal has to be Fenil's son. Jugal is also Kajri's son. Therefore, Kajri is Fenil's wife.

The data mentions "Her cousin Boman" with respect to Aliya.

Hence, Aliya has to be female while Boman's gender is unclear.

Thus, the final family tree is as shown below. Though Hitarth and Chaaru are shown below Esha and Dinkar, they belong to the same generation. This representation is just to ensure ease in drawing the family tree.



Now, Dinkar's daughter is Geet and Jugal's father is Fenil.

From the family tree, Geet is fenil's sister.

137.(D) From the family tree in the solution to the first problem, Esha chaaru, Kahri, Geet and Aliya are definitely females.

However, Boman's gender is not known.

Hence the total number of females in the family can be either 5 or 6.

Hence, a unique number cannot be determined.

138.(D) From the family tree in the solution to the first problem, Hitarth is Boman's paternal grandfather. At the same time, Hitarth is also kajri's father.

139.(D) From the family tree in the solution to the first problem, Aliya, Jugal and Boman are the grandchildren of Esha.

However, Boman's grandfather is not known.

Hence, the exact number of grandsons cannot be found out.

It can be either 1 or 2.

140.(C) From the family tree in the solution to the first problem, Boman's aunt is Kajri.

Kajri's in-laws are Dinkar and Esha.

141.(B) All the people mentioned (except students) repair and mend things.

142.(B) All the items mentioned (apart from books) are edible.

143.(D) Let the heaviest planet is numbered 1 and the lightest planet is numbered 6.

Hence, the third lightest planet will correspond to the number 4.

It is given that the number of planets lighter than Mars was equal to the number of planets heavier than Venus.

Note that if Mars is the lightest and Venus is the heaviest, there is no planet lighter than Mars or heavier than Venus.

Even in this case, the number of planets lighter than Mars is equal to the number of planets heavier than Venus.

Hence, the positions of Mars and Venus can be one of (1,6), (2,5), (3,4), (4,3), (5,2), (6,1).

Let the weights of planets Jupiter, Mars, Mercury, Saturn, Venus and Pluto be j, ma, me, s, v, and p respectively.

From the data given,

s> ma > v and me > p

Hence, Mars cannot be the heaviest planet and the combination (1,6) is ruled out.

Moreover, since Mars is heavier than Venus, the positional number for Mars has to be less than that of Venus.

Hence, the combinations (4,3), (5,2) and (6,1) are also ruled out.

Now, Saturn is not the heaviest planet.

Hence, Mars cannot be the 2nd heaviest planet.

Hence, the combination (2,5) is also not correct.

Hence, the correct positions of Mars and Venus are 3 and 4 respectively.

Hence, Mars is the 3rd heaviest planet; Venus is the 4th heaviest plant is also the 3rd lightest plant.

The 4th heaviest planet is also the 3rd lightest planet.

Hence, Venus is the 3rd lightest planet.

144.(C) If Jupiter is the heaviest planet, j = 1.

Also, from the previous solution s = 2, ma = 3 and v = 4.

Also, it is given that Mercury is heavier than Pluto.

Hence, me = 5 and p = 6.

Hence, Pluto is the lightest planet.

145.(C) Statement 3 and 4 contain contradictory statements regarding Charak securing a rank among the top three. Hence, this is a good starting point for assumption.

Assuming that Charak had secured a rank among top three would imply that the second part in statement 3 would be false, I.e., Deepak would then have secured a rank among top three.

From statement 4: since the second part would be false, it implies that the first part of this statement has to be true, i.e., Ajay must have secured a rank among top three.

Hence, as per the assumption, Ajay, Charak an Deepak should be in the top three while Binoy and Goldy should not.

From statement 5: The first part would then be true thereby leading to a conclusion that Goldy did not secure rank among top three. This is in line with the above conclusion.

From statement 2: The second part has to be false, it was Deepak who must have secured to 2nd rank.

From statement 1: Since Deepak had secured 2nd rank it implies that the second part of the statement is true thereby leading to a conclusion that the first part is false and so Ajay must have secured 1st rank.

Since Ajay, Deepak and Charak are in the top three, Charak secured the 3rd rank.

146.(A) From the solution to the previous question, Ajay secured the 1st rank.

147.(B) It is likely that disputes between two nations would be solved by the United Nations according to the given data. However, there is no direct evidence that they are actually solved by the UN. This statement is highly (not definitely) likely to be true.

148.(B) One can easily notice that arguments A and D are not directly related to the working hours of the government owned banks and hence can be marked as weak arguments. Argument D misses out on establishing a connection between large number of banking customers not able to execute their banking needs within the stipulated time due to overcrowding or their requirement of extended banking hours.

Argument B is both directly connected and also has an important reason, that of the loss of customers to private banks.

Argument C is also directly related as the reduction in efficiency can adversely affect the working of the bank. Therefore, C is also a strong argument.

149.(A) Argument C may appear to have an important reason, but one should understand that though directly connected to the issue, it talks about only one part of the issue at hand. MBA colleges are not the only institutes of higher education.

India having the most coaching institutes in the world is of no consequence to this issue at hand. Hence, arguments C and D are weak.

Argument A talks about the money savings and B talks about the quality of higher educations, both of which are directly connected and are important reasons. Hence, arguments A and B are strong.

150.(C) Football, Players and Field all are different. Therefore,